

LECTURER NOTES ON STRUCTURAL DESIGN-I



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Introduction to design and detailing

✓ Stability

To prevent over turning, sliding or buckling of the structure under the action of load.

(2) Strength

To resist safely the stresses induced by the load in the various structural members.

(3) Serviceability

Serviceability to ensure satisfactory performance of the under service load condition i.e. (that is) providing adequate stiffness to contain deflection, vibration within acceptable limit and also providing impermeability durability.

Date: 17.2.23

Advantages of Reinforced Concrete

(1) Reinforced cement concrete has good compressive stress (because of concrete)

Tensile stress (steel)	compressive stress (concrete)
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(2) RCC also has high tensile stress (because of steel).

(3) It has good resistance to damage by fire & weathering (because of concrete)

(4) RCC protect steel bars (68's piece) from buckling and twisting at high temperature.
(Rotate)

(5) RCC prevents steel from rusting. (68)

(6) Reinforced concrete is durable.

(7) Maintenance of RCC is practically nil (zero).

Different Method of Design

- (1) Ultimate load theory
- (2) Working stress Method (WSM) (Before 1980)
- (3) Limit state Method (IS 456:2000) → (Now)

Working stress Method (Comparison of Limit state)

↳ This method of design is based on linear elastic theory.

↳ In working stress method (WSM) it is assumed that structural material i.e. concrete & steel behave in linear elastic manner & safety can be ensured by restricting the stress in the material induced by working load or service load on the structure.

Imp
↳ The ratio of strength of material to the permissible stress is called as factor of safety.

• Permissible = $\frac{\text{Ultimate stress } f_{ck}}{\text{Factor of safety}}$

↳ While applying WSM the stress under applied loads are analysed by simple bending theory where strain compatibility is assumed.

Ultimate load theory / Method

↳ With increased understanding of the behaviour of reinforced concrete at ultimate load, method of design (ULM) evolved (now) in 1950 and became an alternative to WSM.

↳ This method is also called as load factor method or ultimate strength method.

Imp

In this method the stress condition at the state of collapse of the structure is analysed & non-linear stress-strain curve of concrete & steel and use of modular ratio.

$$m = \frac{E_s}{E_c}$$

The ratio of elastic modulus of steel to the elastic modulus of concrete.

Modular is defined as the ratio of elastic modulus of steel to the elastic modulus of concrete.

The safety measure in the design is introduced by a load factor, defined as (the ratio of ultimate load to the working load) Factor of Safety.

This method generally results in more slender section and more economical design of beams and columns (compared to WSM).

20.2.2023

Limit State Method (LSM)
Limit state method is a design represents for a comprehensive and rational solution to the design problem by considering safety at ultimate load and serviceability at working load.

The LSM method uses a multiple safety factor which attempts to provide adequate safety at ultimate load as well as adequate serviceability at service load by considering all

possible limit state,

Chapter - 1

Working stress method of Design.

Introduction

- Working stress method is based on the behaviour of a section under the load encountered as to be survived period. by it during its concrete in the tension zone of the member is neglected.
- The material both concrete & steel are assumed to behave perfectly elastic.
- The strain distribution across a section is assumed to be linear.
- The section that are plane before bending remain plane after bending.
- Stress at any point is proportional to the distance of the point from the neutral axis.
- The distance between lines of action of resultant compressive forces is known as lever arm.

Material Factor of Safety

i) concrete $\sigma_{cbc} = 3.0$

ii) steel $\sigma_{st} = 1.78$

Assumption of NSM sm imp (same)

i) Concrete E_c assume to be homogeneous

ii) At any cross-section, plane section before bending remain plane after bending.

iii) The stress strain relationship for concrete E_c is a straight line under working load.

iv) The stress strain relationship for steel E_s is a straight line under working load.

v) Concrete area on tension side is assume to be in-effective.

vi) All tensile stress are taken of up by reinforcement.

vii) The steel area is assume to be concentrated at the centroid of the steel.

viii) The modular ratio has the value $m = \frac{E_s}{E_c} = \frac{280}{80} = 3.5$
where, σ_{cbc} is permissible stress in compression due to bending in concrete in N/mm^2 .

Moment of Resistance

(Resistance) \times (Scale)

Reinforced Cement Concrete

Reinforced cement concrete is a composite material comprising of concrete and steel reinforcement in which concrete is relatively low in tensile strength & ductility are counteracted by the inclusion of addition of reinforcement having higher tensile strength and ductility.

Ingredients / composition of RCC

- (1) Aggregate
- (2) Cement
- (3) Steel / Reinforcement
 - ↳ Steel reinforcement used in cement concrete is of following types.
 - (i) Mild steel (IS 432 (part - 1))
 - (ii) Medium tensile steel
 - (iii) HYSR (High yield strength Deformation)
 - (iv) Hard drawn steel wires

Loads on RCC

The various loads acting on the concrete structure are

- (1) Dead load (D.L)
- (2) Live load (L.L) / Imposed load
- (3) ~~Imposed load~~
- (3) Wind load
- (4) Earthquake load (भूकम्प) measure (Richters Scale)
- (5) Erection load
- (6) Snow load
- (7) Temperature load

Draw backs of Working Stress Method

OR Disadvantage

- (i) It fails to provide uniform over load capacity for all the parts & types of structure.
- (ii) It does not take into account the non linear relationship between stress and strain.
- (iii) It also does not consider the redistribution of forces & moments in statically indeterminate structure.
- (iv) This method does not provide true margin for factor of safety.
- (v) Effect of creep and shrinkage of concrete is ignored.

Grade of Concrete

Concrete grades are expressed by letter M followed by a number.

The letter M refers to the mix and the number represents the characteristic compressive strength of concrete in N/mm^2 after 28 days strength.

What is characteristic compressive strength?
Characteristic compressive strength means there is 95% probability of concrete does not fail given compressive strength.

<u>Group</u>	<u>Designation</u>	<u>Characteristics Compressive Strength f_{ck} (N/mm²)</u>
Ordinary Concrete	M10	10
	M15	15
	M20	20
Standard concrete	M25	25
	M30	30
	M35	35
	M40	40
	M45	45
	M50	50
	M55	55
High strength concrete	M60	60
	M65	65
	M70	70
	M75	75
	M80	80

<u>Grade of Steel</u>	<u>Grade</u>	<u>Characteristic Strength (N/mm²)</u>
Mild steel (250)	Fe 250	250
High strength (High) deformed steel	Fe 415	415
	Fe 500	
	Fe 550	
Thermo mechanically treated bars (TMT) or Corrosion Resistant steel (CRS)	Fe 500	500
Steel welded fabric	Fe 480	480

Wrought Iron

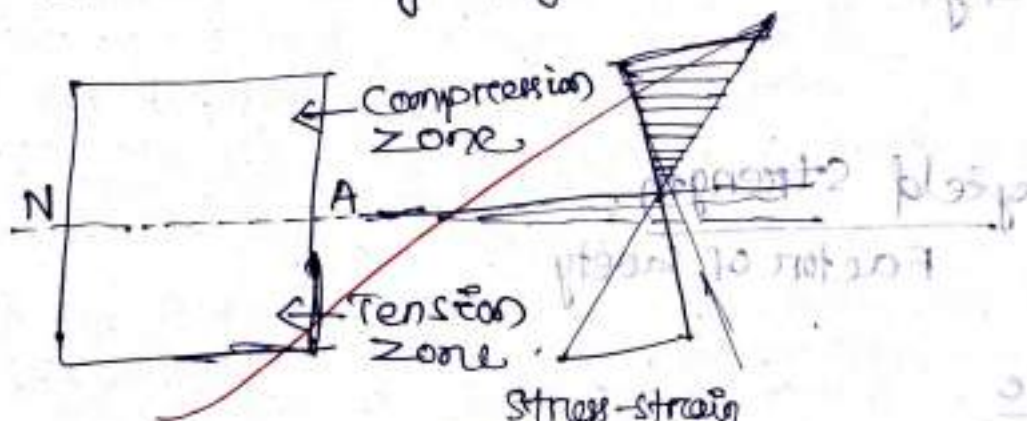
(high carbon

5%)

Dt^o 22.02.23

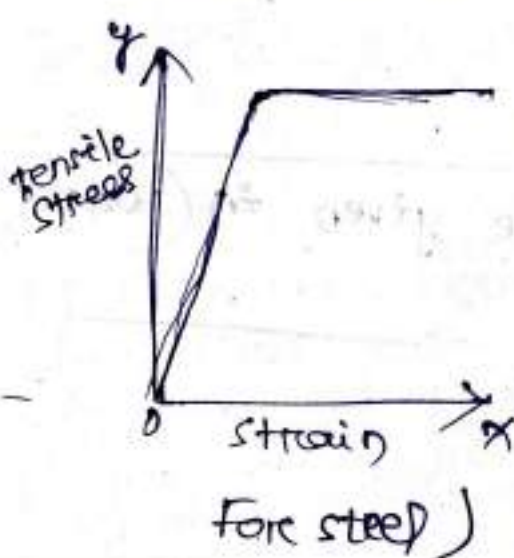
Behaviour of Concrete and steel in W.S.M

- (i) In R.C.C. both concrete and steel resist tension at small loads.
- (ii) As the load increases the concrete at the bottom of the section will reach tensile stress at which cracks occur.
- (iii) The compression stress distribution in concrete as the load increases is given in the following figure.

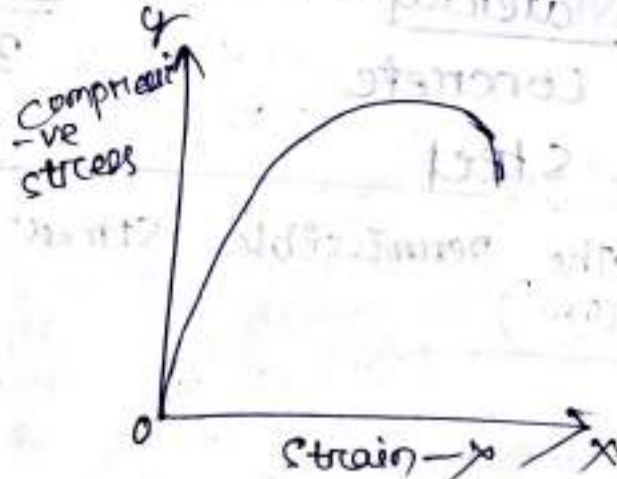


Stress-strain diagram of concrete

- (iii) The stress strain diagram of concrete is not linear. Since stress in the concrete do not increase with increase in strain.



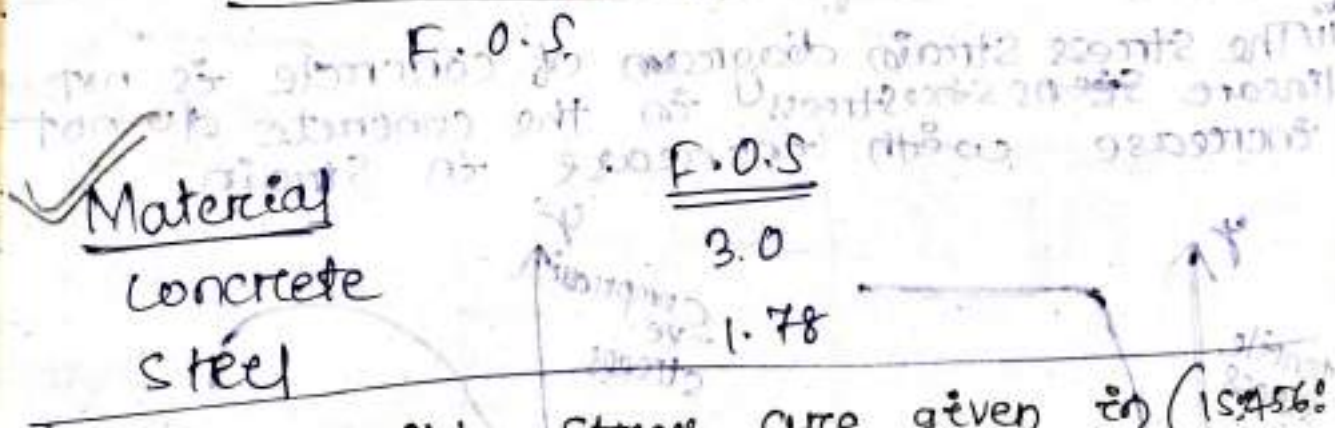
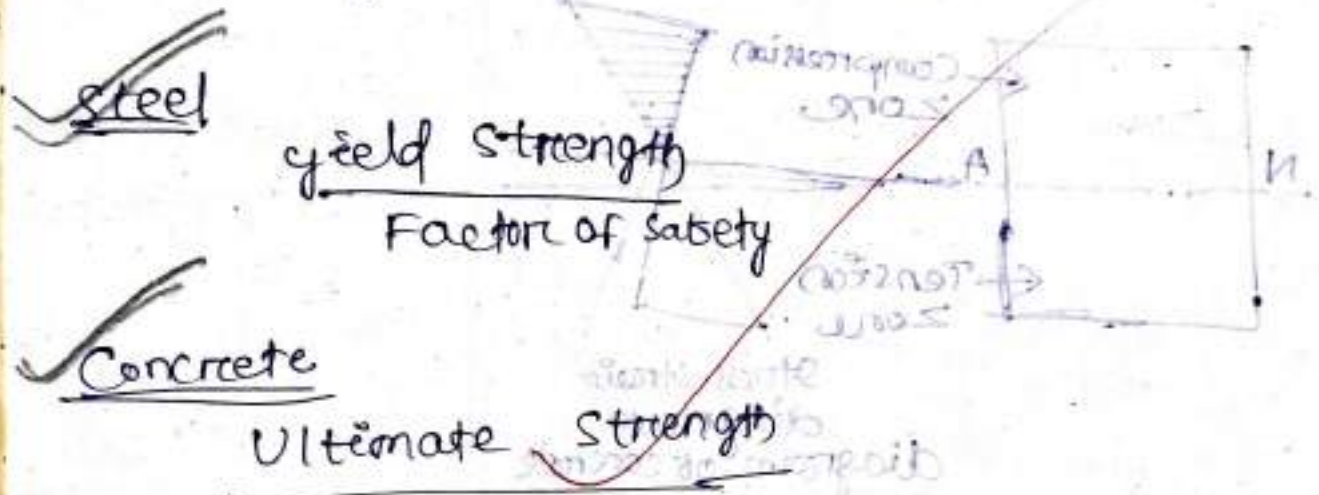
For steel



For concrete

Permissible stress

- i) The failure of the structure will occur at a much higher low.
- ii) The stresses of concrete & steel in a structure designed by working stress method are not allowed to exceed some specified value of stresses known as permissible stress.
- iii) The WSM is based on the concept of permissible stress.
- iv) Permissible stresses are obtained by dividing ultimate strength of concrete or yield strength of steel by factor of safety.



v) The permissible stress are given in IS 456: 2000

Permissible stresses in concrete

1) The permissible stress in concrete in direct tension is denoted by f_{td} . The value of f_{td} for members in direct tension for different grades of concrete are given in table C.1.1 of IS 456 (Pg No. 80)

2) Actual tensile stress of concrete, f_{td} in such members shall not exceed respective permissible value of f_{td} to prevent any crack. The factor safety of concrete in direct tension is from 8.5 to 9.5.

3) The permissible stress of concrete in bending compression f_{bc} in direct compression on RCC and average bond for plain bar in tension f_{bd} are given in table no-21 of IS 456 (Pg No. 81) for different grade of concrete.

4) The factor safety of concrete in bending compression, direct compression & average bond for plain bar are 3.4 & 25 to 35. For plain bar in compression, the value of average bond stress are obtained by increasing respective value in tension by 25%.

5) For deformed bar, the value of table no 21 are to be increased by 60%.

Permissible stresses in steel Reinforcement

1) Permissible stresses in steel reinforcement for different grade of steel, diameter of bar & type of stress in steel reinforcement are given in table no 22 of IS 456
Pg no. 82

(ii) Permissible stresses of steel of grade Fe 250, Fe 415 in tension (σ_{st} & σ_{sh}) & Compression in column (σ_{cc}) are given in that table.

Modular Ratio (m)

Dt: 27.02.23

The ratio of modulus of elasticity to modulus of steel to modulus of elasticity of concrete.

$$m = \frac{E_s}{E_c} \quad (\text{Limit state Method})$$

Consider the long term effect, such as creep, modular ratio is taken as (Working state Method)

$$m = \frac{280}{3 \times \sigma_{cbc}} \quad \sigma_{cbc} = \text{Permissible compressive stress}$$

Advantage of concrete

- ↳ Ingredients of concrete are easily available in most of the places.
- ↳ Concrete can be manufactured to the desired strength with an economical economy.
- ↳ The durability of concrete is very high.
- ↳ It can be cast to any desired shape.
- ↳ The maintenance cost of concrete is almost negligible.
- ↳ Concrete makes a building fire proof.
- ↳ Concrete is resistance to wind & water.

Disadvantages of concrete

- ↳ The tensile strength of concrete is low.
- ↳ Concrete is less ductile.
- ↳ The weight of concrete is very high.

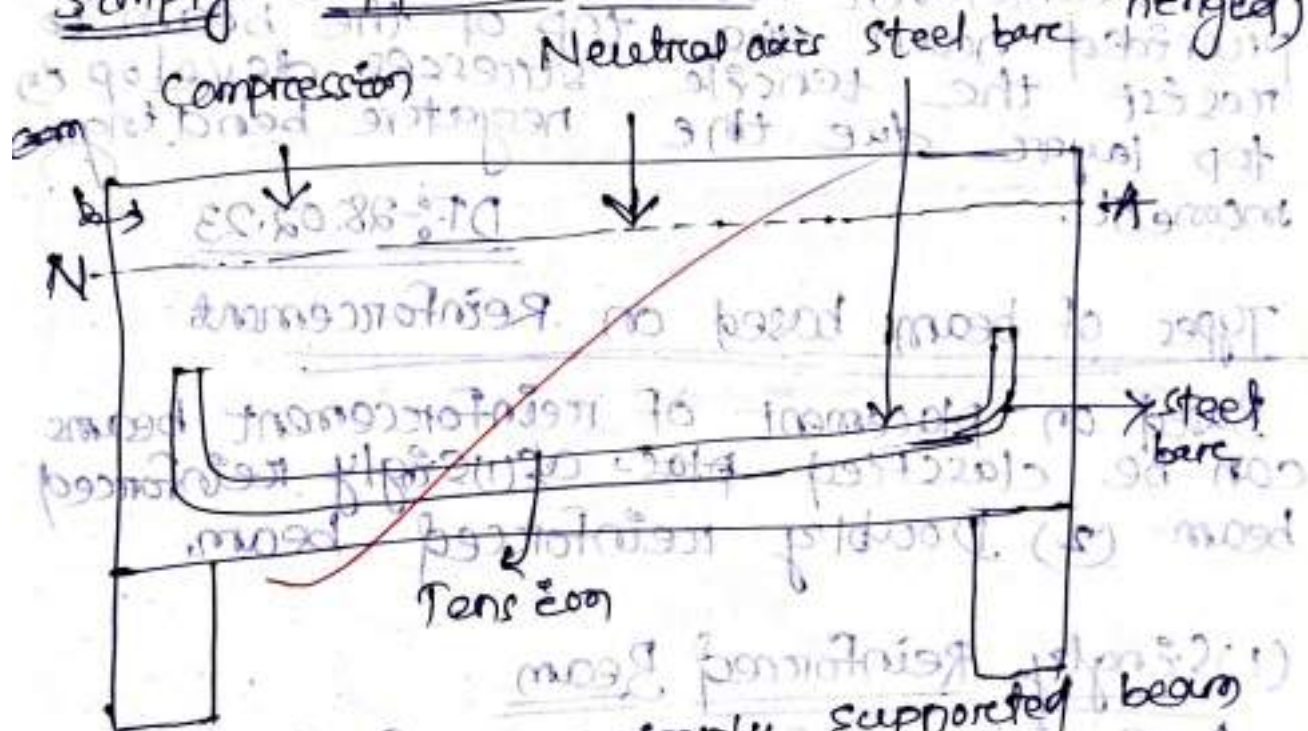
Beam

A beam is a structural element i.e. capable of with standing load primarily by resisting bending.

- (ii) Beams are longer in length compared to its cross-sectional dimension.

Simply supported beam

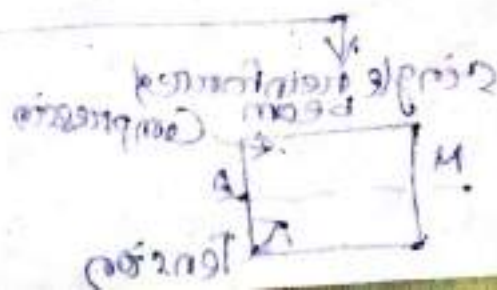
(Both ends supported or hinged)

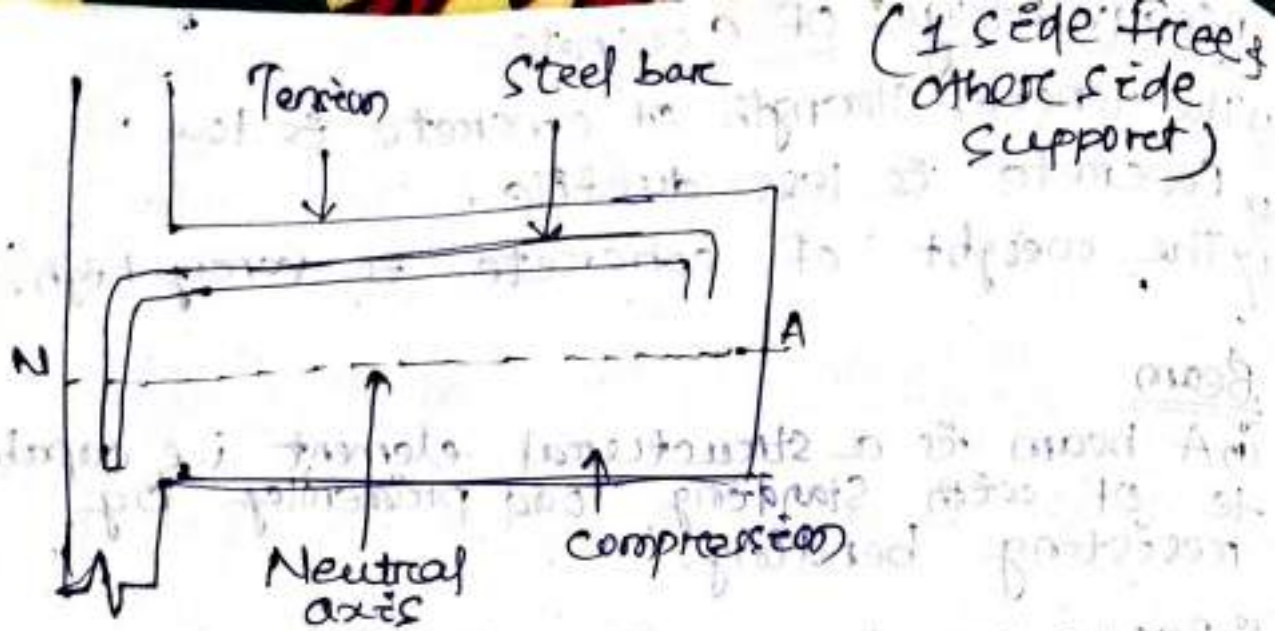


Reinforcement in simply supported beam

In simply supported beam tensile stresses are induced in bottom layers because of positive bending moment of hence steel bars are provided near the bottom of the beam.

Cantilever beam





Reinforcement in cantilever beam

In a cantilever beam steel bars are provided near the top of the beam to resist the tensile stresses develop in top layer due to the negative bending moment.

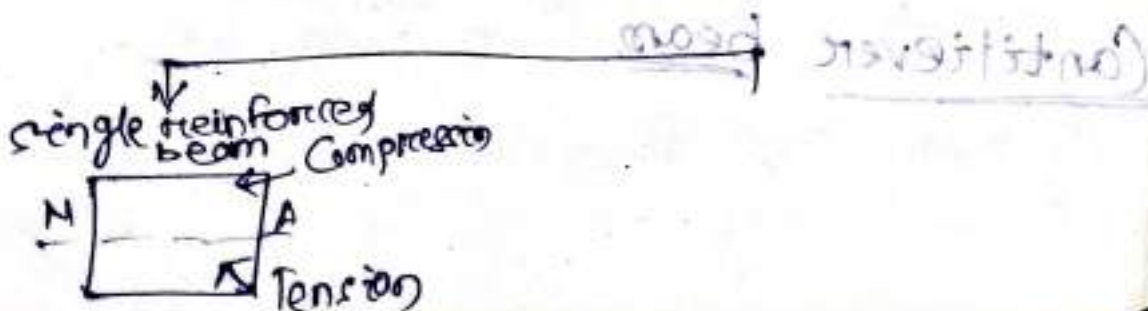
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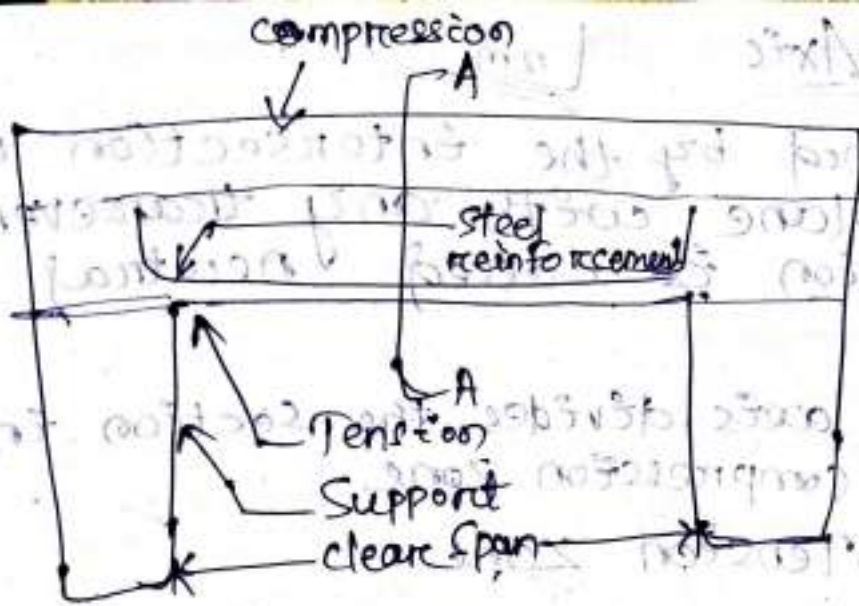
Types of beam based on Reinforcement

Based on placement of reinforcement beams can be classified as (1) singly reinforced beam (2) Doubly reinforced beam.

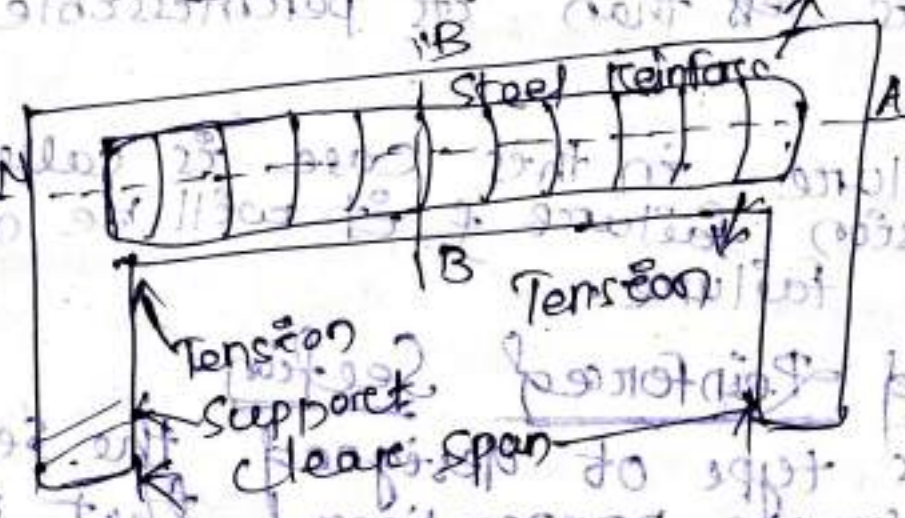
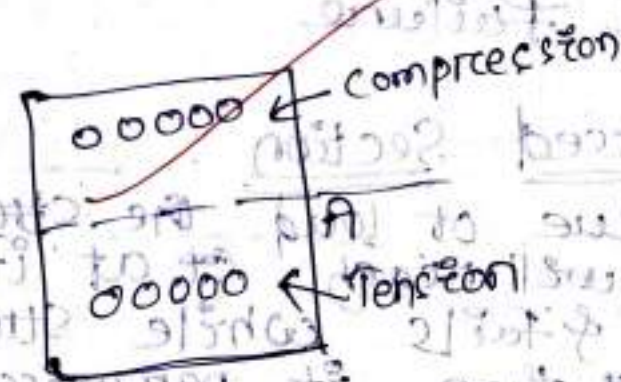
(1) Singly Reinforced Beam

A singly reinforced beam is the type in which concrete element is reinforced only near the tensile zone & that reinforcement is called as tension reinforcement which is designed to resist the tension.





(ii) Doubly reinforced beam
 A doubly reinforced beam is the type of reinforced concrete element in which besides the tensile reinforcement, the concrete zone near the compressive zone is also reinforced with compression steel.




Neutral Axis

2m

Line formed by the intersection of neutral plane with any transverse cross-section is called neutral axis.

Neutral axis divides the section in two zones

- 1) Compression zone
- 2) Tension zone



Under Reinforced Section

Steel failure
Permissible value

i) The stress in steel will reach at its permissible or design value & fails while concrete stress is less than its permissible value.

ii) The failure in this case is a tension or ductile failure.

Over Reinforced Section

i) At some value of load the strain stress in concrete will reach at its permissible value & fails while stress in steel is less than its permissible value.

ii) The failure in this case is called compression failure & it will be a brittle failure.

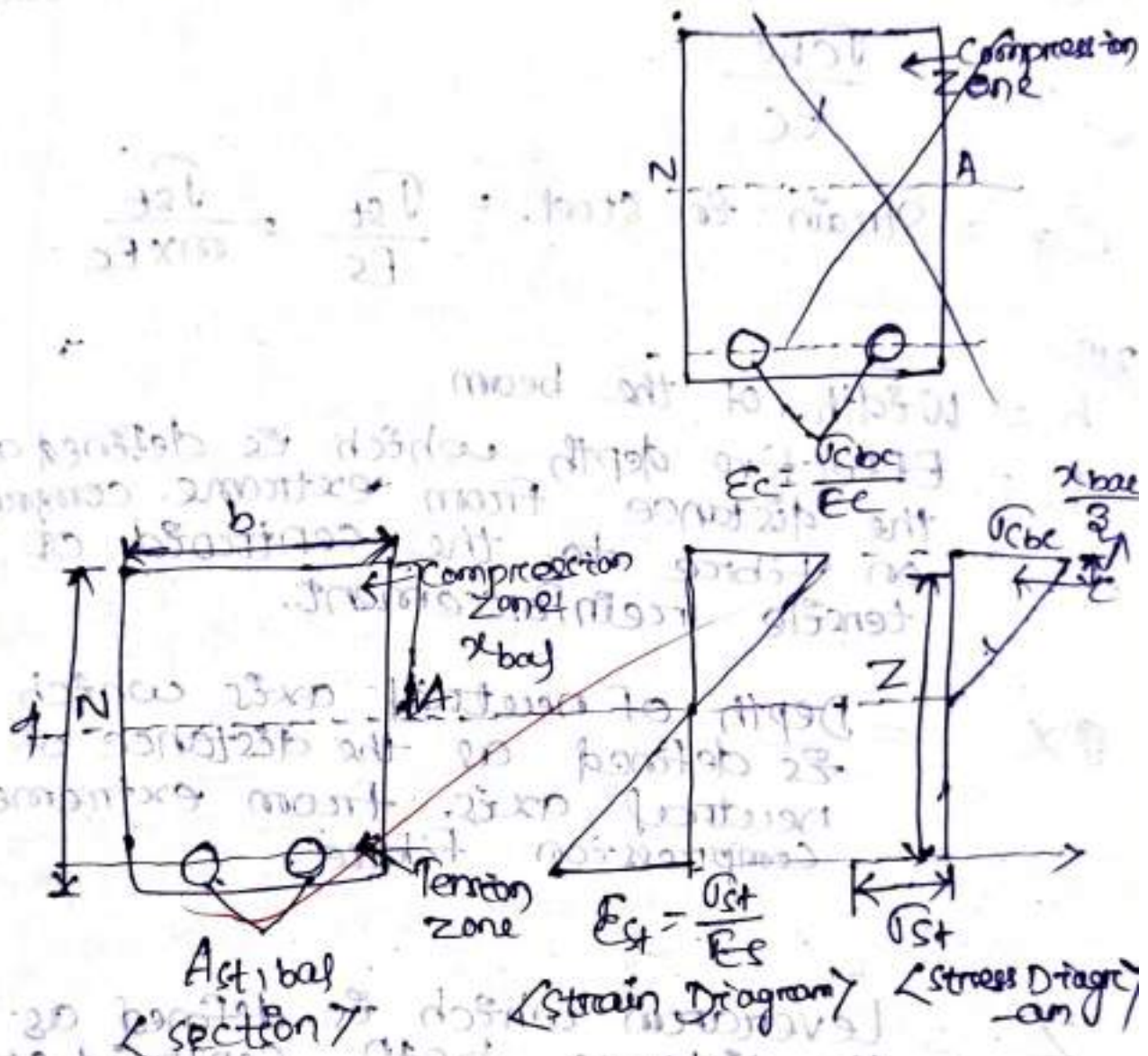
Balanced Reinforced Section

In this type of designed, the section is so proportional that the steel & concrete both reach their max^m value of stress at same time.

Thus at some of load both the material will fail at the same time

Date 1.03.2023

Derivation of formula for Balanced Design



Consider a singly reinforced beam with stress & strain diagram as shown in figure.

As balanced = A_{st}, b, d = Reinforcement are provided for balanced section

σ_{cbc} = permissible stress in concrete in bending compression. (page No. 81)

σ_{st} = permissible stress in steel in tension.

E_c = Modulus of elasticity of concrete

E_s = Modulus of elasticity of steel

ϵ_c = Strain in concrete in extreme fibre

$$\frac{\sigma_{cbc}}{E_c}$$

$$\epsilon_{st} = \text{Strain in steel} = \frac{\sigma_{st}}{E_s} = \frac{\sigma_{st}}{m \times E_c}$$

2m

b = Width of the beam

d = Effective depth which is defined as the distance from extreme compression fibre to the centroid of tensile reinforcement.

x = Depth of neutral axis which is defined as the distance of neutral axis from extreme compression fibre.

Z = Lever arm which is defined as the distance betⁿ centroid of compressive force to the centroid of tensile force.

To find neutral axis

From the strain diagram, $\frac{x_{bal}}{d + x_{bal}} = \frac{\sigma_{cbc}}{\sigma_{st}}$

$$= \frac{m \sigma_{cbc}}{\sigma_{st}}$$

$$x_{bal} = \frac{m \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} x_d$$

or

$$= \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{cbc}}} x_d$$

$$x_{bal} = k x_d$$

where, $k =$ Neutral axis constant which is

$$k = \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{cbc}}}$$

To find lever arm or Z

From the stress diagram,

$$Z = d - \frac{x_{bal}}{3}$$

$$= d - \frac{k \cdot d}{3} = d \left(1 - \frac{k}{3}\right)$$

where,

$$Z = j x_d$$

$$j = \text{Lever arm constant} = \left(1 - \frac{k}{3}\right)$$

To find total forces

$$C = \text{Total compression} \times \frac{1}{2} = \dots$$

$$T = \text{Total tension}$$

$$C = \frac{1}{2} \times \sigma_{cbc} \times (b \times x_{bal}) = \frac{b \times x_{bal} \times \sigma_{cbc}}{2}$$

compression force ✓

$$T = \sigma_{st} \times A_{st, bal} \quad (\text{Tension force})$$

To find Moment of Resistance of section

Moment of Resistance

Capacity of a section to resist the moment is known as its moment of resistance.

$$\text{Moment of resistance} = \text{Total compressive force} \times \text{lever arm}$$

OR

$$M.R = \text{Total tension force} \times \text{lever arm}$$

Considering compressive force $M.R = \text{Total compressive force} \times \text{lever arm}$

$$M.R = \left(\frac{1}{2} \times \sigma_{cbc} \times b \times x_{bal} \right) \times j d$$

$$= \left(\frac{1}{2} \times \sigma_{cbc} \times b \times k x d \right) \times j d$$

$$M.R = \left(\frac{1}{2} \times \sigma_{cbc} \times k \times j \right) \times b d^2$$

$$M = Q_{bal} \times (bd)^2$$

Where,

Q_{bal} = Moment resistance constant

$$= \frac{1}{2} \times \sigma_{cbc} \times K \times j$$

Considering Tensile force

M.R = Total tension \times Lever Arm

$$= (\sigma_{st} \times A_{st, bal}) \times j d$$

To find steel Area

For a balanced section $M_{bal} = A_{st, bal} \sigma_{st} j d$

$$A_{st, bal} = \frac{M_{bal}}{\sigma_{st} \cdot j \cdot d}$$

steel Area

$$\text{Percentage steel, } P_{t, bal} = 100 \times \frac{A_{st, bal}}{bd}$$

$$P_{t, bal} = 100 \times \frac{M_{bal}}{\sigma_{st} \cdot j \cdot d} \times \frac{1}{bd}$$

$$P_{t, bal} = \frac{100 \times (\frac{1}{2} \times \sigma_{cbc} \times K \times j) \times b \times d \times d}{\sigma_{st} \times j \times d \times b \times d}$$

$$P_{t, bal} = \frac{50 \times \sigma_{cbc} \times K}{\sigma_{st}}$$

balance section percentage of steel

To design a balanced section, for a given design moment M , consider $M = M_{bal}$, IF width of the beam, etc assumed,

$$d = \sqrt{\frac{M_{bal}}{b \times Q_{bal}}}$$

Effective depth

steel Area, $A_{st} = A_{st, bal} = \frac{M_{bal}}{\sigma_{st} \times j \times d}$

Calculate the design constant for the following material considering the balanced design for singly reinforced section. The material are M20 grade concrete & mild steel (Fe25) RCC

(Neutral axis constant)

$\sigma_{cbc} = 7 \text{ MPa}$
 $\sigma_{st} = 140$
 $m = \frac{280}{3 \sigma_{cbc}}$

(Pg No-81 of IS 456:2000 table NO-21)

$Z = j \times d$

$j = \left(1 - \frac{k}{3}\right)$



(Pg No-82 table NO-22)

Moment resistance constant

$k = \frac{1 + \frac{m \sigma_{cbc}}{\sigma_{st}}}{2 + \frac{m \sigma_{cbc}}{\sigma_{st}}}$
 $k = \frac{1 + \frac{280}{3 \times 7}}{2 + \frac{280}{3 \times 7}}$

$k = \frac{1 + 140}{280}$

$\frac{1}{2} \times \sigma_{cbc} \times k \times d$

$$m = \frac{280}{3 \times 1000}$$

$$= \frac{280}{3 \times 7}$$

$$= 13.333$$

$$K = \frac{1 + 0.1}{m \times 1000}$$

~~$$= \frac{1}{13.333 \times 7} = 0.0135$$~~

$$= \frac{1}{1 + \frac{140}{13.333 \times 7}}$$

$$= \frac{1}{1 + \frac{140}{93.33}}$$

For a rectangular section, the effective depth is given by $d = \frac{h}{4}$ for a balanced section. In this case, the effective depth is $d = \frac{1000}{4} = 250$ mm. The effective depth is also given by $d = \frac{h}{4} + \frac{h}{4} = \frac{h}{2}$ for a balanced section. In this case, the effective depth is $d = \frac{1000}{2} = 500$ mm. The effective depth is also given by $d = \frac{h}{4} + \frac{h}{4} = \frac{h}{2}$ for a balanced section. In this case, the effective depth is $d = \frac{1000}{2} = 500$ mm.

$$j = \left(1 - \frac{k}{3}\right)$$

$$j = \left(1 - \frac{0.4}{3}\right)$$

$$j = 0.867$$

Q. For a rectangular beam of size 250mm wide x 520mm

Moment resistance constant = $\frac{1}{2} \times \sigma_{cbc} \times K \times j$

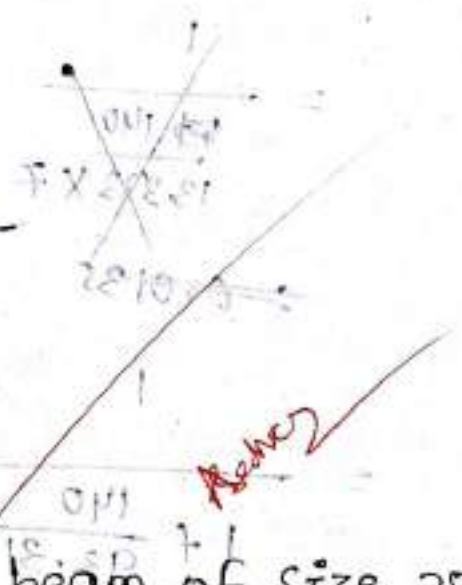
= $\frac{1}{2} \times 7 \times 0.4 \times 0.867$

= 1.213

$P_{t_{bal}} = \frac{50 \times \sigma_{cbc} \times K}{\sigma_{st}}$

= $\frac{50 \times 7 \times 0.4}{140}$

= 1

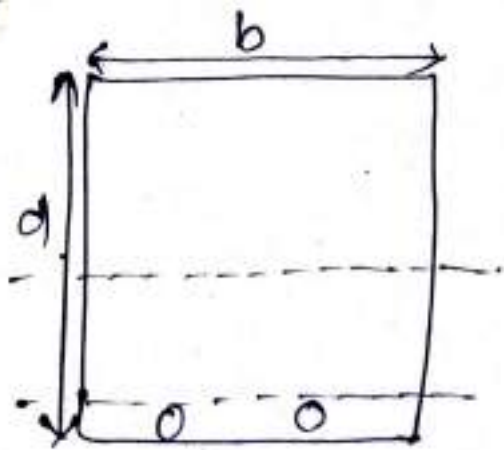


Q. For a rectangular beam of size 250mm wide x 520mm effective depth find out the balanced depth of neutral axis, balanced lever arm, balanced moment of resistance and balanced steel area. The material area M20 grade concrete and Hysd reinforcement of grade Fe 415.

Given data

Effective depth $d = 520\text{mm}$
 Breadth $b = 250\text{mm}$

$f_{ck} = 20$



$b = 250 \text{ mm}$
 $d = 520 \text{ mm}$
 $\sigma_{cbc} = 7$
 $\sigma_{st} = 230$

$$m = \frac{280}{3 \times 7} = 13.33$$

$$K = \frac{10 \cdot b}{1 + \frac{\sigma_{st}}{m \sigma_{cbc}}}$$

$$= \frac{1}{1 + \frac{230}{13.33 \times 7}} = \frac{1}{1 + 2.46}$$

$$= 0.29$$

Percentage of steel to be provided in beam
 $\rho_{bal} = K \cdot d$
 $= 0.29 \times 520$
 $= 150.8$

$$\frac{20 \times 10^3 \times 0.29}{280} = 20.71$$

Lever Arm Constant

$$j = \left(1 - \frac{k}{3}\right)$$

$$= \left(1 - \frac{0.29}{3}\right)$$

$$= 0.90$$



Lever Arm (Z) = j x d

$$= 0.90 \times 520$$

$$= 468$$

Moment resistance constant (Q_{bal}) = $\frac{1}{2} \times \rho_{bc} \times K \times j$

$$= \frac{1}{2} \times 7 \times 0.29 \times 0.90$$

$$= 0.91$$

Moment of resistance resistant = $Q_{bal} \times b d^2$

$$= 0.91 \times 250 \times 520^2$$

$$= 61516000 \text{ N.m.m}$$

$$= \frac{61516000}{10^6} \text{ KN.mt.}$$

P.O. = $\frac{61516}{10^6} = 61.52 \text{ KN.mt.}$

Percentage of steel in balanced section

$$P_{t\text{ bal}} = \frac{50 \times \rho_{bc} \times K}{\sigma_{st}}$$

$$= \frac{50 \times 7 \times 0.29}{280}$$

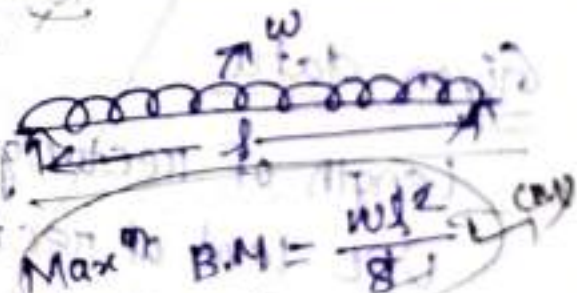
$$= 0.44$$

$$\text{steel Area} = \frac{P_t b a_f \times b d}{100}$$

$$= \frac{0.44 \times 250 \times 520}{100}$$

$$= 572 \text{ mm}^2$$

Q. A simply supported rectangular beam of unit span carries a UDL of 40 kN/m. The width of the beam is 230 mm. Find the depth and steel area for balanced design. Use M20 grade of concrete & mild steel reinforcement.



effective depth = $d = \sqrt{\frac{M_{bal}}{k \times b}}$

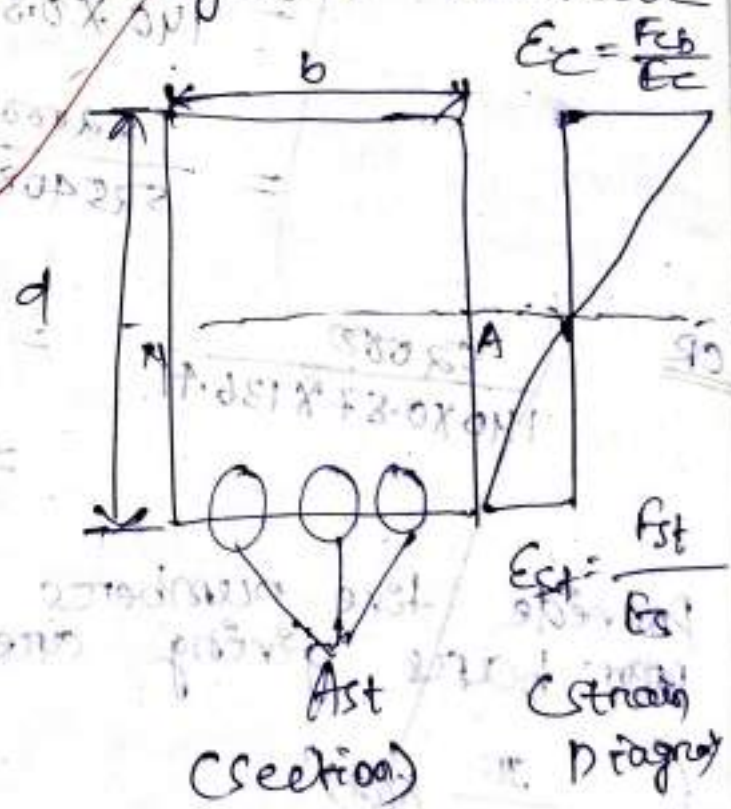
$$Q_{bal} = \frac{1}{2} \times \sigma_{cbc} \times k \times j$$

$$A_{st} = \frac{M_{bal}}{\sigma_{st} \times j \times d}$$

Transformed Area Method Dt: 3.03.23

- (1) A transformed section is a section in which steel area is replaced by an equivalent concrete area.
- (2) A transformed section consists of a single material therefore theory of simply bending can be applied.
- (3) The transformed section may be of steel when concrete is replaced by steel or it may be of concrete when steel area is replaced by concrete.

It is usual to replace steel area by concrete hence a transformed section would mean to a homogenous concrete section.



(Section)

Strain Diagram

Date: 4.03.23

At the section centroid of steel reinforcement, the surrounding concrete being elastic and having perfect bond with steel strain in steel = strain in concrete

Let, F_{st} & F_{cb} be the stresses in concrete respectively of the level of centroid of steel.

\therefore Strain in concrete = strain in steel

$$\frac{F_{cb}}{E_c} = \frac{F_{st}}{E_s}$$

$$\Rightarrow F_{st} = m \times F_{cb}$$

Now force in steel = $A_{st} \times F_{st} = A_{st} \times m \times F_{cb}$
If this steel is to be replaced by an equivalent concrete area, the equivalent concrete will carry the same force

Now the force in equivalent concrete = transformed Area $\times F_{cb}$ (2)

Equating (1) & (2)

$$\begin{aligned} \text{Transformed Area} \times F_{cb} &= A_{st} \times m \times F_{cb} \\ \text{Transformed Area} &= m A_{st} \end{aligned}$$

To Find Neutral Axis

$$b \times x \times \frac{x}{2} = m A_{st} (d - x)$$

$$\text{Lever Arm, } z = d - \left(\frac{x}{3}\right)$$

Max Stress in steel, $F_{st} = \frac{M}{(d - \frac{x}{3}) \times A_{st}}$

Stress in concrete, from strain diagram,

$$\frac{F_{cb}}{E_c} = \frac{F_{st}}{E_s} \times \frac{x}{d-x}$$

$$F_{cb} = \frac{F_{st}}{(E_s/E_c)} \times \frac{x}{d-x}$$

$\therefore F_{cb} = \frac{F_{st}}{m} \times \frac{x}{d-x}$

Max^m stress
in transformed
section
in concrete

M.R. in compression = $F_{cb} \times (b \times \frac{x}{2}) \times \frac{Lever\ Arm}{Arms}$

Moment of resistance in tension = $F_{st} \times A_{st} \times$

Lever Arm

Q. A rectangular beam, of width 200mm, & effective depth (d) 460mm, reinforced with 3 - 16mm dia bars. The section is subjected to a characteristic moment of 30 kNm. Determine the maximum stress in steel and

concrete. The materials are M20 grade concrete & mild steel reinforcement. Also find the moment of resistance of the section.

Ans: Given data

$$d = 460 \text{ mm}$$

$$b = 200 \text{ mm}$$

For M20 grade concrete, $\sigma_{cbc} = 7.0$
 Fe 250 grade steel, $\sigma_{st} = 140$

$$\text{modular ratio } m = \frac{280}{3 \times \sigma_{cbc}}$$

$$= 13.33$$

Transformed area of steel = $m \times A_{st}$

$$= 13.33 \times 3 \times \frac{\pi}{4} \times (16)^2$$

$$= 8038 \text{ mm}^2$$

To find neutral axis taking moment of transformed area about neutral axis

$$100 \times 200 \times 2 \times \frac{x}{2} = 8038 \times (460 - x)$$

$$= 8038 \times (460 - x)$$

$$\Rightarrow 100x^2 = 8038 \times (460 - x)$$

$$\Rightarrow 100x^2 = 36987480 - 8038x$$

$$\Rightarrow 100x^2 + 8038x - 36987480 = 0$$

$$\Rightarrow x = 569.30 \text{ mm}$$

$$\text{Lever Arm } (Z) = d - \frac{2}{3}$$

$$= 460 - \frac{569.30}{3}$$

$$= 270.233 \text{ mm.}$$

∴ Maximum stress in steel

$$f_{st} = \frac{M}{(d - \frac{2}{3}) \times A_{st}}$$

$$= \frac{30 \times 10^6}{(270.233) \times 3 \times \frac{\pi}{4} \times (16)^2}$$

$$= 184.04 \text{ N/mm}^2$$

Max^m stress in concrete, $f_{cb} = \frac{f_{st}}{m} \times \frac{2}{(d-x)}$

$$= \frac{184.04}{19.33} \times \frac{569.30}{(460 - 569.30)}$$

$$= 3.195 \times 10^3 \text{ N/mm}^2$$

$$= 71.91 \text{ N/mm}^2$$

Moment of tension = $\sigma_{st} \times A_{st} \times Z$

Moment of compression = $\sigma_{bc} \times (b \times \frac{x}{2}) \times Z$

= $7 \times (200 \times \frac{169.30}{2}) \times 122004$

= 107690532.8

Moment of tension = $\sigma_{st} \times A_{st} \times Z$

= $140 \times 3 \times \frac{\pi}{4} \times (16)^2 \times 29029$

= 89141.01 N/mm

M.R. of the section = 89141.01 N/mm

Analysis of the Section

Type - I

To find out the depth of neutral axis for a given section & specifying the type of beam.

(i) If the section & steel area are given find out neutral axis by taking moment of transformed area about neutral axis.

$$b \times x \times \frac{x}{2} = m A_{st} \cdot (d - x)$$

(ii) Find out depth of neutral axis for a balanced section.

$$x = K \times d$$

where $K = \frac{1}{1 + m \times \frac{\sigma_{bc}}{\sigma_{st}}}$

(iii) IF ~~actual~~ $x_{act} < x_{bal}$

(The beam section is under reinforced)

IF $x_{act} > x_{bal}$ (The beam section is over reinforced)

IF $x_{act} = x_{bal}$ (The beam section is balanced)

Type-2

To find the moment of resistance for a given section,

(i) Find the position of actual ~~end~~ & balanced neutral axis. as explained above.

(ii) IF $x_{act} < x_{bal}$ (The beam section is under reinforced & moment of resistance is given by)

$$\text{Moment of resistance} = A_{st} \times f_{st} \left(d - \frac{x}{3} \right)$$

(iii) IF $x_{act} > x_{bal}$ (The beam is over reinforced & moment of resistance is given by)

$$M \cdot R = b \times x \times \left(\frac{0.87 f_{ck} b x}{2} \right) \times \left(d - \frac{x}{3} \right)$$

Design of type section - 1 DTG 6.0323

Dimension not given

Find the position of actual & balanced neutral axis as explained above.

(i) The moment of resistance of balanced section $M_{bal} = Q_{bal} \times b d^2$. Out of two variable ~~but b & d~~ b & d one must know to us. It is usual to fix the width b of the section.

(ii) One width is fixed, the depth of well be calculated from following formula.

$$d = \sqrt{\frac{M_{bal}}{Q_{bal} \times b}}$$

(iii) The area of steel A_{st} will be calculated as per the following following $A_{st} = \frac{M}{\sigma_{st} \times j \times d}$

Type - 2

Dimensions are given

(i) Apply moment M , the section dimension b & d are given determine $M_{bal} = Q_{bal} b d^2$

(ii) If $M < M_{bal} \rightarrow$ then the section is designed as under reinforced beam.

(iii) If $M > M_{bal} \rightarrow$ then the section is designed as over reinforced beam.

(iv) If $M = M_{bal} \rightarrow$ the section is designed as balanced reinforced beam.

Q. An RCC beam, 300 mm wide & 460 mm effective depth is reinforced with 4 nos of 12 mm dia bars in tension. Find out the depth of neutral axis & type of the beam. The material are M20 grade concrete & Fe415 grade steel.

Given data

$$b = 300 \text{ mm}$$

$$d = 460 \text{ mm}$$

Tension bar = 4 nos of 12 mm dia

M20 grade, $f_{cbc} = 7.0$ (Pg No - 81 of IS:456)

Fe415 grade, $f_{st} = 230$ (Pg No - 82 of IS:456)

Modular ratio

$$m = \frac{280}{3 f_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

(Area of Steel)

$$A_{st} = 4 \times \left(\frac{\pi}{4} \times 12^2 \right)$$

$$= 452.36 \text{ mm}^2$$

Let x be the depth of neutral axis taking moment of transformed area about neutral axis (N.A).

$$b \times x \times \frac{x}{2} = m A_{st} (d - x)$$

$$\Rightarrow 300 \times \frac{x^2}{2} = 13.33 \times 452.36 (460 - x)$$

$$= 6029.95 (460 - x)$$

$$\Rightarrow \frac{x^2}{2} = \frac{6029.95(460-x)}{300}$$

$$\Rightarrow \frac{x^2}{2} = \dots$$

$$\Rightarrow 150x^2 = 6029.95(460-x)$$

$$\Rightarrow 150x^2 = 2773777 - 6029.95x$$

$$\Rightarrow x^2 = \frac{2773777 - 6029.95x}{150}$$

$$\Rightarrow x^2 + 6029.95x = \dots$$

$$\Rightarrow 150x^2 + 6029.95x - 2773777 = 0$$

~~$$\Rightarrow x = 117.36 \text{ mm}$$~~

$$\Rightarrow x = 117.36 \text{ mm}$$

∴ depth of balanced N.A $x_{bal} = k \times d$

$$k = \frac{\sigma_{st}}{1 + m \times \sigma_{cbc}}$$

$$= \frac{\frac{150}{1.5 \times 7}}{1 + \frac{230}{13.33 \times 7}} = \frac{\frac{100}{10.5}}{1 + \frac{230}{93.31}} = \frac{9.52}{1 + 2.46} = \frac{9.52}{3.46}$$

$$= 0.29$$

$$x_{bal} = k \times d$$

$$= 0.29$$

$$= 133.4 \text{ mm.}$$

IF $x < x_{bal}$
 IF $117.36 < 133.4$ (Under reinforced beam)

Q. Find the moment of resistance of a beam having with 230 mm & 560 mm effective depth reinforced with 3 nos of 20 mm dia bar. Also state the type of the beam. The material are M20 grade concrete & Fe 415 grade steel.

Given data

$$b = 230 \text{ mm.}$$

$$d = 560 \text{ mm.}$$

Tension bar = 3 nos of 20 mm dia
 M20 grade, $f_{ck} = 20$ (Pg no - 81 of 15:456)

Fe 415 grade, $f_{yk} = 415$ (Pg no - 82 of 15:456)

$$\sqrt{m} = \frac{280}{3 f_{ck}} = \frac{280}{3 \times 20} = 4.83$$

$$A_{st} = 9 \times \left(\frac{\pi}{4} \times 20^2 \right)$$

$$= 942.47$$

$$M.R = b \times x \times \left(\frac{\sigma_{bc}}{2} \right) \times \left(d - \frac{x}{3} \right)$$

Let 'x' be the depth of neutral axis taking moment of transformed area about neutral axis (N.A)

$$b \times x \times \frac{x}{2} = m A_{st} (d - x)$$

$$\Rightarrow 230 \times \frac{x^2}{2} = 13.33 \times 942.47 (560 - x)$$

$$\Rightarrow 115 \times \frac{x^2}{2} = 12563.12 (560 - x)$$

$$\Rightarrow 115x^2 = 12563.12 (560 - x)$$

$$\Rightarrow 115x^2 = 7035347.2 - 12563.12x$$

$$\Rightarrow 115x^2 - 7035347.2 + 12563.12x = 0$$

$$\Rightarrow 115x^2 + 12563.12x - 7035347.2 = 0$$

$$\Rightarrow x = 198.67 \text{ mm.}$$

$$\text{Moment of resistance} = A_{st} \times \sigma_{st} \times \left(d - \frac{x}{3} \right)$$

$$= 942.47 \times 230$$

$$\text{depth of neutral axis } x_{bal} = K \times d$$

$$k = \frac{(1.25 \times 10^6)}{1 + \frac{0.35}{m \times \sigma_{cbc}}} \times \sigma_c = 52A$$

$$\left(\frac{x}{d}\right) = 0.29 \times \left(\frac{230}{560}\right) \times \sigma_c = 9.2M$$

$$\left(\frac{x}{d}\right) = \frac{230}{13.33 \times 7} \times \sigma_c$$

$$(x = 0.29) \times \sigma_c = \dots$$

$$(x = 0.29)$$

$$x_{bal} = k \times d$$

$$= 0.29 \times 560$$

$$x_{bal} = 162.4 \text{ mm.}$$

IF $x_{act} > x_{bal}$

IF $198.67 > 162.4$ → Over reinforced section beam.

$$M.R = b \times x \times \left(\frac{\sigma_{cbc}}{2}\right) \times \left(d - \frac{x}{3}\right)$$

$$\left(\frac{x}{d}\right) = \frac{230}{560} \times 198.67 \times \left(\frac{7}{2}\right) \times \left(560 - \frac{198.67}{3}\right)$$

$$= 45694.1 \times 3.5 \times (560 - 66.22)$$

$$= 159929.35 \times 493.78$$

$$= \frac{78969381.35}{106}$$

$$= 78.96 \text{ kN.mf}$$

(Ans)

Limit State MethodDefinition

↳ In the method of design based on limit state concept the structure shall be designed to with stand safely on notes to act on it throughout on its life.

↳ It shall also satisfy the serviceability requirements such as limitation on deflection and cracking. The acceptable limit for safety & serviceability requirement before failure occurs is called limit state.

↳ The aim of design is to achieve acceptable probability that the structure will not become unfit for use.

Limit State of Collapse OR Limit state of Strength

↳ Limit state of strength are those associated with failure under the action of probable & most on unfavorable combination of load on the structure using appropriate partial safety factor which may endanger the safety of life & property.

↳ The limit state of strain includes of limit
 (i) loss of equilibrium of the structure as a whole or any of its parts.

(ii) loss of stability of the structure including supports & foundations.

(iii) failure by excessive deformation, re-upture of the structure.

(iv) Brittle fracture

(v) Fracture due to fatigue

Limit state of serviceability

To satisfy the limit state of serviceability the deflection & cracking in the structure shall not be excessive.

Deflection

The deflection of a structure shall not adversely affect the efficiency of the structure or partition.

Cracking

Cracking of concrete should not adversely affect the durability of the structure.

The acceptable limit of cracking would vary with type of structure & environment.

The surface width of cracks should not exceed 0.3 mm .

Characteristics of strength of material

The term characteristic strength means that value of strength of the material below which not more than 5% of the test result are accepted to fail.

The characteristic strength for concrete shall be according to table no 2 (pg no 16)

The characteristic value shall be assumed as minimum yield stress at 0.2% proof

PROOF STRESS

$$f_{yk} = f_{yk}$$

Characteristic Load Pg no - 67

The term

Characteristic load means the value of load which has 95% probability of not exceeded during the life of the structure

Dead loads are given in IS : 857 (part 1) & imposed load level (part - 2)

IS : 875

Wind load IS : 875 (part - 3)

Seismic load IS : 1893 shall be assume as characteristic load.

pg no - 68

DT : 15.03.23

Design values

Materials

The design strength of the materials, F_d is given by

$$F_d = \frac{f}{\gamma_m}$$

where

f = characteristic strength of the material
and

γ_m = partial safety factor appropriate to the material & the limit state being considered.

Loads
The design load, F_d is given by

$$F_d = F \gamma_f$$

where $F =$ characteristic load and

$\gamma_f =$ partial safety factor appropriate to the nature of loading & the limit state being considered.

Values of partial Safety factor for γ_f loads

Load Combination	Limit state of collapse	Limit states of serviceability
(1) DL + LL	1.5	1.0
(2) DL + WL	1.5 or 0.9	1.0
(3) DL + LL + WL	1.5	1.0
(4) WL	1.0	1.0
(5) DL + LL + WL	1.0	1.0
(6) LL + WL	1.0	1.0
(7) WL	1.0	1.0

OP

Sp No	Load combination	Ultimate limit state	Serviceability limit state
1	DL + LL	1.5 (DL + LL)	DL + LL
2	DL + WL	0.9 DL + 1.5 WL	DL + WL
	(b) DL contribute to stability	1.5 (DL + WL)	DL + WL
3	DL + LL + WL	1.2 (DL + LL + WL)	DL + 0.8 LL + 0.8 WL
	(d) DL assists overturning	1.5 (DL + LL + WL)	DL + 0.8 LL + 0.8 WL

2. Partial safety factor γ_m for material strength

Sr. No.	Material	Ultimate limit state	Serviceability limit state
1	Concrete	1.50	$E_c = 5000 \sqrt{f_{ck}}$ MPa
2	Steel	1.15	$E_s = 2 \times 10^5$ MPa

$1 \text{ N/mm}^2 = 1 \text{ MPa}$

When assessing the strength of a structural member for the limit state of collapse, the values of partial safety factor should be taken as 1.5 for steel.

Q. Find the elastic modulus of concrete for M20 grade.

Ans: $E_c = 5000 \sqrt{f_{ck}}$ MPa

$= 5000 \sqrt{20}$
 $= 22360.67 \text{ MPa (Ans)}$

Limit state of collapse: Flexure

(a) plane sections normal to the axes remain plane after bending.

(b) The maximum strain ϵ_c in concrete at the outermost compression fibre is taken as 0.0035 in bending.

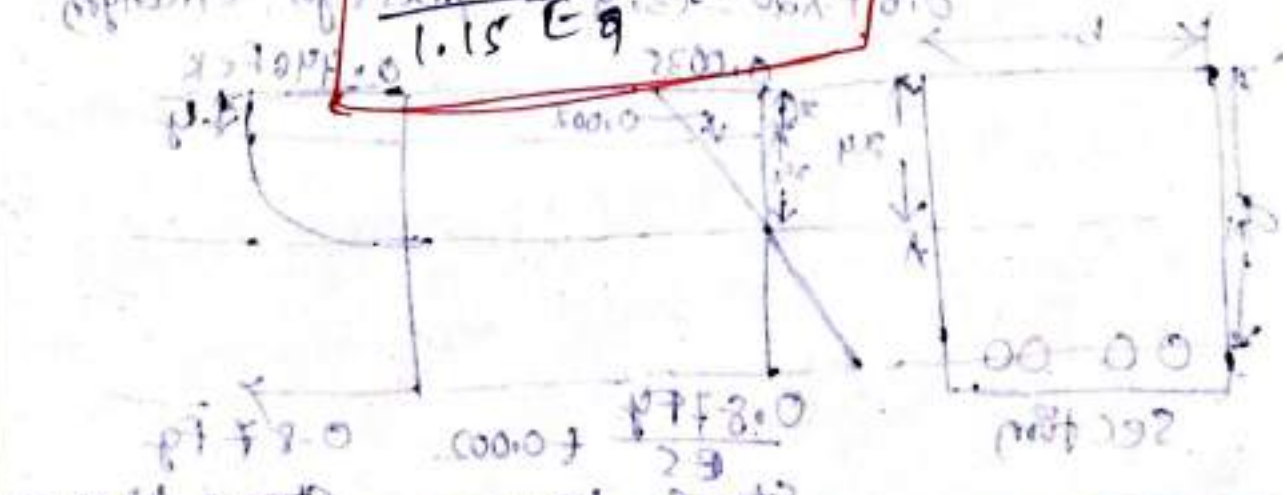
(c) The relationship between the compressive stress distribution in concrete & the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength is substantial agreement with the results of test.

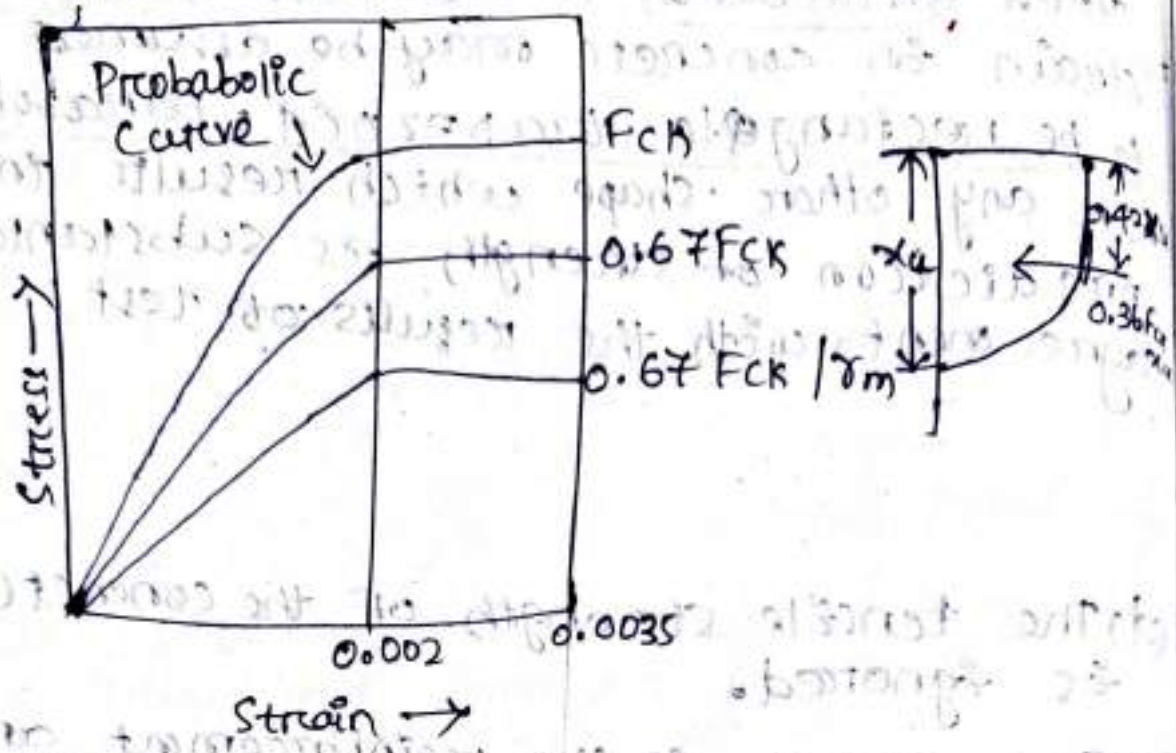
(d) The tensile strength of the concrete is ignored.

(e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given for the type of steel used. Typical curves are given in fig 23. For design purposes the partial safety factor γ_m equal to 1.15 shall be applied.

(f) The maximum strain in the tension reinforcement in the section at failure shall not be less than,

$$\frac{f_y}{1.15 E_s} \geq 0.002$$

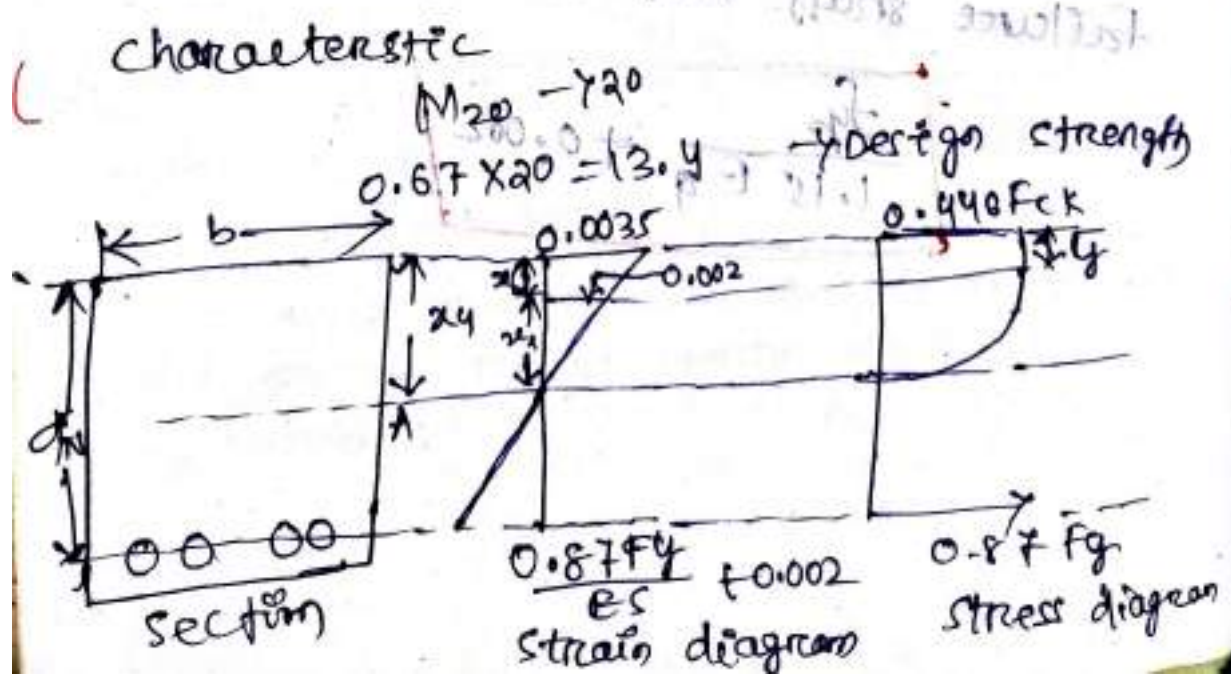




Stress-Strain curve for concrete

For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength.

The partial safety factor $\gamma_m = 1.5$ shall be applied ~~if~~ in addition to this.



Note

For the above stress strain for the design stress block parameters are as follows.

Area of parabolic portion =

$$\left(\frac{2}{3}\right) \times 0.446 F_{ck} \left(\frac{4}{7}\right) x_u$$

Area of rectangular portion =

$$0.446 F_{ck} \left(\frac{3}{7}\right) x_u$$

$$= 0.19 F_{ck} x_u$$

Total area of stress block = $0.17 F_{ck} x_u + 0.19 F_{ck} x_u$

$$= 0.36 F_{ck} x_u$$

Let \bar{y} = be the distance of centroid of stress block from the extreme compression fibre.

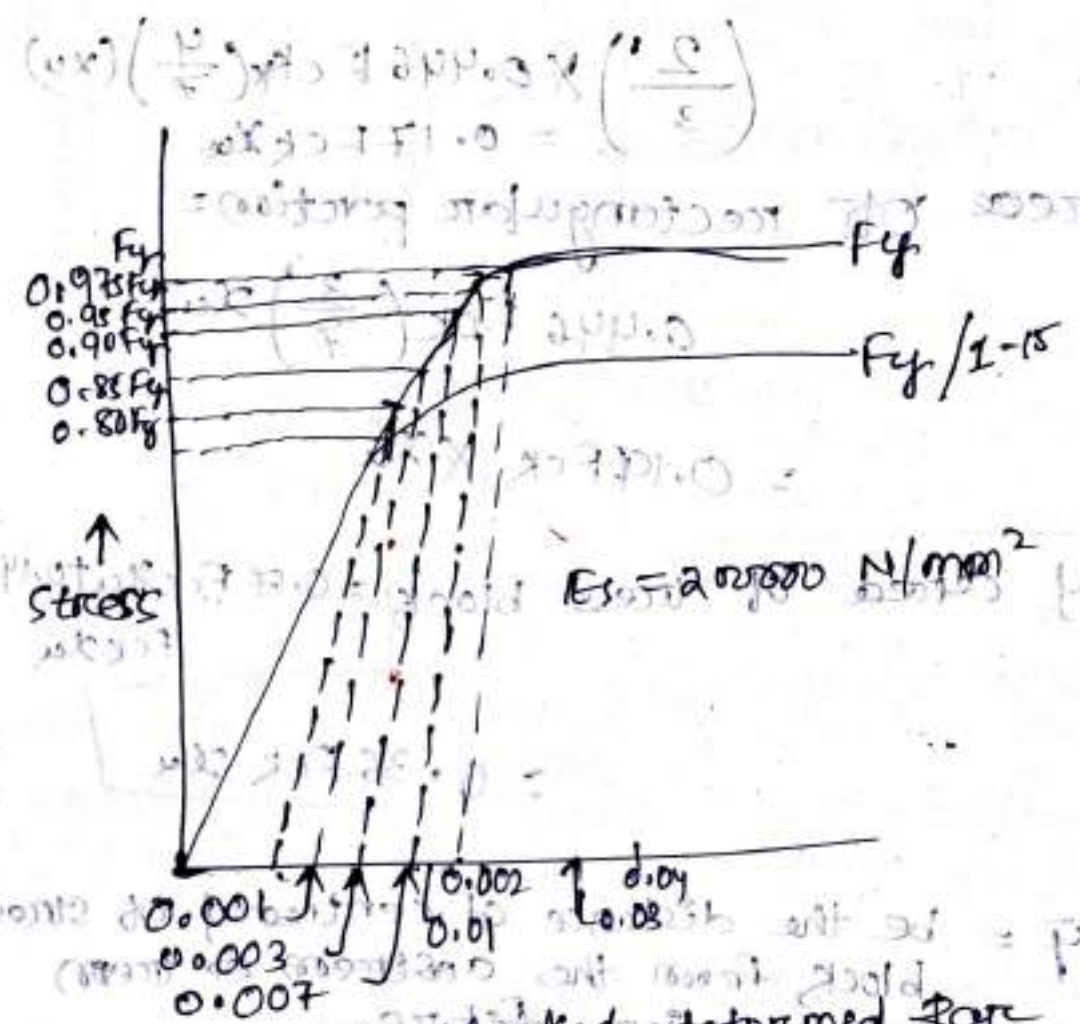
$$\therefore \bar{y} = 0.17 F_{ck} x_u \left(x_1 + \frac{3}{8} x_2\right) + 0.19 F_{ck} x_u \left(\frac{x_1}{2}\right)$$

Substituting $x_1 = \left(\frac{3}{7}\right) x_u$ & $x_2 = \left(\frac{4}{7}\right) x_u$

\therefore depth of centre of compressive force $\bar{y} = 0.42 x_u$
from extreme fibre in compression.

Where,

f_{ck} = characteristic compressive strength of concrete & x_{u2} = depth of neutral axis



cold worked deformed bar

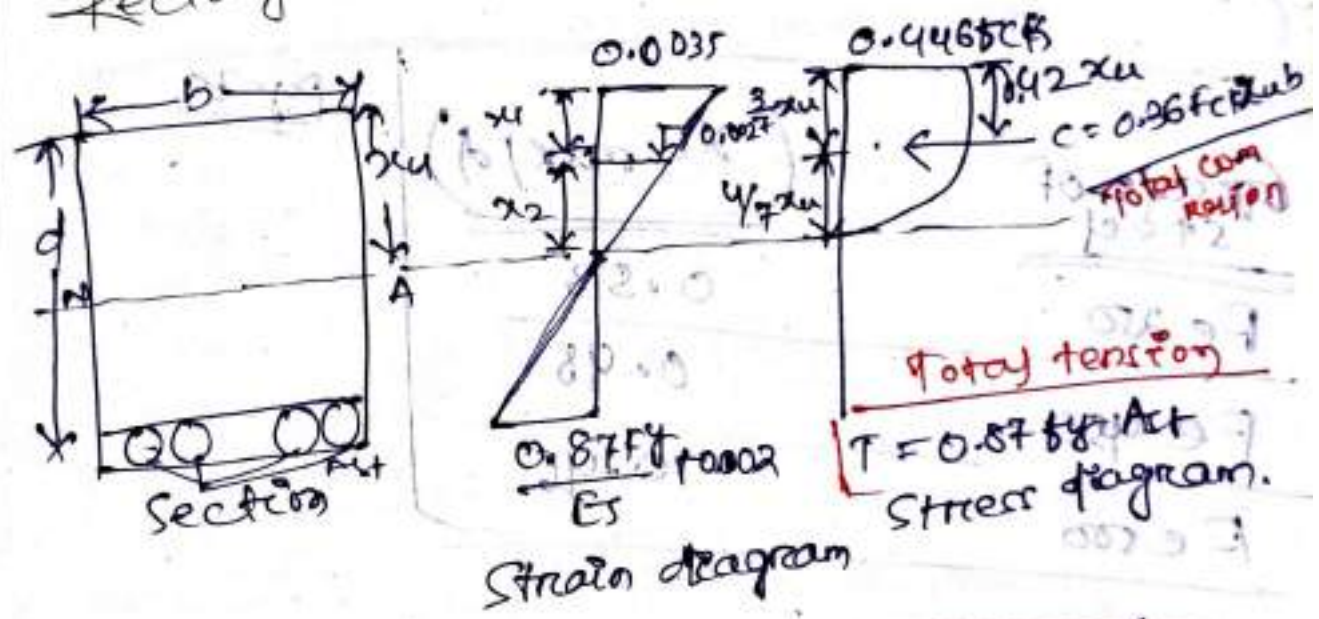
Types of Beam Section

- (i) Section in which tension steel reaches yield steel as the concrete reaches the failure strain in bending are called balanced section.
- (ii) Section in which tension steel reaches yield steel at loads lower than the load at which concrete reaches the failure strain in bending are called under reinforced section.

(iii) Section in which tension steel reaches yield steel at loads higher than the load at which concrete reaches the failure strain in bending are called over reinforced section.

3rd chapter

Analysis of single Reinforced Rectangular beam.



Total Area of stress block = $0.17 f_{ck} x_u$
 $+ 0.19 f_{ck} x_u$
 $= 0.36 f_{ck} x_u$

A singly reinforced rectangular beam section with strain diagram & stress diagram are shown in above figure.

To Find Neutral Axis depth

Total compression = Total tension

$\Rightarrow 0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y A_{st}$

$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b}$

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IF $x_u < x_{u,max}$ → then the section is under reinforced section

IF $x_u > x_{u,max}$ → over-reinforced

IF $x_u = x_{u,max}$ → Balanced section

Grade of steel	$(x_{u,max}/d)$
Fe 250	0.53
Fe 415	0.48
Fe 500	0.46



$x_{u,max} = 0.48d$

$x_{u,max} = 0.478d$

Date: 17.03.23

To find lever arm

From the stress diagram

$Z = d - 0.422x_u$

1.8

0.8

To Find Moment of Resistance

(i) For a balanced section,

M.R = Total compression

$$M_u = 0.36 f_{ck} b x_u \lambda (d - 0.42 x_u)$$

MR

For a under reinforced section

M.R = Total Tension \times lever arm.

$$M_u = 0.87 f_y A_{st} \lambda (d - 0.42 x_u)$$

For limiting values substitute

Substitute $x_{u, max}$ & $M_{u, lim}$ for M_u

$$M_{u, lim} = 0.36 \times f_{ck} \times b \times \frac{x_{u, max}}{d} \times d (d - 0.42 x_{u, max})$$

$$M_{u, lim} = 0.36 \times \left(\frac{x_{u, max}}{d} \right) \times \left(1 - 0.42 \frac{x_{u, max}}{d} \right) \times f_{ck} b d^2$$

$$\times f_{ck} b d^2$$

If $M_u < M_{u, lim}$

Tension

$$M_u = 0.87 f_y A_{st} \lambda \left(1 - \frac{A_{st} f_y}{b d f_{ck}} \right)$$

Type of Problem

Three different types of problem are considered for singly reinforced rectangular beam.

Type-1

To find out the depth of neutral axis of type of the beam.

(i) For a given section & total compression of neutral axis equate total tension & find out the depth of neutral axis using $x_u = \frac{0.87 f_y A_s t}{0.36 f_{ck} b}$

(ii) Also find out the limiting value of depth of neutral axis $x_{u,max}$ using $\left(\frac{x_{u,max}}{d}\right)$

(iii) If $x_u < x_{u,max}$ Then the section is known as under reinforced section.

If $x_u > x_{u,max}$ Then the section is known as over reinforced section.

If $x_u = x_{u,max}$ Balanced section.

Type-2

To find out the moment of resistance of a given section.

(i) For finding out depth of neutral axis & type of beam, as discussed in type-1

(ii) For over reinforced & balanced section, then calculate moment of resistance by using following eqn.

$$M_{u, \text{lim}} = 0.36 \times \left(\frac{x_{u, \text{max}}}{d} \right) \times (1 - 0.42 \frac{x_{u, \text{max}}}{d}) \times f_{ck} \times b d^2$$

(iii) For under reinforced section obtain moment of resistance by using following eqn.

$$M_u = 0.36 f_{ck} b x_u \times (b d - 0.42 x_u)$$

OR

$$M_{u, \text{lim}} = 0.87 f_y A_{st} \cdot d \left(1 - \frac{A_{st} \cdot f_y}{b d f_{ck}} \right)$$

Type-3

To find out the To design a singly reinforced rectangular section, for given width & applied factored moment.

(i) The width is decided by the following formula

$$d = \sqrt{\frac{M}{Q_{\text{lim}} \times b}}$$

where, $Q_{\text{lim}} = 0.36 \times \left(\frac{x_{u, \text{max}}}{d} \right) \times (1 - 0.42 \frac{x_{u, \text{max}}}{d}) \times f_{ck}$

(ii) The steel area can be calculated by using following formula.

$$A_{st} = \frac{M}{0.87 f_y (d - 0.42 x_{u, max})}$$

Q. A rectangular beam 230 mm wide & 520 mm effective depth is reinforced with four nos. of 16 mm diameter bars. Find out the depth of neutral axis & specify the type of beam. The material are M20 grade concrete & Fe 415 grade steel.

Given data

$b = 230 \text{ mm}$
 $d = 520 \text{ mm}$
 4 nos. of 16 mm dia

$$A_{st} = 4 \times \frac{\pi}{4} \times (16)^2$$

$$= 804.24 \text{ mm}^2$$

Total compression = Total tension

$$\Rightarrow 0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b} = \frac{0.87 \times 415 \times 804}{0.36 \times 20 \times 230}$$

$$= 175.3 \text{ mm}$$

For Fe 415, $\frac{x_{u,max}}{d} = 0.48$

$\Rightarrow x_{u,max} = 0.48 \times 520 = 250 \text{ mm}$

$x_u < x_{u,max} \rightarrow$ under reinforced section

Q. A rectangular beam 200 mm wide & 450 mm effective depth is reinforced with 4 nos of 20 mm diameter bars. Find out the depth of neutral axis & specify the type of beam & the material moment of resistance of the beam. The material are M30 grade concrete & Fe 250 grade steel.

Ans.

Given data

$b = 200 \text{ mm}$

$d = 450 \text{ mm}$

4 nos of 20 mm dia.

$F_{ck} = 30 \text{ N/mm}^2$
 $F_y = 250 \text{ N/mm}^2$

$A_{st} = 4 \times \frac{\pi}{4} \times (20)^2 = 1256.63 \text{ mm}^2$

Total Compression = Total tension

$= 0.36 F_{ck} \cdot b \cdot x_u = 0.87 F_y A_{st}$

$x_u = \frac{0.87 F_y A_{st}}{0.36 F_{ck} \cdot b} = \frac{0.87 \times 250 \times 1256.63}{0.36 \times 30 \times 200}$

$x_u = 126.53 \text{ mm}$

For $f_e = 250$, $\frac{x_{u,max}}{d} = 0.53$

$\Rightarrow x_{u,max} = 0.53 \times 450$

$= 238.5 \text{ mm}$

$x_u < x_{u,max} \rightarrow$ under reinforced section

$M_u = 0.36 F_{ck} b x_u (d - 0.42 x_u)$

$= 0.36 \times 30 \times 200 \times 238.53 (450 - 0.42 \times 238.53)$

$= 108463032 \text{ N}\cdot\text{mm}$

$\frac{108463032}{106} = 108463032$

Type-3 $= 108.46 \text{ kNm/m}$ (Ans)

Q3. Design a singly reinforced rectangular beam for an applied moment of 120 kNm. Assume moment of the width of the section as 230 mm. The material are M20 grade concrete & Fe415 grade steel.

Given data

(fact moment)
req

$M_u = 120 \text{ kNm} = 120 \times 10^6 \text{ N}\cdot\text{mm}$

$b = 230 \text{ mm}$

$F_{ck} = 20 \text{ N/mm}^2$

$$F_y = 415 \text{ N/mm}^2$$

$$d = \sqrt{\frac{M_u}{Q_{lim} \times b}}$$

$$Q_{lim} = 0.36 \times \left(\frac{x_{u \max}}{d} \right) \times \left(1 - 0.42 \frac{x_{u \max}}{d} \right) \times f_{ck}$$

For Fe 415, $\frac{x_{u \max}}{d} = 0.48$

$$x_{u \max} = 0.48 \times 435 = 209.07 \text{ mm}$$

$$Q_{lim} = 0.36 \times 0.48 \times (1 - 0.42 \times 0.48) \times 20$$

$$= 2.75$$

$$\therefore d = \sqrt{\frac{120 \times 10^6}{2.75 \times 230}} = 435.57 \text{ mm}$$

Adopt 500 mm depth. (D)

M_u

$$A_{st} = \frac{M_u}{0.87 F_y (d - 0.42 x_{u \max})}$$

$$= \frac{120 \times 10^6}{0.87 \times 415 \left(500 - 0.42 \times 209 \right)}$$

$$= 957.21 \text{ mm}^2$$

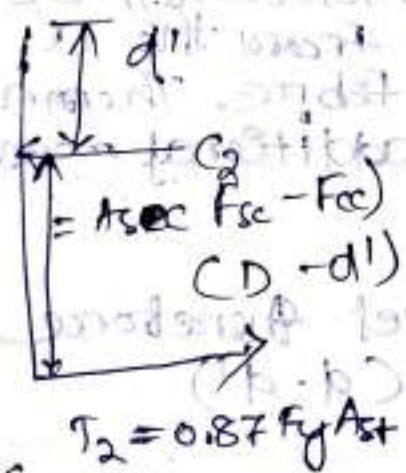
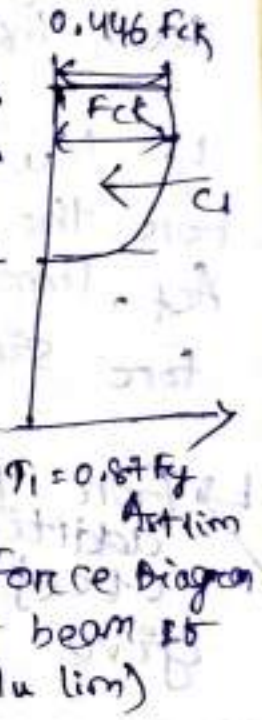
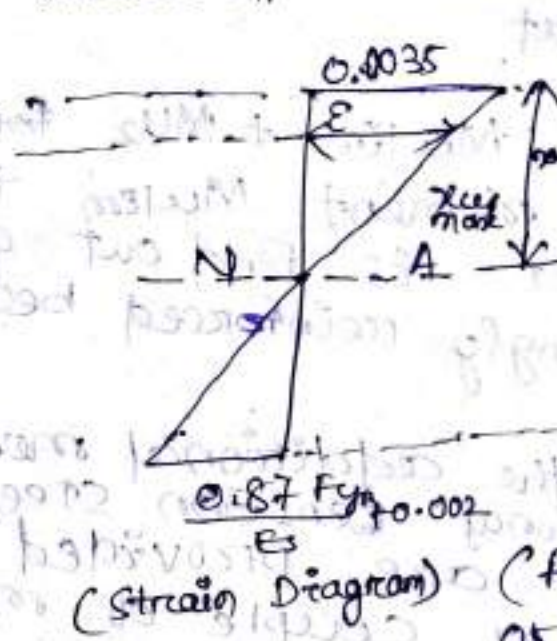
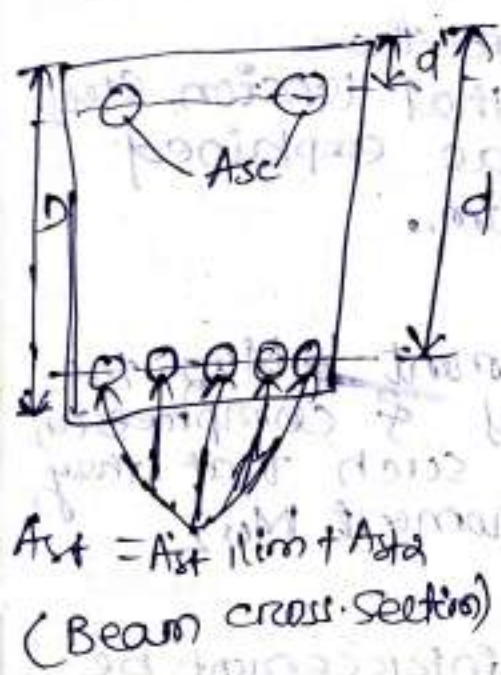
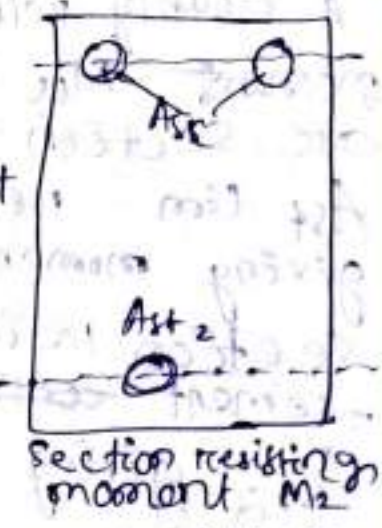
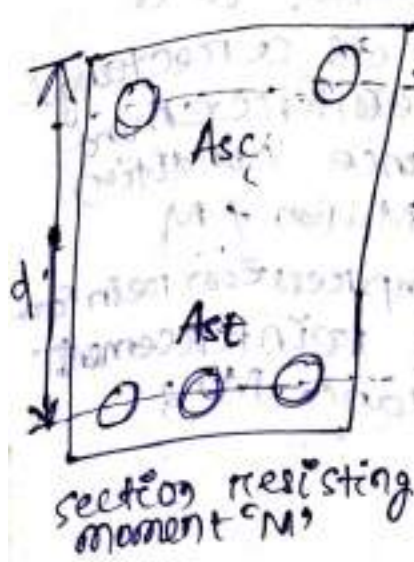
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Doubly Reinforced Section

If the applied moment is greater than the moment of resistance of a singly reinforced section then there can be three alternatives:

- (i) If possible, increase the dimension of the section, preferably depth.
- (ii) Higher grade concrete can be used to increase the moment of resistance of the section.
- (iii) Steel reinforcement may be added in compression zone to increase the moment of resistance of the section. This is known as doubly reinforced section.

Derivation of the formula



(force diagram of beam at M_{u2})

Area of additional reinforcement

A doubly reinforced beam subjected to a moment M_u can be expressed as a rectangular section with tension reinforcement A_{st} (lim reinforced for balance condition) giving moment of resistance M_{u1} and M section reinforced with compression reinforcement A_{sc} & tensile reinforcement A_{st2} giving moment of resistance M_{u2} such that

↳ $M_u = M_{u1} + M_{u2}$ For the moment M_{u1} that tension steel A_{st} limit is found out as explained for singly reinforced beam.

↳ For the additional moment M_{u2} the additional tension steel & compression steel are provided such that they give a couple of moment M_{u2} .

↳ Let the compression reinforcement be provided at a depth d' from the top fibre. Then the lever arm for additional moment will be $(d - d')$.

↳ Considering tension steel therefore
 $M_{u2} = 0.87 f_y A_{st2} (d - d')$

↳ Considering compression steel M_{u2}
 $= A_{sc} (f_{sc} - f_{cc}) (d - d')$ where,

A_{st} = Area of additional tensile reinforcement.

A_{sc} = Area of compression reinforcement

f_{sc} = Stress in compression reinforcement

f_{cc} = Compressive stress in concrete at the level of compression reinforcement.

Since the additional reinforcement is balanced by additional compressive force.

$$\therefore A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st2}$$

$$\Rightarrow A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y}$$

Pg-96

Total Area of reinforcement shall be obtained by

$$A_{st} = A_{st1} + A_{st2}$$

when, A_{st1} = Area of reinforcement for a singly reinforced section for M.u.c.m.

The value of f_{sc} in N/mm^2 can be obtained from the following table for different value of $\frac{d}{d'}$ & grade of steel.

f_y in N/mm^2	d/d'			
	0.05	0.1	0.15	0.20
250	217	217	217	217
355	353	353	342	329
500	424	412	395	370
550	458	441	419	380

The value of f_{cc} is very small so can be neglected

~~TYPE 1~~

TYPE OF PROBLEM

TYPE - 1

To find out moment of resistance of a given section.

Total compression = Total tension.

$$C_1 + C_2 = T$$

$$\Rightarrow 0.36 f_{ck} b x_u + A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

Find out x_u

Find $x_{u,max}$ & type of beam.

If $x_u > x_{u,max}$ Over reinforced section, then take $x_u = x_{u,max}$.

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} f_{sc} (d - d')$$

TYPE - 2

To find out reinforcement to fracture flexure for a given section & factored moment.

Find out a given section by using following eqn, $M_{u,lim}$ for

$$M_{u,lim} = 0.36 f_{ck} b X_{u,max} (d - 0.42 X_{u,max})$$

$$A_{st,lim} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 X_{u,max})}$$

Obtain $M_{u2} = M_u - M_{u,lim}$

Find compression steel from following eqn $M_{u2} = A_{sc} \times f_{sc} (d - d')$

$$\Rightarrow A_{sc} = \frac{M_{u2}}{f_{sc} (d - d')}$$

Corresponding tension steel can be found out by following formula. $A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y}$

$$A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y}$$

$$A_{st,lim} = A_{st,lim} + A_{st2}$$

$$\frac{A_{st2}}{A_{st,lim}}$$

$$A_{st,lim} + A_{st2}$$

Q. Find the factored moment of resistance of a beam section 230 mm wide & 460 mm effective depth reinforced with 2 nos. of 16 mm dia bars as compression reinforcement and 4 nos. of 20 mm dia bars as tension reinforcement. The material are M20 grade concrete & Fe 250 grade steel.

Given data

$$b = 230 \text{ mm}, f_{ck} = 20 \text{ N/mm}^2$$

$$d = 460 \text{ mm}, f_y = 250 \text{ N/mm}^2$$

$$d_1 = 40 \text{ mm}$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times (16)^2$$

$$= 402 \text{ mm}^2$$

$$A_{st} = 4 \times \frac{\pi}{4} \times (20)^2$$

$$\frac{d_1}{d} = \frac{40}{460} \approx 0.08 = 0.1$$

$$f_{sc} = 217 \text{ N/mm}^2$$

Total compression = Total tension

$$\Rightarrow 0.36 f_{ck} b x_u + A_{sc} (f_{sc}) = 0.87 f_y A_{st}$$

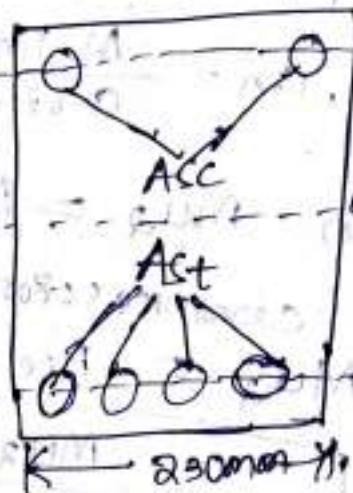
$$\Rightarrow 0.36 \times 20 \times 230 x_u + 402 \times 217 = 0.87 \times 250 \times 1256$$

$$\Rightarrow 1656 x_u + 87234 = 627318$$

$$\Rightarrow 1656 x_u = 627318 - 87234$$

$$\Rightarrow x_u = \frac{185946}{1656}$$

$$\Rightarrow x_u = 112.29 \text{ mm}$$



For F_{e250} , $x_{u,max} = 0.53 \times d$

$$= 0.53 \times 460$$

$$= 243.8 \text{ mm.}$$

$x_u < x_{u,max} \rightarrow$ under reinforced section

$$M_u = 0.36 f_{ck} \cdot b x_u (d - 0.42 x_u) + A_{sc} f_{sc} (d - d')$$

$$= 0.36 \times 20 \times 230 \times 112.29 (460 - 0.42 \times 112.29) + 1000 \times 112.29 (460 - 40)$$

$$= 185952.24 (412.84) + 234 \times 420$$

$$= 113406468 \text{ N}\cdot\text{mm}$$

\Rightarrow #

$$= 113406468 \text{ N}\cdot\text{mm}$$

$$= \frac{113406468}{10^6} \text{ kN}\cdot\text{m}$$

Q.2 A rectangular beam

of size 230 mm wide & 500 mm depth is subjected to a factored moment of 200 kNm. Find the reinforcement. For flexure. The materials are M20 grade concrete & Fe415 grade steel.

Prbl
③

Q. Calculate the area of steel of grade Fe415 required for a section of width 250 mm & overall ~~500~~ depth 500 mm (effective depth 460 mm) in M20 at the limit state moment to be carried by the beam section is 150 kNm.

Given data

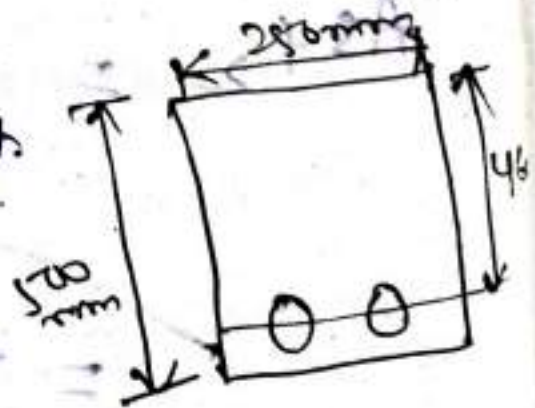
$$b_{\text{eff}} = 250 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$f_{\text{ck}} = 20$$

$$M_u = 150 \text{ kNm}$$

$$f_y = 415$$



For Fe415

$$\frac{x_{u, \text{max}}}{d} = 0.48$$

$$\Rightarrow x_{u, \text{max}} = 0.48 \times d$$
$$= 0.48 \times 460 = 220.8 \text{ mm}$$

$$M_{u,lim} = 0.36 F_{ck} b \times x_{u,max} (d - 0.42 x_{u,max})$$

$$= 0.36 \times 20 \times 250 \times 0.48 \times \frac{460}{200} \left(520 - 0.42 \times \frac{460}{200} \right)$$

$$= 172454400 \text{ ~~KN-mm~~ N.mm.}$$

$$= \frac{172454400}{10^6} \text{ KN.m} = 172.45 \text{ KN.m}$$

$$= 172.45 \text{ KN.m}$$

$$A_{st,lim} = \frac{M_{u,lim}}{0.87 F_y (d - 0.42 x_{u,max})}$$

$$= \frac{172454400}{0.87 \times 415 \left(520 - 0.42 \times 0.48 \times \frac{460}{200} \right)}$$

$$= 1196.51 \text{ mm}^2$$

for $\rho_{t,FTS}$

$$\frac{d'}{d} = \frac{58}{520} = 0.10 \quad F_{sc} = 353 \text{ N/mm}^2$$

$$M_u = M_{u,lim}$$

$$= 150 - 172.45 = 145.96$$

$$= 22.45 \text{ KN.m}$$

(Muller & P. 140) $M_{u2} = 145.96 \text{ kNm}$

$$A_{sc} = \frac{M_{u2}}{F_{sc}(d-d')}$$

$$= \frac{145.96 \times 10^6}{353(500-50)}$$

$$= 1.44 \text{ mm}^2 \quad 27.91 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc}(F_{sc} - F_{cc})}{0.87 f_y}$$

$$= \frac{1.44(353 - 0)}{0.87 \times 415}$$

$$= 1.37 \text{ mm}^2 \quad 27.28 \text{ mm}^2$$

$$A_{st} = \frac{M_{u1}}{0.87 f_y (d - 0.42 x_{u,max})}$$

$$= \frac{145.96 \times 10^6}{0.87 \times 415 (500 - 0.42 \times 0.48 \times 500)}$$

$$= 1198.47 \text{ mm}^2 \quad 1100.34 \text{ mm}^2$$

$A_{rt} = A_{rt1} + A_{rt2}$

$= 1100 + 27.28$
 $= 1127.28 \text{ mm}^2$

$= 1196.47 + (-1.37)$
 $= 1195.1 \text{ mm}^2 \text{ (Ans)}$

Dt: 02.03.23

Specification for beam:

Effective span (l)

* The effective span of a simply supported beam shall be taken as clear span plus effective depth of the beam or center to center distance between the supports whichever ever is less.

* The effective span of a cantilever shall be taken as its length to the face of the support plus half the effective depth except where it forms the end of a continuous beam where the length to the centre of support shall be taken.

* Limiting stiffness (25) &

* The stiffness of beams is governed by the span to depth ratio.

* As per clause 456:2000 (pg no. 37) for spans not exceeding 10m, the span to effective depth ratio should not exceed

<u>Types of support</u>	<u>Span / depth ratio</u>
Cantilever	7
Simply supported	20
Continuous	26

* For spans above 10m, the above values may be multiplied by 10/span in m.

* Depending on the amount and type of steel, the above values shall be modified by multiplying with the modification factors obtained from figure of IS 456: 2000.

Minimum reinforcement



* The minimum area of tension reinforcement should not be less than the following (clause 26.5.1 of IS 456: 2000) (pg No - 46)

$$\frac{A_{st}}{bd} \geq \frac{0.85}{f_y}$$

Maximum reinforcement

* The maximum area of tension reinforcement should not exceed 4% of the gross cross sectional area (clause 26.5.1 of IS 456: 2000) (pg No - 47)

~~$$\frac{A_{st}}{bd} \leq \frac{0.85}{f_y}$$~~

$$P_{t,max} < 0.04 b D$$

where

D = Gross depth of the beam

b = width of the beam

Simply supported

continuous

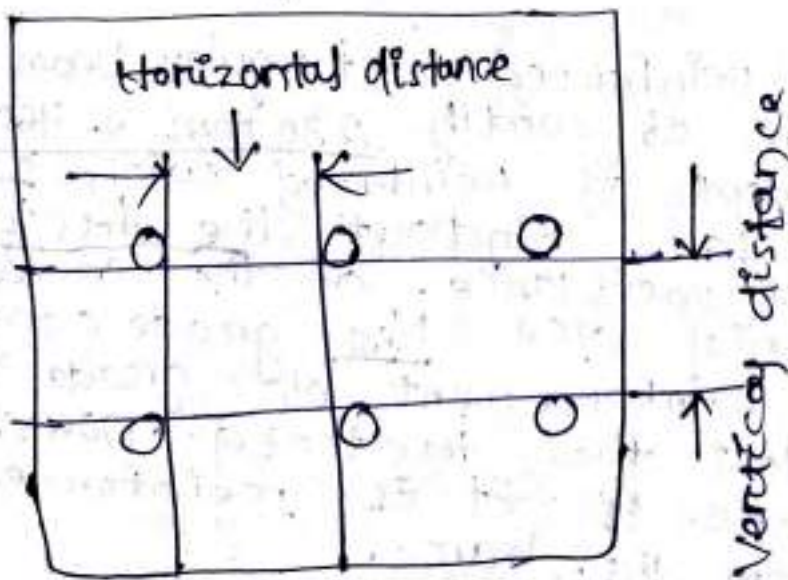
spacing of bars (45)

* The horizontal distance between two parallel main reinforcing bars shall usually be not less than the greater of

- * Diameter of the bar, if the diameters are equal.
- * Diameter of the largest bar, if the bars are unequal.
- * 5mm, more than the nominal maximum size of the aggregate.

* When there are two or more rows of bars, the bars shall be vertically in line and the minimum vertical distance between the bars shall be 15mm, two thirds of nominal maximum size of aggregate or the maximum size of the bars, whichever ever is greater.

* The maximum spacing of bars in tension for beams is taken from table 15 of IS 456:2000 (pg no - 46) depending on the amount of redistribution carried out in analysis & f_y



Concrete cover for reinforcement

↳ Thickness of concrete cover for reinforcement is assumed based on:

- * At each end of reinforcement bar, the concrete cover should not less than 25 mm not less than twice the diameter of such bar.
- * For longitudinal reinforcing bars in beam, the concrete cover should not less than 25 mm nor less than the diameter of such bar.

Side face reinforcement

- * Side face reinforcement shall be provided along the two faces, where the depth of the beam exceeds 750 mm.
- * The total area of such reinforcement shall be not less than 0.1% of the beam area, and shall be distributed equally on two faces at a spacing not exceeding 300 mm or width of the beam which ever is less.

Q. A singly reinforced rectangular beam of width 230 mm & 450 mm. effective depth is reinforced with 5 no. 20 mm dia. bar. Find out the factored moment of resistance of the section. The material are M₂₀ grade concrete & Fe₂₅ reinforcement of grade Fe₄₁₅. Also find out the factored moment of resistance if it is reinforced with 5 no. 20 mm. dia. bar.

Given data

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

3 nos of 20mm dia

$$F_{ck} = 20$$

$$F_y = 415$$

$$A_{st} = 3 \times \frac{\pi}{4} \times (20)^2$$

$$= 942.47 \text{ mm}^2$$

Total Compression = Total tension

$$= 0.36 F_{ck} \cdot b \cdot x_u = 0.87 F_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 F_y A_{st}}{0.36 F_{ck} \cdot b}$$

$$= \frac{0.87 \times 415 \times 942.47}{0.36 \times 20 \times 230}$$

$$= 205.48 \text{ mm}$$

$$\text{For Fe 415} = \frac{x_{u, \text{max}}}{d} = 0.48$$

$$\Rightarrow x_{u, \text{max}} = 0.48 \times d$$

$$= 0.48 \times 460$$

$$= 220.8 \text{ mm}$$

$$\text{PF. OF 21 X 21 X 8.0}$$

$$0.87 \times 0.8 \times 25.0$$

$x_u < x_{u\max} \rightarrow$ under reinforced beam

$$M_{ue} = 0.36 F_{ck} b x_u (d - 0.42 x_u)$$
$$= 0.36 \times 20 \times 230 \times 205.48 (460 - 0.42 \times 205.48)$$

$$= \frac{127160178.2}{10^6} \text{ N.m}$$

$$= \frac{127160178.2}{10^6}$$

$$= 127.16 \text{ kN.m}$$

Case-2

5 nos of 20 mm

$$A_{st} = 5 \times \frac{\pi}{4} \times (20)^2$$

$$= 1570.79 \text{ mm}^2$$

Total compression = Total tension

$$= 0.36 F_{ck} b x_u = 0.87 f_y A_{st}$$

$$\Rightarrow x_u = \frac{0.87 f_y A_{st}}{0.36 F_{ck} b}$$

$$= \frac{0.87 \times 415 \times 1570.79}{0.36 \times 20 \times 230}$$

$$= 342.47 \text{ mm.}$$


$$\text{fore, } \rho_e \text{ 415} = \frac{x_{u, \max}}{d} = 0.48$$

$$x_{u, \max} = 0.48 \times 460 \\ = 220.8 \text{ mm.}$$

$x_u < x_{u, \max}$ → Over reinforced beam.
 $x_u > x_{u, \max}$ → Over reinforced beam.

$$M_{u, \text{lim}} = 0.36 \times \left(\frac{x_{u, \max}}{d} \right) \times (1 - 0.42 \times \frac{x_{u, \max}}{d}) \times f_{ck} \times b d^2$$

$$= 0.36 \times \left(\frac{0.48 \times 220.8}{460} \right) \times (1 - 0.42 \times \frac{220.8}{460}) \times 20 \times 230 \times (460)^2$$


 A ^{doubly} singly reinforced rectangular beam of width 200 mm & 400 mm effective depth is reinforced with 5 no. 20 mm dia. bars in tension & reinforced with compression steel of 2 no. 16 mm dia bars at $d' = 40$ mm. Find out the factored moment of resistance of the section (M20 grade of concrete & Fe 500).

Given data

$$b = 200 \text{ mm}$$

$$d = 400 \text{ mm}$$

5 no. of 20 mm dia

2 no. of 16 mm dia

$$d' = 40 \text{ mm}$$

$$F_{ck} = 20$$

$$F_y = 500$$

$$\begin{aligned}
 A_{sc} &= 5 \times \frac{\pi}{4} \times (20)^2 \\
 &= 1570.79 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 A_{st} &= 2 \times \frac{\pi}{4} \times (16)^2 \\
 &= 402.12 \text{ mm}^2
 \end{aligned}$$

$$\frac{d'}{d} = \frac{40}{400} = 0.1 \quad F_{sc} = 412 \text{ N/mm}^2$$

Total Compression = Total Tension

$$0.36 F_{ck} (b x_u + A_{sc} (f_{sc})) = 0.87 f_{yk} A_{st}$$

$$= 0.36 \times 20 \times 200 \times x_u + 1570.79 (412) = 0.87 \times 500 \times 402.12$$

$$= 61440 x_u + 647165.48 = 174922.2$$
$$\Rightarrow 1440 x_u = 174922.2 - 647165.48$$
$$= -472243.28$$

$$= 1440 x_u = -647165.48 + 174922.2$$
$$\Rightarrow 1440 x_u = 472243.28$$

$$\Rightarrow x_u = \frac{472243.28}{1440}$$

$$\Rightarrow x_u = 327.94$$

for, f_{e500} , $x_{u,max} = 0.46 \times 400$
 $= 184 \text{ mm.}$

$x_u > x_{u,max} \rightarrow$ over reinforced section.

$$34.0 = \frac{\text{compression}}{d}$$
$$d = 230 \text{ mm}$$
$$f_{ck} = 20$$
$$f_{yk} = 500$$
$$A_{sc} = 1570.79$$
$$A_{st} = 402.12$$

$$M_{ue} = 0.36 f_{ck} b x_{u,eq} (d - 0.42 x_{u,eq}) + A_{sc} (f_{sc}) (d - d')$$

$$= 0.36 \times 20 \times 200 \times 327.94 (400 - 0.42 \times 327.94) + 1570.79 \times 412 (400 - 40)$$

~~356830012.4 N.mm~~

356830012.4 N.mm

$$= \frac{356830012.4}{10^6} = 356.83 \text{ KN.mt. (Ans)}$$

Q. A rectangular cantilever beam of size 230 mm width x 500 mm effective depth is subjected to a bending moment of 80 kN/m. at working loads. Find the steel area required. The materials are M20 grade concrete & HYSD reinforcement of grade Fe415.

Shear Band of Development length

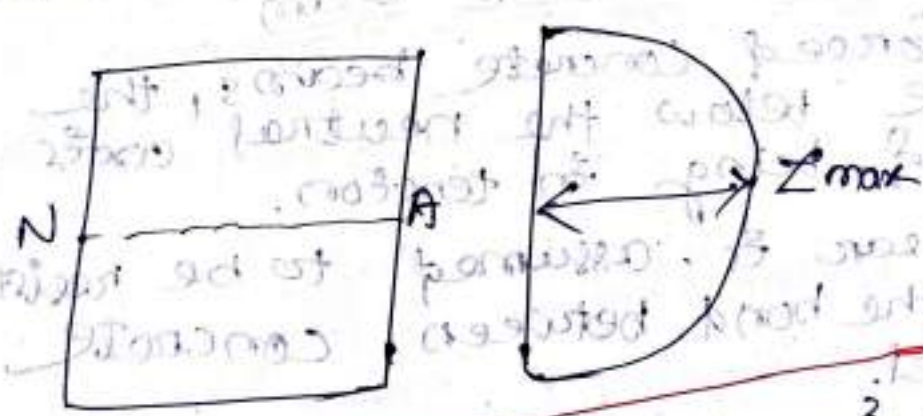
Diagonal shear (or) Flexure crack

Shear & Singly reinforced beam

- (i) Bending of singly reinforced beam is usually accompanied by shear.
- (ii) The combination of shear & bending stresses produces the principle stress which causes diagonal tension in the beam.

shear stress in beam

homogeneous beam



for rectangular beam $Z_{max} = \frac{3}{2} \cdot \frac{F}{A}$
 OR $1.5 \frac{F}{A}$

the theoretical shear stress distribution is parabolic and its value is zero at the top and bottom edges and maximum at the center of gravity of the beam.

(i) In the case of homogeneous beam that i.e. a concrete beam without reinforcement, the shear stress at any point is given by the elastic theory.

(ii) The distribution of shear stress across the homogeneous beam of rectangular section is parabolic in nature.

(iii) It is zero at the top & bottom & is maximum at the neutral axis.

(iv) The maximum shear stress $= 1.5 \times$ Average shear stress

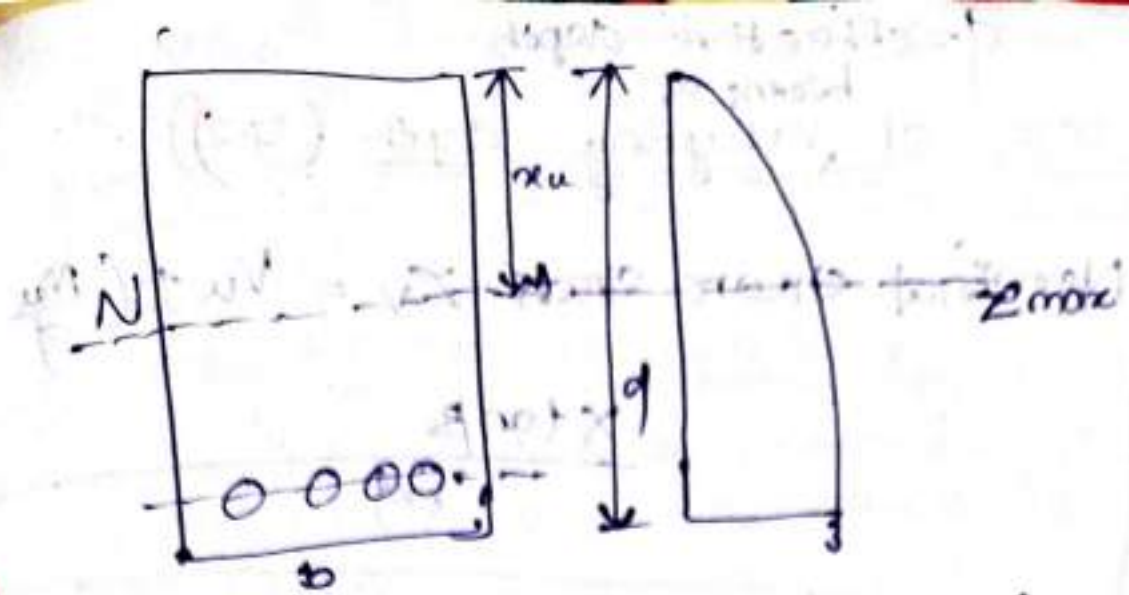
$$\tau_{max} = 1.5 \times \frac{F}{A}$$

Reinforced Concrete beam

In reinforced concrete beams, the concrete below the neutral axis is neglected being in tension.

The shear is assumed to be resisted by the bond between concrete & steel.

Hence, the shear stress in reinforced concrete beam varies parabolically with zero at the top, compression face, reaching maximum value at the neutral axis and constant from neutral axis up to the centre of gravity of steel bars.



(Reinforced Concrete Section)
 DT: 29.03.23

The maximum shear stress

$$Z_{max} = \frac{V}{b \times Z}$$

Z = lever arm
 V = shear force

In IS 456:2000 Code for shear stress is based on average shear stress across the section.

Arg: shear stress / Nominal shear stress

$$Z \cdot \frac{V_u}{bd}$$
 (Pg no - 72)

Where V_u = shear force due to design load
 b = Breadth of the member
 For flanged section b shall be taken as breadth of the web.

d = effective depth

In case of varying depth (72)

Nominal shear stress $Z_v = V_u \pm \left(\frac{M_u}{d} \right) \times \tan \beta$

where

M_u = Bending moment at the section

β = Angle betⁿ top & bottom edges of the beam

(73)

Design shear strength of concrete

Concrete is capable of resisting shear force to some extent. The shear resistance value depends mainly on grade of concrete & area of steel provided for resisting bending moment.

The design shear strength of concrete in beams is given without shear reinforcement in table - 19, pg No - 73 of IS: 456: 2000

Maximum shear strength in concrete (73)

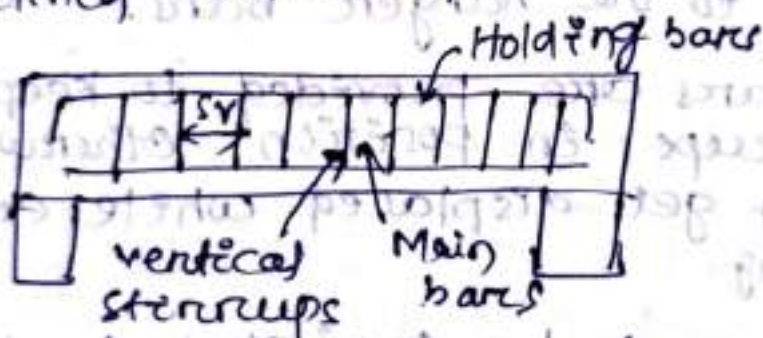
To avoid compression failure of the section in shear the nominal shear stress should not exceed the maximum shear stress in concrete.

the max^m shear strength in concrete given in table No 20 Pg - 73 of IS 456: 2000.

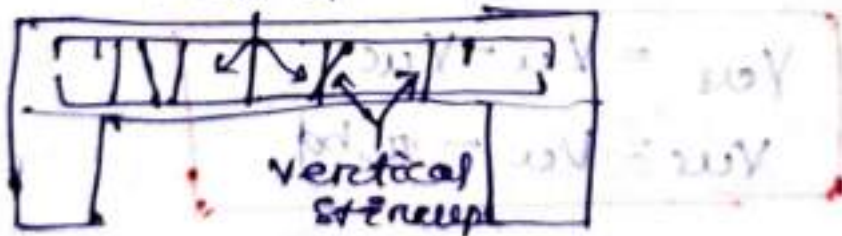
Forms of shear reinforcement

shear reinforcement is necessary if the nominal shear stress exceeds the design shear stress.
 In general, shear reinforcement is provided in any one of the following three forms.

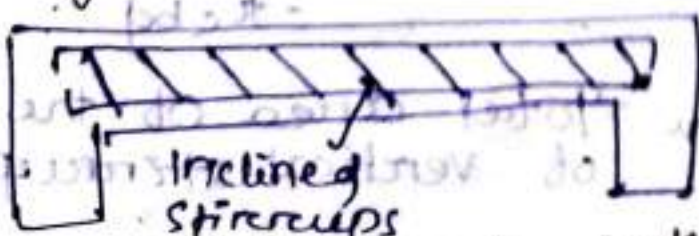
- (i) Vertical stirrups
- (ii) Bent-up bars with stirrups
- (iii) Inclined stirrups



shear reinforcement - vertical stirrups
 Bent-up bar

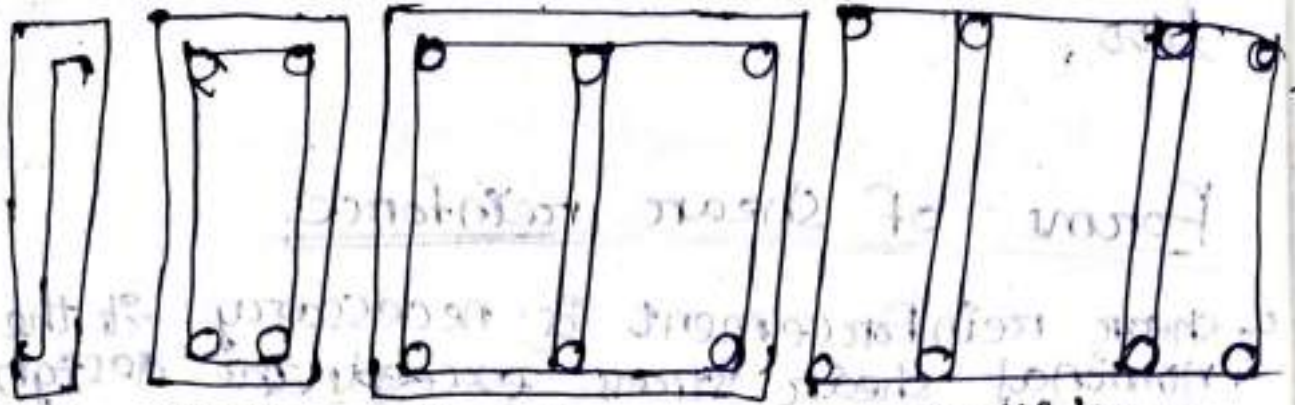


shear reinforcement - bent-up bars along with vertical stirrups



shear reinforcement - inclined stirrups

Vertical stirrups



One leg, Two leg, Four leg, Multi leg

Vertical stirrups

Generally the vertical stirrups are provided as two-legged or four-legged stirrups bend round the tensile reinforcement and taken to the compression zone and anchored to the hanger bars.

Hanger bars are provided to keep vertical stirrups in position otherwise they may get displaced while concrete is being cast.

The shear to be resisted by the shear reinforcement is given by

$$V_{us} = V_u - V_{uc}$$

$$V_{us} = V_u - \pi c b d$$

The diagram shows a rectangular beam cross-section with vertical stirrups and hanger bars. The stirrups are represented by vertical lines with hooks at the top and anchors at the bottom. The hanger bars are represented by horizontal lines connecting the stirrups at the top.

where,

V_{uc} = shear resistance of concrete
 $= \pi c b d$

A_{sv} = Total area of the legs of vertical stirrups

d = Effective depth of the section.

for vertical stirrups

$$(1) V_{us} = \frac{0.87 f_y A_{sv} d}{s_v}$$

for inclined stirrups

$$(2) V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} (\sin \alpha + \cos \alpha)$$

For single bar or single group of parallel bars.

$$V_{us} = 0.87 f_y A_{sv} \sin \alpha$$

Maximum shear reinforcement:

The minimum quantity of shear reinforcement should be provided for all beams which is given as 456:2000 (close no 8.5.1.6.) (Pg no. 48)

By the equation

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y}$$

Where,

A_{sv} = Total cross-sectional area of stirrup legs effective in shear.

s_v = Stirrup spacing along the length of the member.

b = Breadth of the beam or breadth of the web of flanged beam, and

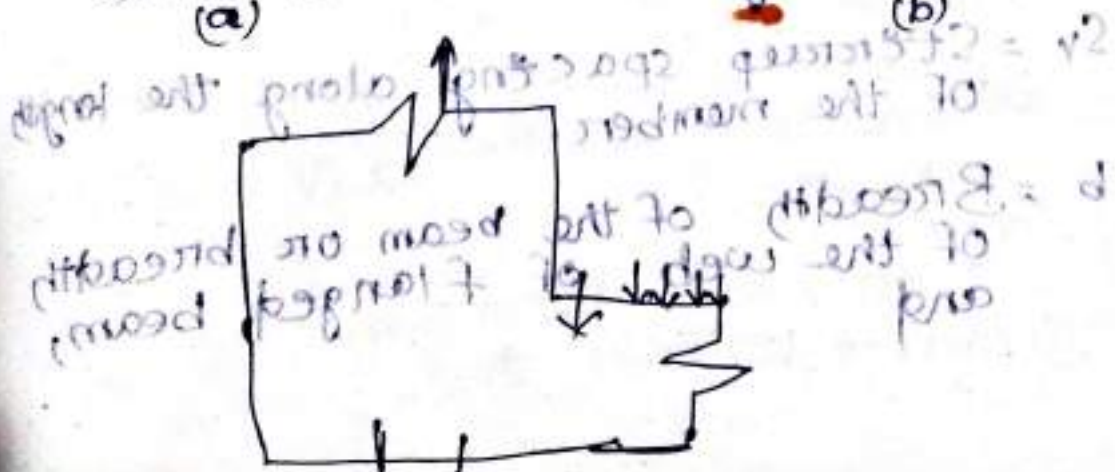
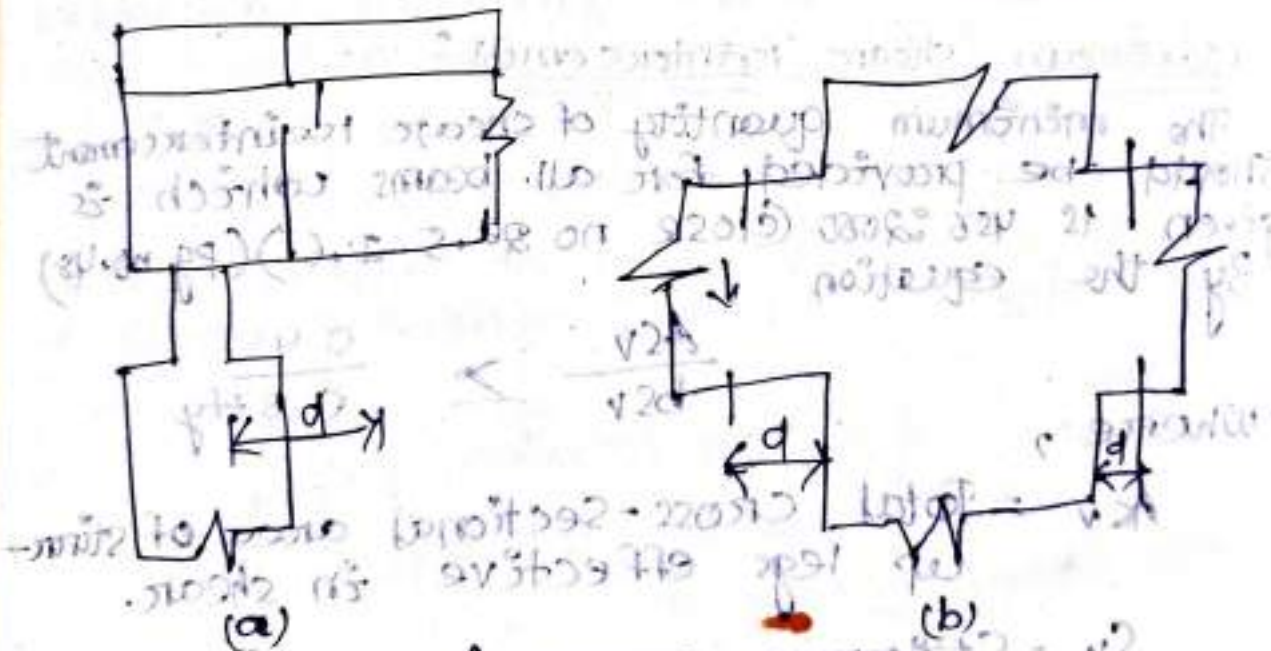
f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than $415 N/mm^2$

Maximum Spacing of Shear Reinforcement

The maximum spacing of shear reinforcement measured along the axis of the member shall not exceed $0.75d$ for vertical stirrup and D for inclined stirrup at 45° , where D is the effective depth of the section under consideration. In no case shall the spacing exceed $300 mm$.

To qualify stirrups are used $D \geq 3/4/33$

Critical Section for shear



4) For beams generally subjected to UDL
 of where the principal load in lateral
 load is located further than $2d$ from
 the face of the support where d
 is effective depth of the beam
 the critical section depend on the
 condition of supports as given in
 figure A, B, C

when the reaction in the direction
 of the applied shear introduces
 compression into the end region
 of the member section located
 at a distance than d from
 the face of the support may
 be design for shear (fig A & B)

Q. A T Beam section having 230mm
 width of rib and 460mm
 effective depth is reinforced
 with 5 nos of 16mm dia bar
 as tension reinforcement. The
 section is subjected to a factored
 shear of 52.5 kN. check the
 shear stress and design the
 shear reinforcement. The
 materials are M20 grade
 concrete and Fe 415 grade
 steel. For stirrup mild steel
 bar may be used. (6mm bar)

Given data

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$V_u = 52.5 \text{ kN}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times (16)^2 = 1005.31 \text{ mm}^2$$

$$Z_v = \frac{W_t}{b_d}$$

$$= \frac{5215 \times 10^3}{230 \times 460} = 0.496 \text{ N/mm}^2$$

$$\therefore \text{Imp} = \frac{100 A_{st}}{b_d} = \frac{100 \times 1005.31}{230 \times 460}$$

$$= 0.95 \quad \left(\begin{array}{l} \text{Pg 40-73} \\ \text{Table-19} \end{array} \right)$$

$$Z_c = 0.56 + 0.048 = 0.608 \text{ N/mm}^2$$

Here, $Z_v < Z_c$

\therefore shear reinforcement is required, select 6 mm dia mild steel bar for stirrup.

(Assume 2 leg)

$$A_{sv} = 2 \times \frac{\pi}{4} \times (6)^2$$

$$= 56 \text{ mm}^2$$

For minimum shear reinforcement

(Pg-48)

$$\frac{A_{sv}}{b_s v} \geq \frac{0.4}{0.87 f_y} \quad (a) \quad 0.75d = 0.75 \times 460$$

$$\frac{56}{230 \times s_v} \geq \frac{0.4}{0.87 \times 415} \quad (b) \quad 300 \text{ mm}$$

$s_v \leq 219.77 \text{ mm}$ \therefore provide 6 mm dia two legged stirrup 219.77 mm

Spacing shall not exceed

Dt: 4/4/23

Q. A T beam section having 230 mm width and 460 mm effective depth is reinforced with 5 Nos. of 16mm dia bars as tension reinforcement. The section is subjected to a shear stress of 90 kN. Check the shear stress, and design the shear reinforcement. The materials are M20 grade of concrete and Fe415 grade steel for stirrups, mild steel bar may be used.

Given data

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$V_u = 90 \text{ kN}$$

$$A_{st} = 5 \times \frac{\pi}{4} \times (16)^2 = 1605.31 \text{ mm}^2$$

$$\tau_v = \frac{V_u}{bd}$$

$$= \frac{90}{230 \times 460} = 0.85 \text{ N/mm}^2$$

$$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 1605.31}{230 \times 460} = 0.95$$

from table no-19 is 456 (2000)

$$\tau_c = 0.608 \text{ N/mm}^2$$

$$\tau_v > \tau_c$$

∴ Shear reinforcement shall be provided.

$$\begin{aligned}
 V_{us} &= V_u - Z_c \cdot b \cdot d \\
 &= (90 \times 10^3) - 0.608 \times 230 \times 460 \\
 &= 27673.6 \text{ N} = 25.67 \text{ kN}
 \end{aligned}$$

Using 6mm dia two legged mild steel bars for stirrup

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v} \quad \text{--- (1)}$$

$$\begin{aligned}
 A_{sv} &= 2 \times \frac{\pi}{4} \times (6)^2 = 56 \text{ mm}^2 \\
 \Rightarrow 25673.6 &= \frac{0.87 \times 250 \times 56 \times 460}{S_v}
 \end{aligned}$$

$$S_v = 218.23 \text{ mm}$$

Spacing shall not exceed

$$(i) 0.75 \times d = 0.75 \times 460 = 345 \text{ mm}$$

$$(ii) 300 \text{ mm}$$

$$\therefore \text{Spacing} = 218 \text{ mm}$$

provide 6mm dia two legged stirrup
at 218 mm spacing c/c

Q. A T-beam section having 230 mm width of rib, and 460 mm effective depth is reinforced with 5 Nos of 16 mm dia bars as tension reinforcement, and out of which 2 Nos bars are bent up at 45°. The section is subjected to a factored shear of 120 kN. Check the shear stress and design the shear reinforcement. The material are M20 grade concrete and Fe415 grade steel for stirrup mild steel bar may be used.

Given data

$$b = 230 \text{ mm}$$

$$d = 460 \text{ mm}$$

$$V_u = 120 \text{ kN}$$

∴ Nominal shear stress, $\tau_v = \frac{V_u}{bd}$

$$= \frac{120 \times 10^3}{230 \times 460} = 1.13 \text{ N/mm}^2$$

2 bars are bent up

$$\therefore A_{sv} = 2 \times \frac{\pi}{4} \times (16)^2 = 402 \text{ mm}^2$$

$$A_{sv} = 0.87 f_y A_{sv} \sin \theta$$

$$= 0.87 \times 415 \times 402 \times \sin 45^\circ$$

$$= 102630.9 \text{ N} = 102.63 \text{ kN}$$

Force remaining 3 bars

$$A_{st} = 3 \times \frac{\pi}{4} \times (16)^2 = 603 \text{ mm}^2$$

$$p_t = \frac{100 A_{st}}{bd} = \frac{100 \times 603}{230 \times 460} = 0.57$$

Table no - 19 (page no - 73) $Z = 0.50 \text{ N/mm}^2$

$$V_{us} = V_u - Z \cdot b \cdot d$$

$$= (120 \times 10^3) - (0.50 \times 250 \times 460)$$

$$= 67100 \text{ N}$$

$$= 67.1 \text{ kN}$$

$$0.56 \rightarrow 0.48$$

$$0.75 \rightarrow 0.56$$

$$0.25 \rightarrow 0.08$$

$$1 \rightarrow \frac{0.08}{0.25}$$

$$0.07 \rightarrow \frac{0.08}{0.25} \times 0.08$$

$$= 0.02$$

$$0.57 \rightarrow 0.48 + 0.02 = 0.50$$

∴ Shear resistance provided by bentup bar = $\frac{67.1}{\sqrt{2}} = 33.55 \text{ kN}$

∴ Shear resistance provided by vertical stirrup = $67.1 - 33.55$

$$= 33.55$$

∴ Using 6mm dia 2 legged mild steel bars for stirrup.

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{\sqrt{2}}$$

pg no - 73

$$A_{sv} = 2 \times \frac{\pi}{4} \times (6)^2 = 56 \text{ mm}^2$$

$$33.55 = \frac{0.87 \times 250 \times 56 \times 460}{33.55 \times 10^3} \geq 167 \text{ mm}$$

$$f_2 \cdot 0 = \frac{250 \times 56}{33.55 \times 10^3} = \frac{14000}{335500} = 0.0417$$

spacing shall not exceed.

$$(i) 0.75 \times d = 0.75 \times 460 = 345 \text{ mm.}$$

$$(ii) 300 \text{ mm.}$$

$$\therefore \text{spacing} = 167 \text{ mm.}$$

• provide 6 mm dia two legged stirrup at 167 mm spacing c/c (centre to centre)

Q. A rcc beam of span 5 m. is 250 mm wide and 500 mm deep effective. It has 4 bars of 22 mm tensile reinforcement. The beam carries a load of 30 kN/m. include self-weight. Design the beam for shear use M20 grade concrete and Fe 415 grade steel.

Given data

$$b = 250 \text{ mm, } w = 30 \text{ kN/m.}$$

$$d = 500 \text{ mm.}$$

$$L = 5 \text{ m.}$$

$$\text{Maximum shear force } (V_u) = \frac{wL}{2}$$

$$= \frac{30 \times 5}{2} = 75 \text{ kN.}$$

Nominal Shear stress

$$\tau_v = \frac{V_u}{bd} = \frac{75 \times 10^3}{250 \times 500} = 0.60 \text{ N/mm}^2$$

$$A_{st} = 4 \times \frac{\pi}{4} \times (22)^2 = 1520 \text{ mm}^2$$

Bond And Bond Stresses Dt: 10/04/23

The plane section before bending remains plane after bending.

The design has to take care of the following two cases of bond failure

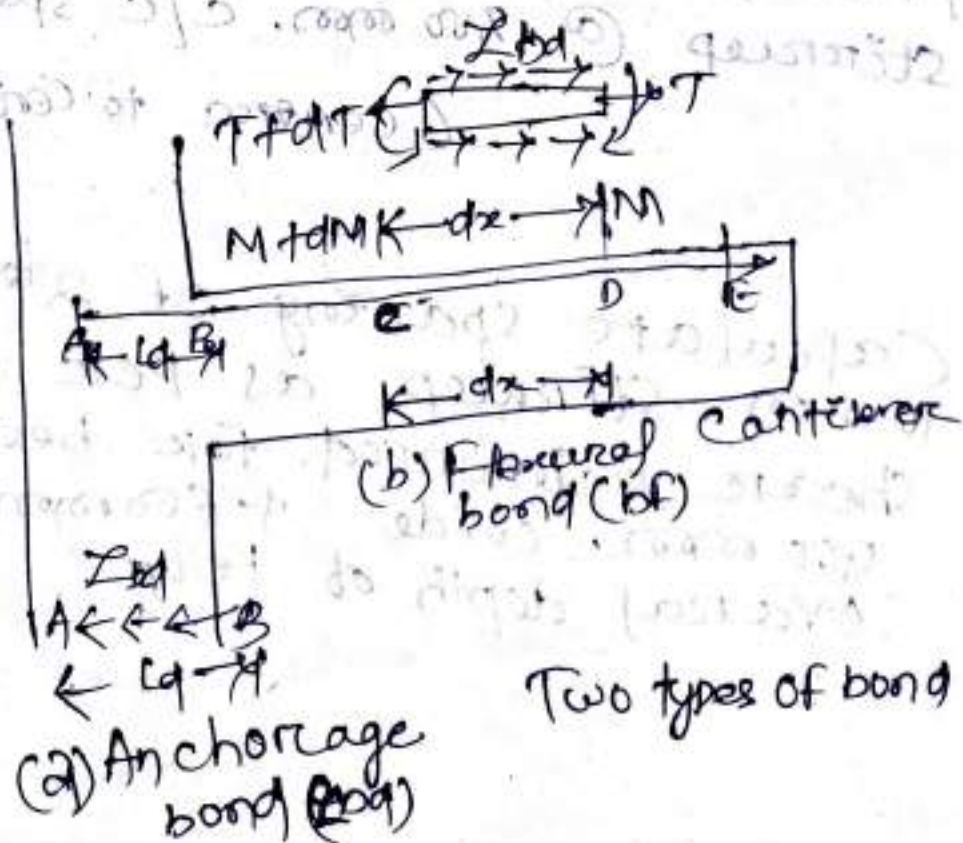
- (i) Anchorage bond
- (ii) Flexural bond

Flexural Bond

Flexural bond is one which arises from the change in tensile force carried by the bar along its length due to change in bending moment along the length of the member.

Flexural bond is critical at points where the shear is more.

This occurs at a particular section. There fore flexural bond stress is known as local bond stress.



Q. Calculate the spacing of 2 legged 10mm stirrup as per min^m shear reinforced for beam 400mm wide & 600mm overall depth of Fe 415.

Given data

$$b = 400 \text{ mm}$$

$$d = 600 \text{ mm}$$

min^m shear reinforcement

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y} \text{ (Pg no. 48)}$$

$$A_{sv} = 2 \times \frac{\pi}{4} \times (10)^2$$

$$= 157.07 \text{ mm}^2$$

$$\frac{157.07}{400 \times s_v} \geq \frac{0.4}{0.87 \times 415}$$

$$s_v = 354.43 \text{ mm}$$

Spacing shall not exceed

(i) 300 mm

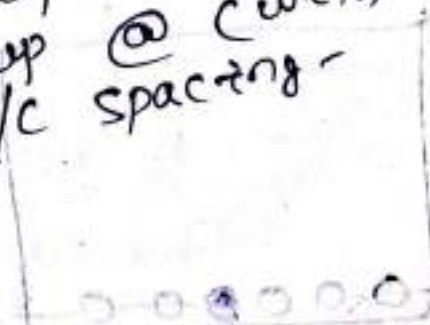
$$\text{(ii) } 0.75 \times d$$

$$= 0.75 \times 600$$

$$= 450 \text{ mm}$$

provided stirrup 300 c/c

2 legged 10mm dia @ c with respect to spacing -



Anchorage Bond

↳ The min^m anchorage length required to resist design force in the bar is called anchorage length.

↳ The length of the bar L_d required to transfer the force in the bar to the surrounding concrete through bond is called development length.

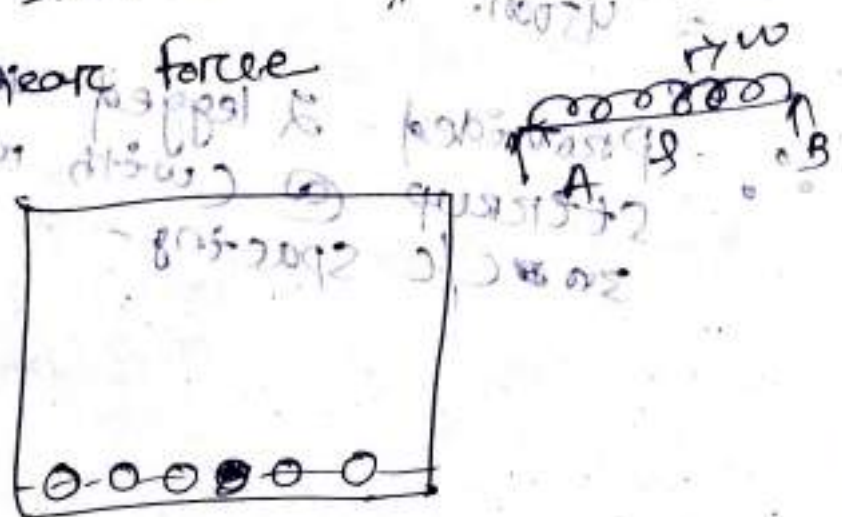
↳ The design bond stress in limit state method for plain bars in tension is given

$\tau_{bd} = \frac{f_y}{\sigma_{bc}}$ of IS 456:2000

Q. A singly reinforced rectangular beam 300 x 600 mm carries a uniformly distributed load 50 kN/mt including self weight over a simply supported beam of span 6 mt. 2 bars of 25 mm dia of which are cranked up near the support. Design shear reinforcement at support.

Nominal shear stress $\tau_v = \frac{V_u}{b d}$

Shear force $V_u = \frac{w L}{2}$



Given data

L = 6m
W = 50 kN/m
fy = 415
fck = 20

b = 300mm
d = 600mm

Vu = Factored load

D.L = F.O.S = 1.5
Limit state method

$$\frac{WL}{2} = V_u = \frac{50 \times 6}{2} = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

Nominal shear stress

$$\tau_v = \frac{V_u}{bd} = \frac{150 \times 10^3}{300 \times 600} = 0.83 \text{ N/mm}^2$$

UDL = 50 kN/m

Factored load = 50 x 1.5 = 75

$$\tau_v = \frac{V_u}{bd} = \frac{75 \times 6}{300 \times 600} = 225 \text{ kN}$$

$$V_u = \frac{WL}{2}$$

$$\tau_v = \frac{V_u}{bd} = \frac{225 \times 10^3}{300 \times 600} = 1.25$$

$$A_{st} = \frac{\tau_v}{\sigma_{st}} \times \frac{L}{4} \times (25)^2 \times 3.0 = 1472.62$$

$$\rho_t = \frac{100 A_{st}}{bd}$$

$$= \frac{100 \times 1472}{300 \times 600}$$

$$= 0.81\%$$

Table - 19 (Pg. 73)

$$Z_c = 0.58 N/mm^2$$

$$Z_v > Z_c$$

∴ Shear reinforcement

$$V_{usb} = 0.87 F_y A_{sv} \sin \alpha$$

$$A_{sv} = 2 \times \frac{\pi}{4} \times (25)^2$$

$$= 981.7477 \text{ mm}^2$$

$$V_{us} = V_u - Z_c b d$$

$$= \frac{225 \times 10^3}{1000} - 0.58 \times 300 \times 600$$

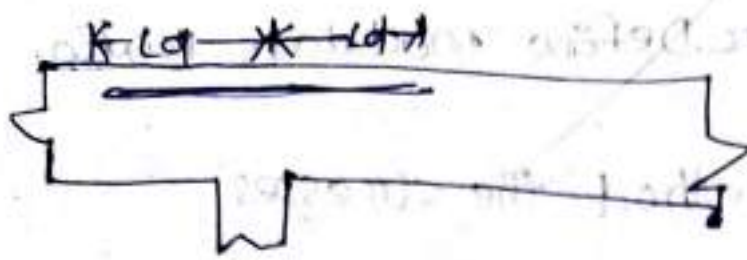
$$= 120600 \text{ N}$$

104175
Reel Area
120600 N

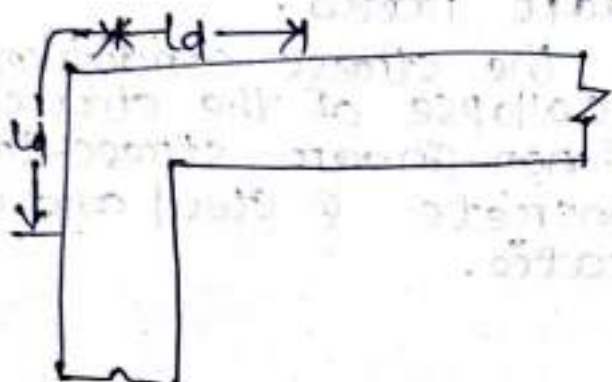
$$V_{usb} = 0.87 F_y A_{sv} \sin \alpha$$

$$= 0.87 \times 415 \times 981.7477 \times \sin 45^\circ$$

$$= 250600 \text{ N}$$

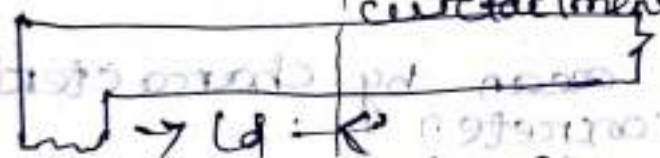


(a) For negative reinforcement near support

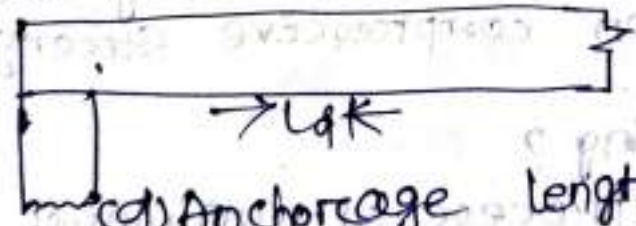


(b) Anchorage length in columns for Anchorage Bond

-ve reinforcement of beam, Theoretical point of curtailment for tension steel.



(c) Anchorage length for curtailed beam



(d) Anchorage length for lap splicing Anchorage Bond (L_d)

↳ Anchorage bond is the which increases over the length of anchorage provided for a bar.

↳ It also increases near the end or bottom point of reinforcing bar.

↳ The anchorage bond must resist the pulling out of the bar if it is tension or pushing in of the bar if it is in compression.

↳ Anchorage bond stress is developed over a specified length L_d .

↳ Anchorage is provided so that the steel is not under tension load can be reached & popout will not occur.

↳ Anchorage of reinforcing bar is necessary when the development length of the reinforcement is larger than the structure.

Dt: 19/04/23

Anchorage bond stress

↳ The required length necessary to develop pull resisting force is called anchorage length in case of axial tension or compression & development length in case of flexural tension ~~of~~ & is denoted by L_d .

↳ Design bond stress for deformed bar in tension is as per IS 1786 & these values shall be increased by 60%.

↳ Design bond stress for bars in compression shall be increased by 25% as compare to bond stress in tension.

Note

$Q \cdot L_d = 1.2 \text{ for } F_{250}$
(Design bond stress) (for tension)

- (i) Z_{bd} for tension for deformed bar?
- (ii) Z_{bd} for compression for deformed bar?

(i) $Z_{bd} = 1.2 \times 1.6 = 1.92$

(ii) $Z_{bd} = 1.2 \times 1.6 \times 1.25 = 2.4$

~~(i) Z_{bd} for compression for Fe250~~

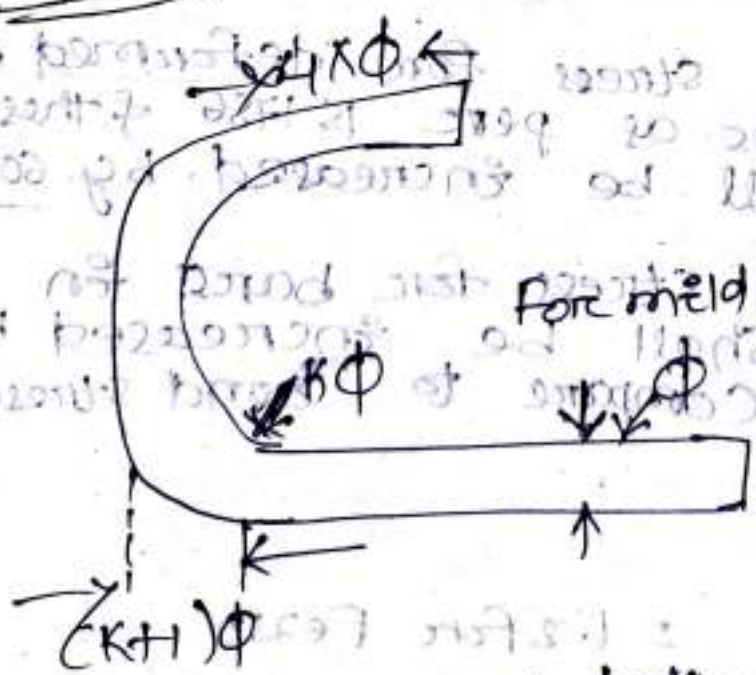
Z_{bd} for tension for deformed bar?

(i) Z_{bd} for tension for deformed bar?

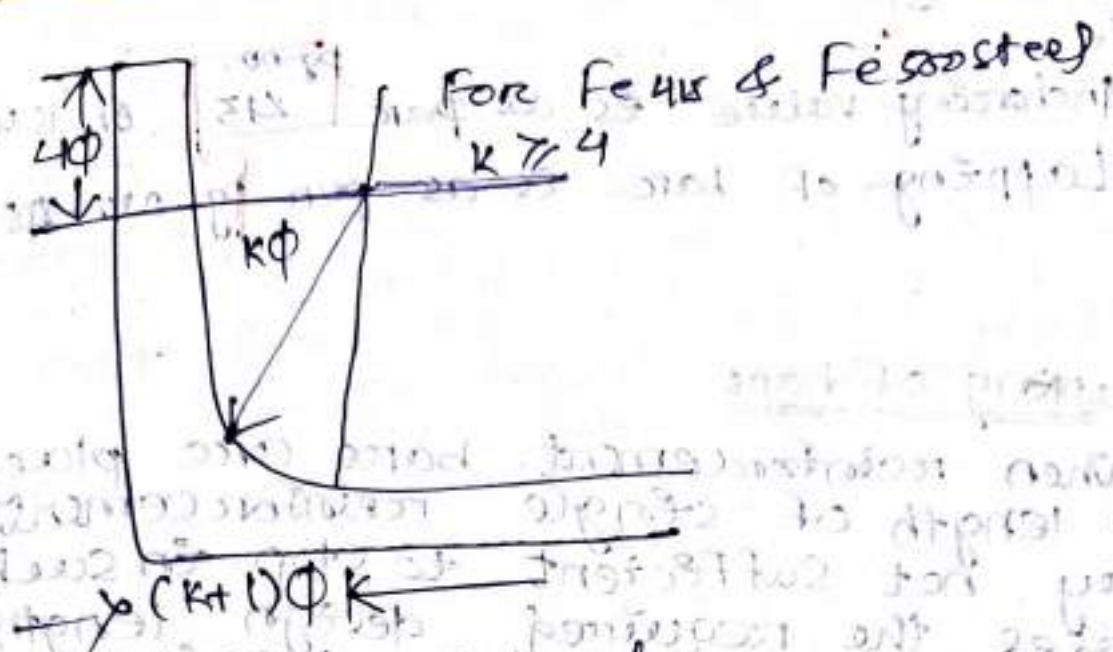
(ii) Z_{bd} for compression for Fe250?

(i) $Z_{bd} = 1.2 \times 1.25 = 1.5$

Equivalent Development length of hook & Bend.



(a) standard hook



(b) standard bend.

(i) To improve ~~time~~ anchorage of the bars many hooks are provided in standard & high yield bend bars. ~~types~~ ~~are~~ ~~provided~~ ~~in~~ ~~plain~~ ~~bars~~ ~~are~~ ~~provided~~ ~~in~~ ~~bars~~.

(ii) The equivalent development length for these standard hook & bends may be taken as per pg no-43 of IS 456.

Bend & hook :- Bends and hooks shall conform to IS 2502.

1) Bends :- The anchorage value of bend shall be taken as 4 times the diameter of the bar for each 45° bend. Subject to a maximum of 16 times the diameter of the bar.

2) Hooks :- The anchorage value of a standard type hook shall be equal to 16 times the diameter of the bar.

Note

- ↳ Anchoring value is as per pg no 43 of IS 456:2000
- ↳ Lapping of bar is as per pg No-45

Lapping of bars

↳ When reinforcement bars are placed the length of single reinforcement may not be sufficient to span in such cases the required design length is achieved by overlapping of 2 bars is called lap length.

↳ This lap length ensures safe & efficient transfer of loads from one bar to another bar.

↳ This lap length is necessary for reinforced concrete structure to allow the transfer of both tensile & compressive loads from one reinforced bar to another by means of skin friction.

Rules For lap length Determination

(a) Lap splices shall not be used for bars larger than 36mm; for larger diameter bars may be welded (see 12.4); in cases where welding is not practicable, lapping of bars larger than 36mm may be permitted in which case additional spirals should be provided around the lapped bars.

(b) Lap splices shall be considered as staggered if the centre to centre distance of the splices is not less than 1.3 times the lap

length calculated as described in (c)

(c) Lap length including anchorage value of hooks for bars in flexural tension shall be L_d (see 26.2.1) or 30ϕ whichever is greater and for direct tension shall be $2L_d$ or 30ϕ whichever is greater. The straight length of the lap shall not be less than 15ϕ or 200 mm. The following provision shall also apply.

Where lap occurs for a tension bar located at:

1) Top of section as cast and the min^o cover is less than twice the diameter of the lapped bar, the lap length shall be increased by a factor of 1.4.

2) Corner of a section and the minimum cover to either face is less than twice the diameter of the lapped bar or where the clear distance between adjacent laps is less than 75 mm or 6 times the diameter of lapped bar, whichever is greater, the lap length should be increased by a factor of 1.4.

Where both condition (1) & (2) apply, the lap length should be increased by a factor of 2.0.

Notes Splices in tension members shall be enclosed in spirals made of bars not less than 6 mm diameter with pitch not more than 100 mm.

(d) The lap length in compression shall be equal to the development length in compression, calculated as described in 26.2.1 but not less than 24ϕ .

(e) When bars of two different dia are to be spliced, the lap length shall be calculated on the basis of diameter of the smaller bar.

(f) When splicing of welded wire fabric is to be carried out, lap splices of wires shall be made so that overlap measured between the extreme cross wires shall be not less than the spacing of cross wires plus 100 mm.

(g) In case of bundled bars, lapped splices of bundled bars shall be made by splicing one bar at a time, such individual splices within a bundle shall be staggered.

Lap length in Beam, Column & Slab Pg-48

According to IS 456:2000 lap length of columns can be calculated using following formula.

$$\text{Lap length of column} = 40 \phi \times d$$

Where ϕ = diameter of bar

According to IS 456:2000 the lap length of slab can be calculated using following formula.

$$\text{Lap length of slab} = 60 \phi \times d$$

According to IS 456:2000 the lap length of beams can be calculated using following formula.

$$\text{Lap length of } \phi = 60 \phi \times d$$

Q. Calculate the anchorage length in tension & compression.

- (i) single mild steel bar of diameter 20 mm in concrete of grade M20.
- (ii) An hysd bar of grade Fo 415 of diameter 16 mm in concrete of grade M20.

(i) M.S bar (Fe 250)

Tension
 Design stress for M.S bar
 $\sigma_s = 0.87 f_y = 0.87 \times 250 = 217.5 \text{ N/mm}^2$

\therefore Anchorage length / Development length

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} \quad (\text{Pg-42})$$

Design bond stress $\tau_{bd} = 1.2 \text{ N/mm}^2$ (Pg-43)

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{20 \times 217.5}{4 \times 1.2} = 906.25$$

Compression

$$\tau_{bd} = 1.2 \times 1.25 = 1.5 \text{ N/mm}^2$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{20 \times 217.5}{4 \times 1.5} = 725$$

(ii) Design bond stress $L_{bd} = 1.2 \times 1.6 \times 1.28$
 ~~$= 2.4 \times 1.92 \text{ N/mm}^2$~~

Tension

$$\sigma_s = 0.87 f_y$$

$$= 0.87 \times 415$$

$$= 361.05 \text{ N/mm}^2$$

$$L_d = \frac{10 \times 361.05}{4 \times 2.4} = 601.75$$

$$L_d = \frac{16 \times 361.05}{4 \times 1.92} = 752.18$$

Compression

$$\sigma_s = 0.87 \times 415 = 361.05$$

$$L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{16 \times 361.05}{4 \times 2.4}$$

$$= 601.75$$

$$\frac{2.4 \times 1.92 \times 0.5}{1.2 \times 1.28}$$

$$\frac{\phi \sigma_s}{4 \tau_{bd}} = L_d$$

$$L_{pd} = 1.5 \times 1.28 \times 1.28$$

$$\frac{2.4 \times 1.92 \times 0.5}{1.2 \times 1.28} = \frac{\phi \sigma_s}{4 \tau_{bd}} = L_d$$

Setting

Checking Development length of Tension Bar

- ↳ The stress in reinforcing bar at every cross-section must be developed on both side of the section.
- ↳ This is done by providing development length L_d to both sides of the section. Such as development length is usually available at mid span where positive bending moment is maximum for simply supported beam. Similarly, such a development length is usually available at the intermediate supports of a continuous beam where negative bending moment is maximum. Special checking for development length is essential at following location.

- (i) At simple support
- (ii) At cantilever support
- (iii) At point of contraflexure
- (iv) At point of bar cut off

Requirement of Development length

The IS 456 code saying that at the simple support the positive moment tension reinforcement shall be limited to a diameter such that $L_d \leq \left(\frac{M_i}{V}\right) + l_0$

Where,

L_d = Development length.

M_i = Moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d ;

$f_d = 0.87 f_y$ in the case of limit state design and the permissible stress σ_{at} in the case of working stress design.

f_y
yield stress

V = Shear force at the section due to design loads.

L_0 = Sum of the ~~all~~ anchorage beyond the centre of the support and the equivalent anchorage value of any hook or mechanical anchorage at simple supports; and at a point of inflection, to be limited to the effective depth of the member or 12ϕ whichever is greater and

ϕ = diameter of bar.

The value of M_1/V in the above expression may be increased by 30% when the ends of the reinforcement are confined by a compressive reaction.

$$\text{Compression} \geq (1.3) \times \left(\frac{M_1}{V} \right) + L_0$$

Internal

Q. A simply supported beam is 25 cm. ~~by~~ $\times 250$ cm and has two nos 90 mm hyd bars going into the support. If the shear force at the centre of support is 110 kN at working

load. Determine the anchorage length (M20 bars of Fe415 grade steel).

Soln →

Factored shear force = 1.5×110
 $= 165 \text{ kN}$

$$A_{st} = 2 \times \frac{\pi}{4} \times (20)^2$$

$$= 628 \text{ mm}^2$$

Note: clear cover
 Beam = 25 mm < 2000
 Column = 40 mm
 Footing = 75 mm

Assuming as end clear cover to the longitudinal bar

Effective length = ~~2500 mm~~
 $= 500 \text{ mm}$

Effective length = $500 - 25 - \frac{20}{2}$
 $= 465 \text{ mm}$

$z_u = 0.87 f_y A_{st} C_d = 0.42 z_u$

pg-94

$z_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$
 $= \frac{0.87 \times 415 \times 628}{0.36 \times 20 \times 250}$
 $= 126 \text{ mm}$

$$x_{u,max} = 0.48d \quad (\text{pg no-70})$$

$$0.11X = 0.48 \times 465$$

$$M_x = 223.2 \text{ mm}$$

$x_u < x_{u,max}$ (under reinforced section)

$$M_1 = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 628 (465 - 0.42 \times 126)$$

$$= 95.43 \times 10^6 \text{ N/mm}^2$$

$$= 95.43 \text{ kN/m}^2$$

Anchorage length, $l_d = \frac{0.87 \times f_y \times \phi}{4 \sigma_{bd}}$

$$l_d = 1.2 \times 1.6 = 1.92$$

$$l_d = \frac{0.87 \times 415 \times 20}{4 \times 1.92}$$

$$= 940.28$$

If the bar is given at 90° bend at centre of support therefore this anchorage value $l_0 = 8\phi$

max. $l_0 =$

$$= 8 \times 20 = 160 \text{ mm}$$

$$L_d \leq 1.3 \left(\frac{M_1}{V} + L_0 \right) \quad [\text{checking for development length}]$$

$$\Rightarrow 47\phi \leq 1.3 \left(\frac{93.43 \times 10^6}{165 \times 10^3} + 160 \right)$$

$$\Rightarrow \phi \leq \underline{\cancel{19mm}} \quad 20.08 \text{ mm.}$$

Since actual bare dia ϕ of 20mm, is ~~greater~~ ^{less} than 20mm, therefore there is no need to increase the anchorage length L_0 to 12ϕ i.e. 240mm.

~~$$L_d \leq 1.3 \left(\frac{M_1}{V} + L_0 \right)$$~~

~~$$47\phi \leq 1.3 \left(\frac{93.43 \times 10^6}{165 \times 10^3} + 240 \right)$$~~

Internal Question

~~$12 \times 5 = 10 \text{ m}$~~

~~$15 \times 2 = 10 \text{ m.}$~~

Answer all Questions

2) A cantilever supported beam is 500 mm x 1000 mm and has 5-16mm HYSD bars going into the supports. If the shear at the support is 150 kN factored load. Determine the anchorage length (Use M30 concrete and Fe 500 grade steel).

or

1) A singly reinforced rectangular beam 400 x 750 mm total depth carries a point load of 75 kN including over a simply supported beam of span 7 m and is reinforced with 4 no. of 20mm.

1/2

$$V_{usb} = 0.87 f_y A_{sv} \cdot s \sin \alpha$$

$$A_{sv} = 2 \times \frac{\pi}{4} \times (25)^2$$

$$= 981.74 \text{ mm}^2$$

$$\therefore V_{usb} = 0.87 \times 415 \times 981.74 \times \sin 45^\circ$$

$$= 250639.10$$

$$= 250640 \text{ N}$$

Shear Resistance
Design of vertical stirrup

\therefore Shear resistance by vertical stirrup

$$(V_{us1}) \therefore \frac{V_{us}}{2} = \frac{120600}{2}$$

$$= 60300 \text{ N}$$

$$V_{us1} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}$$

Assume 10 mm dia stirrup

$$\Rightarrow 60300 = \frac{0.87 \times 415 \times 3 \times \frac{\pi}{4} \times 10^2 \times 600}{S_v}$$

$$\Rightarrow S_v \times 60300 = 319014007.8$$

$$\Rightarrow S_v = \frac{319014007.8}{60300}$$

$$= 5290.44 \text{ } 846.47 \text{ mm}$$

Min^m shear reinforcement

$$\frac{A_{sv}}{bsv} \geq \frac{0.4}{0.87f_y} \quad (\text{pg NO-48})$$

$$\frac{3 \times \frac{\pi}{4} \times (10)^2}{300 \times s_v} \geq \frac{0.4}{0.87 \times 415}$$

$$\Rightarrow 3 \times \frac{\pi}{4} \times (10)^2 \times 0.87 \times 415 = 0.4 \times 300 \times s_v$$
$$\Rightarrow 85070.40 = 120 s_v$$

$$\Rightarrow s_v = \frac{85070.40}{120}$$

$$\Rightarrow s_v = 708.92$$

$$\Rightarrow 708.92 \geq s_v$$

\therefore Max^m spacing $\rightarrow 0.75 \times d = 0.75 \times 600 = 450 \text{ mm}$
or 300 mm .

\therefore Spacing = 300 mm (Ad)

18.18

5th Analysis and Design of T-Beam

Date: 27.04.23

Introduction

↳ A T-beam is a structural element able to withstand large loads by resistance in the beam or by internal reinforcements.

↳ T-beam are the section in which the greater width is at the compression side and reduced width below the neutral axis.

↳ Since the concrete below the neutral axis does not resist any bending moment but simply serves to embed steel and also intensity of compressive force above neutral axis is less.

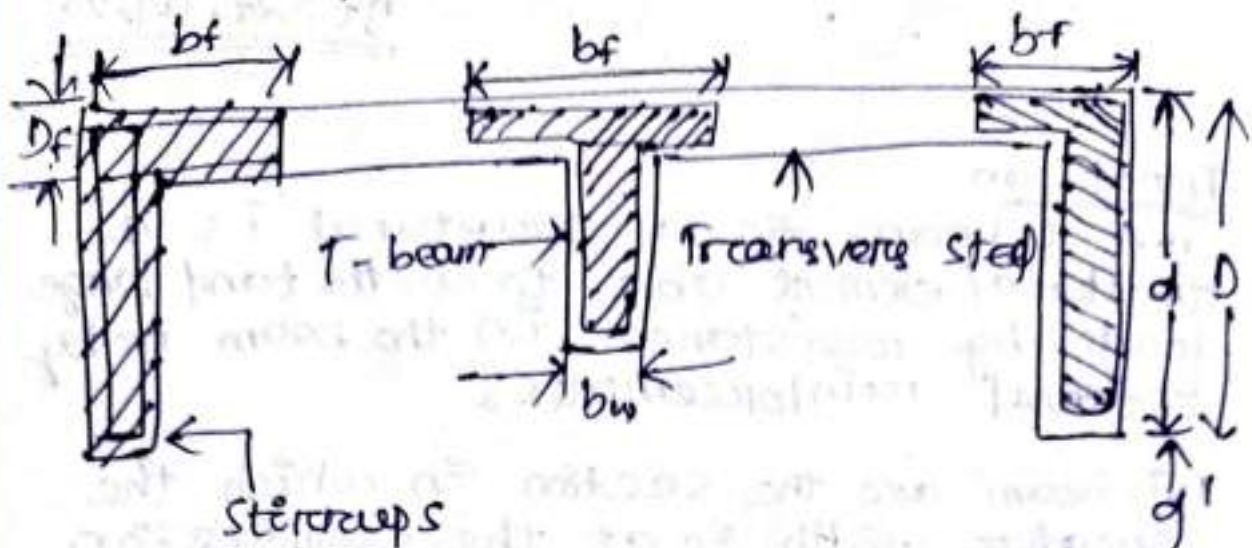
↳ Beams in R.C buildings are casted monolithically, some portion of the slab is often considered to act together with the beam. If the slab is present on both the sides, the beam is called a T-beam, if the slab is present only one side, it is called an L-beam.

Date: 1/5/23

Advantages of T-beams

- (i) It increases moment of resistance of the beam.
- (ii) T-beam able to withstand large loads.
- (iii) As most of the compressive force is shared by the flange the depth of the beam require is less.

Therefore maximum deflection is also less.



flanged beam.

Dimension of T-beam

Effective width of flange

It is the portion of the slab which acts monolithically with the beam & resists compressive stress. 23.62

As per clause no. 23.62 it is 46 (pg-36) the effective width of flange should be taken as for T-beam.

a) For T-beam, $b_f = \frac{l_0}{6} + b_w + 6D_f$

b) For L-beams, $b_1 = \frac{l_0}{12} + b_w + 3D_1$

b_f should not be greater than breadth of web + half the sum of clear distance to the adjacent beam on either side.

For isolated beams, the effective flange width shall be obtained taken as follows.

For, T-beam, $b_1 = \frac{l_0}{\left(\frac{l_0}{b_1}\right) + 4} + b_w$

For, L-beam, $b_f = \frac{0.5 l_0}{\left(\frac{l_0}{b_f}\right) + 4} + b_w$

The effective flange width of an isolated beam should not be greater than actual width of the flange where

b_f = effective width of flange.

l_0 = distance between points of zero moments in the beam.

b_w = breadth of the web,

D_f = thickness of flange, and

b = actual width of the flange.

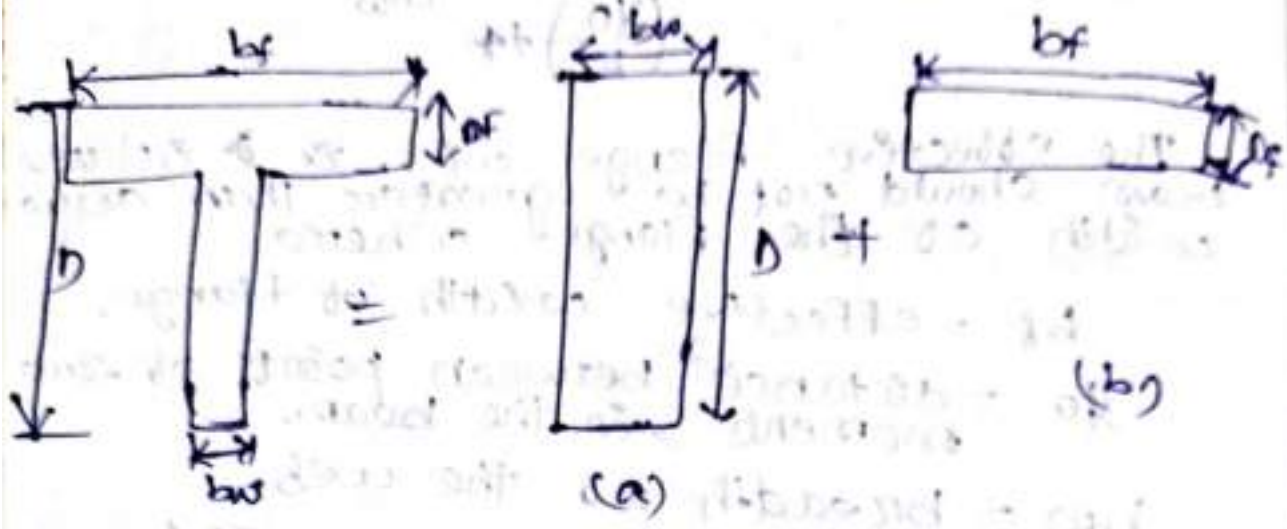
Date: 2/05/23

The flanged beam analysis and design are similar to doubly reinforced rectangular beam. In doubly reinforced beam additional compressive is provided by adding reinforcement in compression zone where additional reinforcement is provided by the slab concrete.

If the span of the slab is parallel to that of beam, the span of slab can be made to span in the direction perpendicular to that of the beam by adding some reinforcement in the slab.

The moment of resistance of a T-beam is the sum of moment of resistance of beam (A) and moment of resistance of beam (b).

Similarly steel area required for beam shall be equal to the sum of steel required for beam (a) & steel required for beam (b)

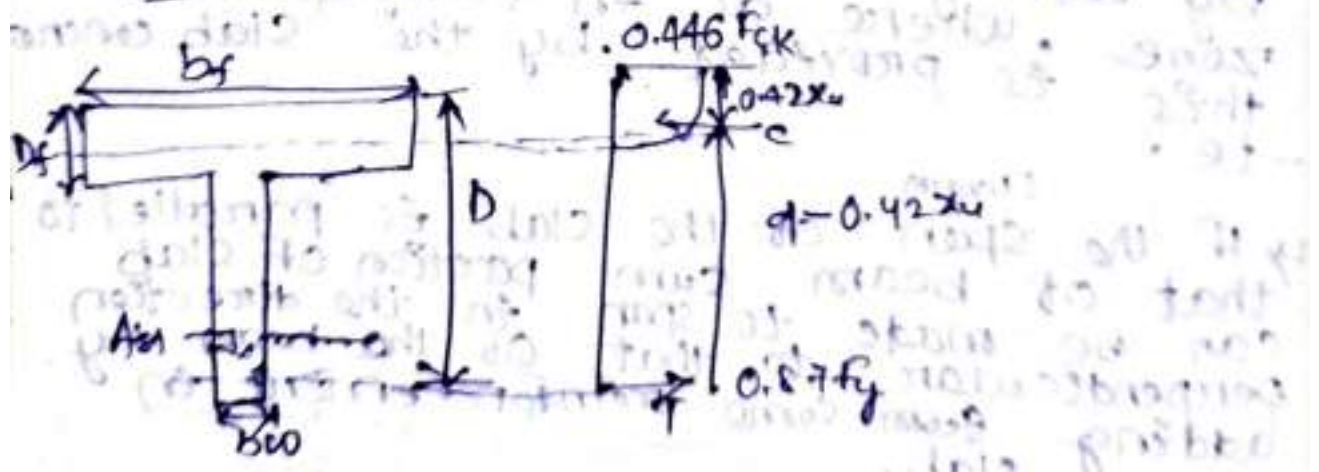


Position of Neutral Axis

For a flanged beam neutral axis either lies in flange or lies in web.

(1) Neutral axis lies in flange

(2) Neutral axis lies in flange (\$x_u < t_f\$)



When a neutral axis lies in the flange the size of compression zone becomes \$b_f \times x_u\$

As concrete doesn't resist any tension the width of tension zone has no effect on the moment of resistance of the section.

Therefore this beam can be analysed as a rectangular beam of dimension $(b_f \times d)$ & the formula derived for rectangular beam shall be apply.

For a singly reinforced rectangular beam equating total compression & total tension

$$\therefore x_u = \frac{0.87 f_y A_{st}}{0.36 F_{ck} b_f}$$

$$x_u \leq x_{u,max}$$

To find out type of beam $x_{u,max}$ shall be find out & compared with actual value of neutral axis i.e x_u .

- IF $x_u < x_{u,max}$ → Under reinforced beam
- IF $x_u > x_{u,max}$ → Over reinforced beam
- IF $x_u = x_{u,max}$ → Balanced

For under reinforced beam,

$$M_u = 0.36 F_{ck} b_f x_u (d - 0.42 x_u)$$

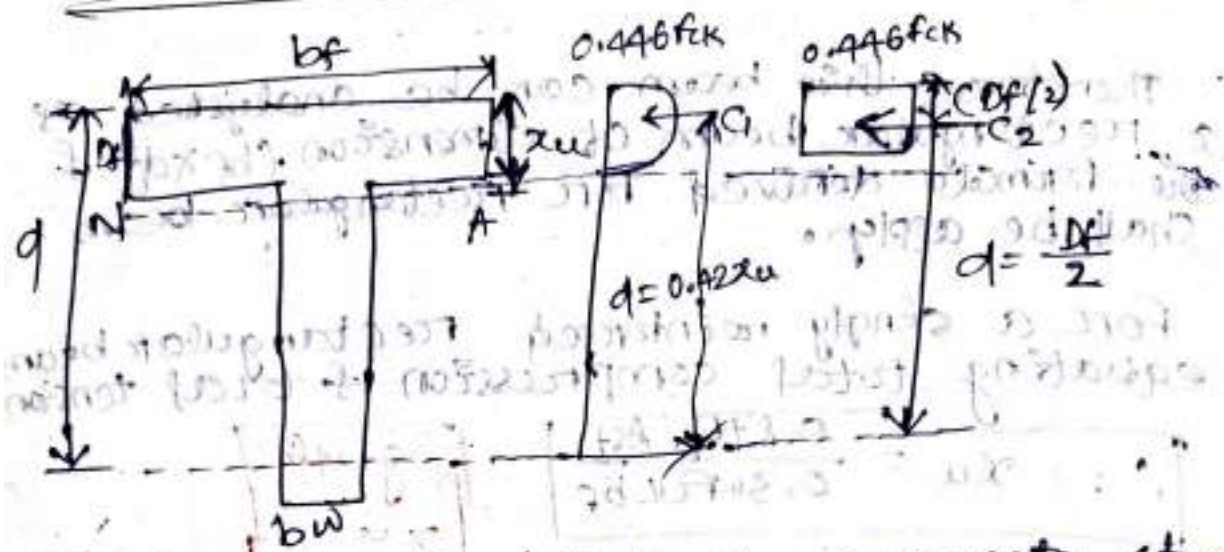
$$= 0.87 f_y A_{st} (d - 0.42 x_u)$$

For over reinforced beam,

$$M_{u,lim} = 0.36 F_{ck} b_f x_{u,max} (d - 0.42 x_{u,max})$$

Dt: 4/05/23

Neutral axis lies in web ($2x_u > D_f$)



That stress ~~is~~ block in concrete show that the stress is uniform upto $3/7x_u$ & them parabolic.

Case-1

$(D_f \leq \frac{3}{7}x_u)$

In this case the stresses are uniform in flange.

∴ Total tension = $0.87 f_y A_{st}$

∴ Total compression = $0.36 f_{ck} bw x_u + 0.446 f_{ck} (bf - bw) \times D_f$

Moment of resistance, $M_{ur} = 0.36 f_{ck} bw x_u (d - 0.47 x_u) + 0.446 f_{ck} (bf - bw) \times D_f (d - \frac{D_f}{2})$

Case-2 ($D_f > \frac{3}{7}x_u$)

The rectangular stress block in this case is assume to be equal to ' y_f '.

Total tension = $0.87 f_y A_{st}$

∴ Total compression = $0.36 f_{ck} b_w x_u + 0.446$

$f_{ck} (b_f - b_w) X_{DF}$

Moment of resistance, M_u

$= 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.446$
 $f_{ck} (b_f - b_w) X_{DF} (d - \frac{y_f}{2})$

where, $y_f = 0.15 x_u + 0.65 D_f$

$D_f = \frac{l_0}{16} + b_w + 6 D_f$

$\frac{l_0}{16} + b_w + 3 D_f \rightarrow L\text{-beam}$

Q. A T-beam of effective flange width 1200 mm, thickness of slab 100 mm, width of web 300 mm, & effective depth 560 mm is reinforced with 4 nos. of 25 mm dia - bars. Calculate factored moment of resistance. The materials are M20 grade concrete & Fe415 grade steel.

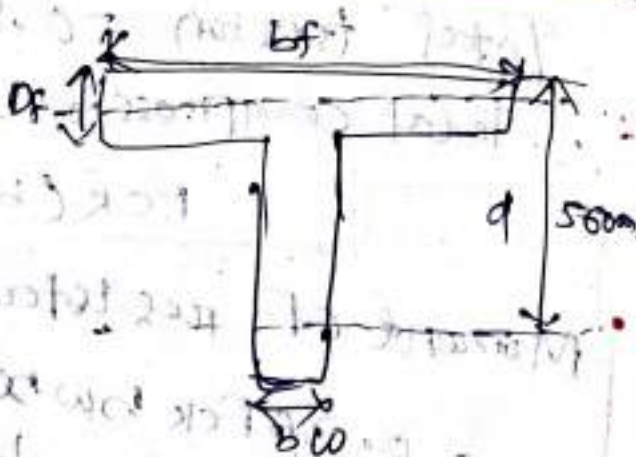
Soln: Given data

$$bf = 1200 \text{ mm}$$

$$Df = 100 \text{ mm}$$

$$bw = 300 \text{ mm}$$

$$d = 560 \text{ mm}$$



$$\therefore A_{st} = 4 \times \frac{\pi}{4} \times (25)^2 = 1964 \text{ mm}^2$$

\therefore Assume neutral axis lies in flange ($x_u < Df$)

$$\therefore \text{Total tension} = 0.87 f_y A_{st}$$

$$\therefore \text{Total compression} = 0.36 f_{ck} b_f x_u$$

Equating total compression = Total tension

$$\Rightarrow 0.36 f_{ck} b_f x_u = 0.87 f_y A_{st}$$

$$\Rightarrow 0.36 \times 20 \times 1200 \times x_u = 0.87 \times 415 \times 1964$$

$$\Rightarrow 8640 x_u = 709102.2$$

$$\Rightarrow x_u = \frac{709102.2}{8640}$$

$$= 82.07 \text{ mm} < 100 \text{ mm (Df)} \text{ (OK)}$$

\therefore Neutral axis lies in flange only

$$\text{For } f_e \text{ ULS, } x_{u, \max} = 0.48 \lambda_f$$

$$= 0.48 \times 560$$

$$= 268.8 \text{ mm}$$

$\therefore x_u < x_{u, \max}$ (Under reinforced section)

$$\begin{aligned} M_u &= 0.36 F_{ck} b_f x_u (d - 0.42 x_u) \\ &= 0.36 \times 20 \times 1200 \times 82.07 \times (560 - 0.42 \times 82.07) \\ &= 372645760.4 \text{ ~~N.m~~ N.m} \\ &= 372.64 \times 10^6 \text{ KN.m (Ans)} \end{aligned}$$

Q. A T-beam of effective flange width 800 mm, thickness of slab 70 mm, width of rib 150 mm, & effective depth of 400 mm. It is reinforced with 5 nos. of 25 mm dia. bars. Calculate the factored moment of resistance. The material are M30 grade concrete & Fe250 grade steel)

Dt: 9/08/23

Design of T-beam

Step-1

Depth of the beam

- ↳ Select the depth (d) in range of $l/12$ to $l/15$ based on stiffness.
- ↳ Overall depth of beam is taken as $d + 50$.

Step-2

↳ Self weight of slab.

↳ Live load acting on beam.

↳ Total load on slab = Self weight of slab + live load.

↳ Load from the slab per meter run of the beam = Load on slab per $m^2 \times c/d$ distance between Beams.

↳ Self wt. of the beam,

↳ Total load on beam = Load from slab + Self weight of beam.

↳ Factored load = $1.5 \times$ Total load on beam.

Step-3 Factored bending moment.

$$M_u = \frac{W_u l^2}{8}$$

Step-4 Effective width of flange

↳ Calculate effective width using the below formula

$$b_f = \frac{l_0}{6} + b_w + 6D_f$$

pg-37

Step-5 Assuming x_u is within the flange
↳ Equate compressive force in concrete to tensile force in steel.

$$0.36 F_{ck} b_f x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 F_{ck} b_f}$$

Step-6 Reinforcement

Calculate the reinforcement by using moment of resistance formula.

$$M_u = 0.36 F_{ck} b_f x_u (d - 0.42 x_u)$$

↳ Min^m area of tensile steel.

$$A_{st, \min} = \frac{0.85 b_w d}{f_y}$$

$A_{st, \min}$ should be less than A_{st} provided.

Step-7 Max^m area of tension steel.

↳ Calculate the reinforcement by using moment of resistance formula.

$$A_{st, \max} = 0.04 b_w D.$$

$A_{st, \max}$ should be greater than A_{st} provided.

↳ Hence determine no of bars and diameter of the bars.

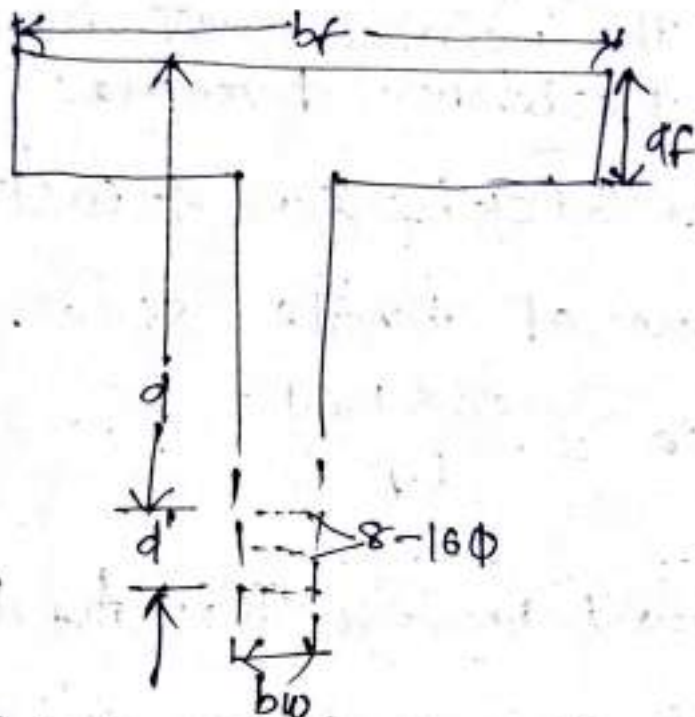
Step-8 Using SP16

↳ Determine $\frac{M_u}{bd^2}$

↳ Refer, Table 2 of SP 16 and read out the value of percentage of reinforcement corresponding to f_y and f_{ck} .

$$A_{st} = \frac{P \times b d}{100}$$

↳ Hence determine the no of bars and diameter of bars.



Reinforcement details

Q. A T-beam floor consists of 150 mm thick R.C.C slab monolithic with 300 mm wide beams. The beams are spaced at 3.5 m. Centre and their effective span is 6 m. If the super imposed load on the slab is 5 kN/m^2 design or live on intermediate T-beam. Use M20 mix and Fe 415 grade steel.

Given data

Effective span (l_d) = 6 m. = 6000 mm.

D_f = 150 mm.

b_w = 300 mm.

C/c Spacing of beams = 3.5 m

∴ Live load = 5 kN/m²

$f_{ck} = 20$, $f_y = 415$

Step-1

Depth of the beam

Assume, $d = \frac{l}{12}$ to $\frac{l}{15}$

$$= \frac{6000}{15} = 400 \text{ mm.}$$

∴ Overall depth $D = d + 50$
 $= 400 + 50 = 450 \text{ mm.}$

DT: 12/05/23

Step-2 Loads

Self wt. of concrete
 $= 2400 - 2500 \text{ kg/m}^3$

∴ Dead load of slab = $0.15 \times 25 = 3.75 \text{ kN/m}^2$

∴ Live load of slab = 5 kN/m^2

∴ Total load = $3.75 + 5 = 8.75 \text{ kN/m}^2$

∴ Load from the slab per mt. of the beam = load on slab per m² × C/c distance betⁿ beam

$$= 8.75 \times 3.5 = 30.63 \text{ kN/mt.}$$

∴ Self wt. of beam = $0.3 \times 0.3 \times 25 \times 1$
 $= 2.25 \text{ kN/mt.}$

∴ Factored load = 1.5×32.88
 $= 49.32$

Step-3 Factored bending moment.

$$M_u = \frac{w_u \times l^2}{8} = \frac{49.32 \times 6 \times 6}{8} = 221.94 \text{ kNm}$$

Step-4

Effective width of flange

T-beam, $b_f = \frac{l_0}{6} + b_w + 6D_f$

$$\frac{6000}{6} + 300 + 6 \times 150$$

$$= 2200 \text{ mm}$$

Step-5 Assuming x_u is within the flange

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_c k b_f}$$

$$= \frac{0.87 \times 415 \times A_{st}}{0.36 \times 20 \times 2200}$$

$$= 0.0228 A_{st}$$

Step-6 Reinforcement

$$M_u = 0.36 f_c k b_f x_u (d - 0.42 x_u)$$
$$\Rightarrow 221.94 \times 10^6 = 0.36 \times 20 \times 2200 \times 0.0228 A_{st} (400 - 0.42 \times 0.0228 A_{st})$$

$$\Rightarrow 221.94 \times 10^6 = 361.152 A_{st} (400 - 9.576 \times 10^{-3} A_{st})$$

$$\Rightarrow 221.94 \times 10^6 = 361.152 A_{st} \times 400 - 361.152 A_{st} \times 9.576 \times 10^{-3} A_{st}^2$$

$$\Rightarrow 221.94 \times 10^6 = 144460.8 A_{st} - 3.458 A_{st}^2$$

$$\Rightarrow 3.458 A_{st}^2 - 144460.8 A_{st} + 221.94 \times 10^6 = 0$$

$$\Rightarrow A_{st} = 221940000$$

$$\therefore x_u = 0.0228 \times 221940000 = 5060232 \text{ mm}$$

0.36
0.87

OP

$$M_{iu} = 0.087 F_y A_{st} (d - 0.42 x_u)$$

~~$$\Rightarrow 221.94 \times 10^6 = 0.087 \times 20 \times 2200 \times 0.42$$~~

$$\Rightarrow 221.94 \times 10^6 = 0.087 \times 415 \times A_{st} (400 - 0.42 \times 0.0228 A_{st})$$

$$\Rightarrow 221.94 \times 10^6 = 361.05 A_{st} (400 - 0.042 \times 9.576 \times 10^{-3} A_{st})$$

$$\Rightarrow 221.94 \times 10^6 = 361.05 A_{st} \times 400 - 361.05 A_{st} \times 9.576 \times 10^{-3} A_{st}$$

$$\Rightarrow 221.94 \times 10^6 = 144420 A_{st} - 3.4574148 A_{st}^2$$

$$\Rightarrow 3.4574148 A_{st}^2 - 144420 A_{st} + 221.94 \times 10^6 = 0$$

$$\Rightarrow A_{st} = 221940000 \text{ mm}^2$$

$$\therefore x_u = 0.0228 \times 221940000 = 5060232 \text{ mm}$$

Step-7

Minimum Area of tension steel,

~~Step 8~~

$$A_{st \text{ min}} = \frac{0.85 b w d}{f_y}$$

$$\frac{0.85 \times 300 \times 400}{415}$$

$$= 245.78 \text{ mm}^2 < A_{st \text{ provided}} (0.5)$$

Step-8

∴ Max^m Area of tension steel,

$$A_{st \text{ max}} = 0.04 \times b w D$$

$$= 0.04 \times 300 \times 450$$

$$= 5400 \text{ mm}^2$$

∴ provide 8 nos of 16 mm ϕ bar

$$A_{st} = 8 \times \frac{\pi}{4} \times 16^2$$

$$= 1608.49 \text{ mm}^2$$

Design of beams

Basic Rules for design

(i) Effective span (Pg no- 34-22.2)

Simply supported beam or slab

The effective span of a member that is not built integrally with its supports shall be taken as clear span plus the effective depth of slab or beam or centre to centre of supports, whichever is less.

Control of deflection (Pg no- 37)

(a) Basic values of span to effective depth ratios for spans up to 10m.

Cantilever	7
Simply supported	20
Continuous	26

(b) For spans above 10m, the values in (i) may be multiplied by $10/\text{span}$ in meters except for cantilever in which case deflection calculations should be made.

Reinforcement in beam (Pg no-46)

Tension Reinforcement

(a) Min^m reinforcement - The min^m area of tension reinforcement shall be not less than that

given by the following.

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y}$$

where

A_{st} = Minimum area of tension reinforcement,

b = Breadth of beam or the breadth of the web of T-beam,

d = effective depth, and

f_y = characteristic strength of reinforcement cement in N/mm^2 .

(b) Maximum reinforcement - The maximum area of tension reinforcement shall not exceed $0.04 bd$.

Compression reinforcement

The maximum area of compression reinforcement shall not exceed $0.04 bd$. Compression reinforcement in beams shall be enclosed by stirrups for effective lateral restraint. The arrangement of stirrups shall be as specified in 26.5.3.2.

Criteria For development length pg-44

The IS code saying that at the simple support the positive moment tension reinforcement shall be limited to a diameter such that $L_d \leq \left(\frac{M_1}{V}\right) + L_0$

Where,

L_d = Development length

M_1 = Moment of resistance of the section assuming all reinforcement at the section to be stressed to f_d .

$f_d = 0.87 f_y$ in the case of limit state design and the permissible stress design.

V = Shear force at the section due to design loads.

L_0 = Sum of the anchorage beyond the centre of the support and the equivalent anchorage value of any hooks or mechanical anchorage at simple support, and at a point of inflection, L_0 is limited to the effective depth of the members or 12ϕ , whichever is greater &

ϕ = Diameter of bar.

The value of M_1/V in the above expression may be increased by 30% when the ends of the reinforcement are confined by a compressive reaction.

$$\text{Compression} < (1.3) \times \left(\frac{M_1}{V} \right) + L_0$$

slenderness limit for beam (Pg no-39)

A simply supported or continuous beam shall be so proportioned that the clear distance between the lateral restraints does not exceed $60 b$ or $\frac{250b^2}{d}$ whichever is less, where d is the effective depth of the beam and b the breadth of the compression face at midway between the lateral restraints.

For a cantilever, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed $25 b$ or $\frac{100b^2}{d}$ whichever is less.

The design of a beam is based on the following assumptions:

1. The beam is homogeneous and isotropic.

2. The stress is proportional to the strain.

3. The beam is initially straight.

4. The deflection is small.

5. The temperature is uniform.

6. The beam is supported by a pin and a roller.

7. The beam is subjected to a uniformly distributed load.

8. The beam is subjected to a point load.

9. The beam is subjected to a moment.

10. The beam is subjected to a combination of loads.

Q. A simply supported rectangular beam of 6m span carries a uniformly distributed characteristic load of 24 kN/m. including its self wt. Design the beam. The materials are M20 grade concrete & HYSD reinforcement of grade Fe 415.

DT: 17/05/20

A simply supported rectangular beam of 6m span carries a uniformly distributed characteristic load of 24 kN/m. including its self wt. Design the beam. The materials are M20 grade concrete & HYSD reinforcement of grade Fe 415.

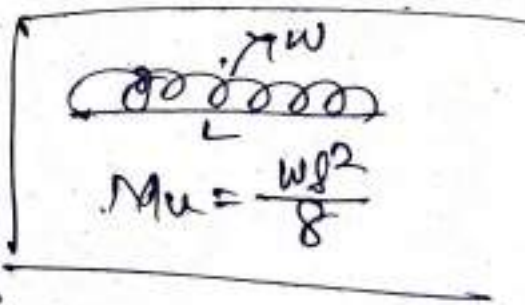
Soln:

Factored load = $24 \times 1.5 = 36 \text{ kN/m}$

$$M_u = \frac{wl^2}{8}$$

$$= \frac{36 \times 6^2}{8} = 162 \text{ kN.m}$$

$$= \frac{162 \times 10^6}{1000} \text{ N.mm}$$



$$\therefore V_u = \frac{wl}{2} = \frac{36 \times 6}{2} = 108 \text{ kN}$$

Max^m. load

Step-1 Depth of the beam.

Assume, width of the beam, $b = 300\text{mm}$.

$$d_{\text{req}} \text{ (depth required)} = \sqrt{\frac{M_u}{Q_{lim} \times b}}$$

$$= \sqrt{\frac{162 \times 10^6}{2.75 \times 300}} = 443.12 \text{ mm}$$

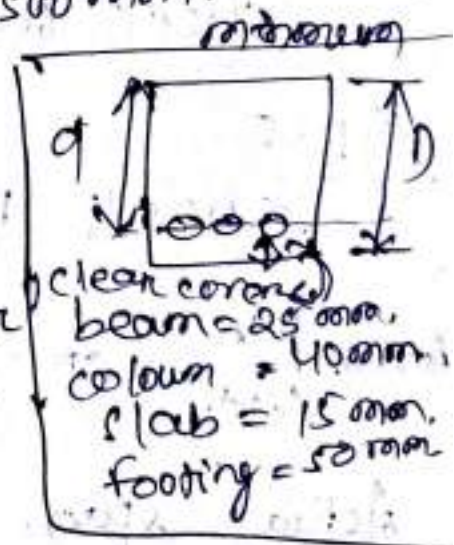
$$Q_{lim} = 0.36 \times \left(\frac{x_{u,max}}{d}\right) \left(1 - 0.42 \frac{x_{u,max}}{d}\right) f_{ck}$$
$$= 0.36 \times 0.48 \times \left(1 - 0.42 \times 0.48\right) \times 20$$
$$= 2.75$$

Provide effective depth $d = 500 \text{ mm}$.

Assume clear cover of 30mm & 20mm dia. steel bar

$$\text{Overall depth} = d + \text{clear cover} + \frac{\text{bar dia}}{2}$$

$$= 500 + 30 + \frac{20}{2}$$
$$= 540 \text{ mm}$$



Step-2

Steel area,

$$A_{st} = \frac{M_u}{0.87 F_y (d - 0.42 x_{u, max})}$$

$$= \frac{162 \times 10^6}{0.87 \times 415 (500 - 0.42 \times 0.48 \times 500)}$$

$$= 1123.97 \text{ mm}^2$$

∴ Provide 4 no. of 20 mm. dia bar
giving $A_{st} = 4 \times \frac{\pi}{4} \times (20)^2$

$$= 1256.63 \text{ mm}^2$$

Step-3

check for development length

Step-4

check for shear.

Step-5

check for deflection:

$$\frac{\text{Actual span}}{\text{depth}} \text{ or } \frac{l}{d} = \frac{6000}{500} = 12$$

~~Actual span~~

up to 1000.
Simply supported = 20
Cantilever = 7
con floor = 26

$$\therefore \text{Basic } \frac{\text{span}}{\text{depth}} = 20$$

$$\therefore \text{Service stress} = 0.58 \times f_y \times \frac{A_{tr}}{A_{tr} + p_{tr} d}$$

$$= 0.58 \times 415 \times \frac{1123.97}{1256.63}$$

$$= 215.28 \text{ N/mm}^2$$

$$p_t = \frac{100 A_{tr}}{bd}$$

$$= \frac{100 \times 1256.63}{300 \times 500}$$

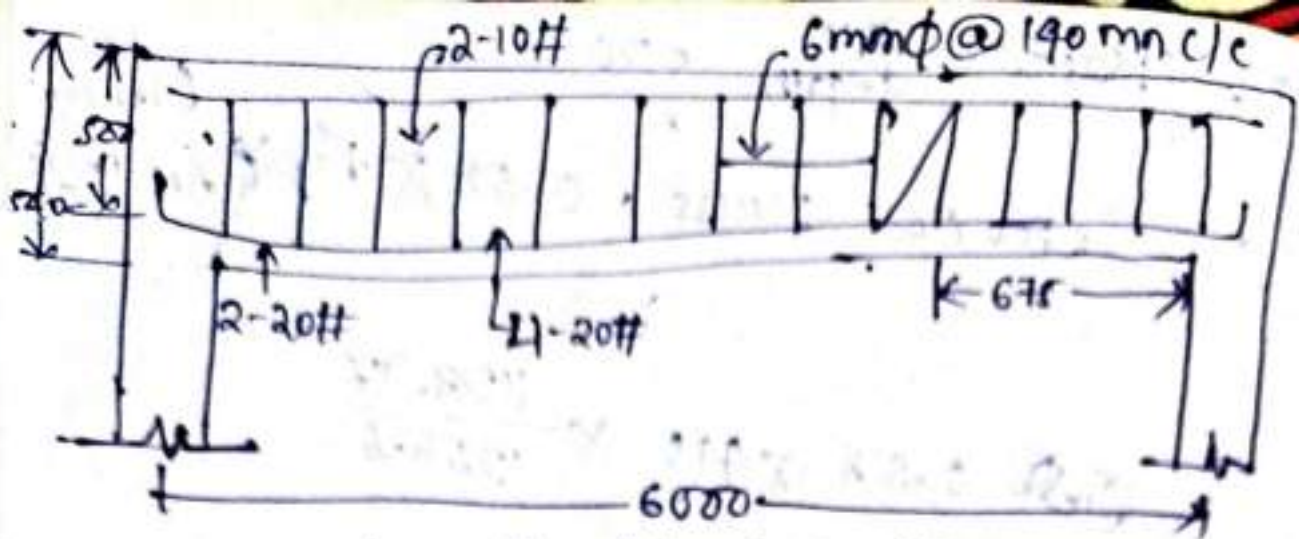
$$= 0.84$$

$$\text{Modification Factor} = 1.5$$

$$\therefore \text{Permissible } \frac{\text{span}}{\text{depth}} = 20 \times 1.5 = 23$$

$$\text{Actual } \frac{\text{span}}{\text{depth}} = 12 \text{ (ok) on safe}$$

⑤ Detailing of a beam. Sketch



Longitudinal section

Analysis & design of simply supported slab

Slab

Slabs are plate elements having the depth d is much smaller than its span & width. They usually carry a uniformly distributed load ~~of~~ from the floor or roof of the building. They are generally of two types

- (1) One way slab
- (2) Two way "

One way slab

The slab supported on all four edges but

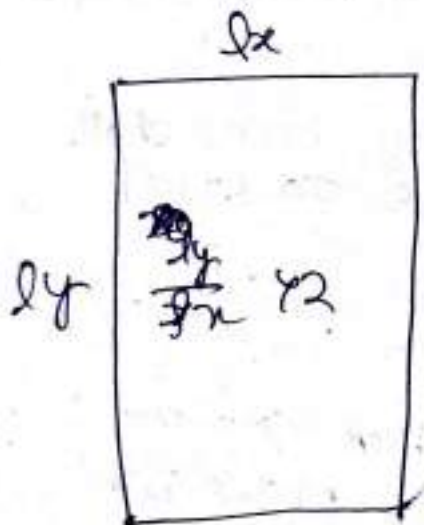
$$\frac{d_y}{l_x} > 2$$

is called as one way slab.
 Here l_y is much more than l_x than there will be a tendency of the slab to bend in one direction (about l_x only)

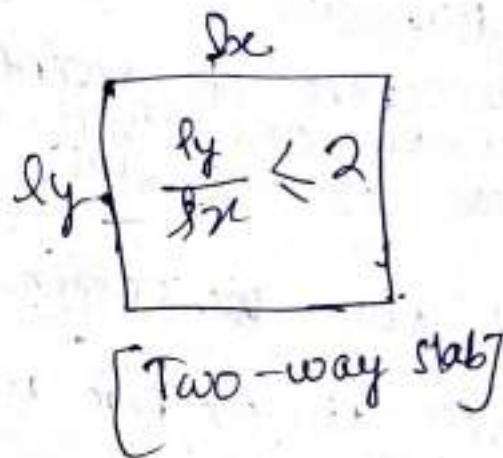
Dt: 20/05/23

Two way slab

If the slab is supported on all 4 edges & if $\frac{l_y}{l_x} \leq 2$ then the tendency of the slab is to bend in both directions. Such slabs are called as two way slab.



[One way slab]



Design
Analysis of one slab

Effective span Step-1

(pg-34)

The effective span of a member that is not built integrally with its supports shall be taken as clear span plus the effective depth of slab or beam or centre to centre of supports whichever is less.

Control of deflection (pg-37) Step-2

Step-3

Reinforcement requirement (UR)

The mild steel reinforcement in either direction in slabs shall not be less than 0.15 percent of the total cross-sectional area. However, this value can be reduced to 0.12% when high strength deformed bars or welded wire fabric are used.

Max^m diameter

The diameter of reinforcing bars shall not exceed one-eighth of the total thickness of the slab.

(1) Max^m dia - ~~100~~ 50 mm.

(2) Min^m dia → For main bar → 10 mm
(For plain bar)
(Fe 250)

8 mm (For HYSD bar)

For distribution bar, 6 mm (For plain)
HYSD)

Step-4

Shear stress

In normal cases the shear in slab is not critical however shear shall be checked as per clause 40.2 of IS 456

[Pg no - 72]

For solid slab the design shear strength in concrete shall be $\phi \tau_c$

$\phi \tau_c$

Step-5

Cracking

To ensure that cracking of the slab is not excessive, spacing of the reinforcement shall be limited to following:

For main bar spacing \nless (not greater than) $3d$ or \nless 300mm

For distribution bar spacing \nless or \nless 450 mm.

where d = Effective depth of slab

Step-6

Cover

For mild exposure clear cover is 20 mm. this can be reduce by 5 mm, where reinforcement of 12mm dia or less is use.

Step-7

Development length

Q. A simply supported one way slab of clear span 3 mt. is supported on masonry wall of thickness 350 mm. Slab is used for residential load. Design the slab. The materials are M20 grade & HYSD reinforcement of grade Fe415. Live load shall be 2 kN/m^2 .

6 prax (22)
7 no. p (19)

Soln! -

Assume overall depth of slab (D) = 130 mm.

Step-1 Load calculation. $= 0.13\text{ mt}$

$\therefore L.L = 2\text{ kN/m}^2$

\therefore Dead load = 0.13×25
 $= 3.25\text{ kN/m}^2$

(Assume)
 \therefore Floor Finish = 1.00 kN/m^2

\therefore Total load = 6.25 kN/m^2

\therefore factored load = 6.25×1.5
 $= 9.47\text{ kN/m}^2$

\therefore Effective span = $3000 + 110 = 3110\text{ mm.}$

(approx)
Effective depth of slab = $D - \text{clear cover}$
 $130 - 20$
 $= 110\text{ mm.}$

OTC

$$3000 + 350 = 3350 \text{ mm.}$$

1088
3110 to 3350

∴ Effective span = 3110 mm,

Step-2 Calculation of bending moment & shear force.

$$(Mu) \text{ Bending moment} = \frac{wl^2}{8}$$

$$9.47 \times (3110)^2$$

$$\frac{9.47 \times (3.110)^2}{8}$$

$$\frac{11.36 \text{ KN.m}}{8}$$

$$= 11.36 \text{ KN.m}$$

$$\text{Shear Force} = \frac{wl}{2} = \frac{9.47 \times 3.11}{2}$$

$$= 14.72 \text{ KN}$$

Step-3 Calculation of depth

$$d_{\text{requi}} = \sqrt{\frac{Mu}{Q_{\text{lim}} \times b}}$$

Assume, 1 m width of slab

$$Q_{\text{lim}} = 0.36 \times \left(\frac{x_{u,\text{max}}}{d}\right) \times \left(1 - 0.42 \frac{x_{u,\text{max}}}{d}\right) f_{ck} b$$
$$= 0.36 \times 0.48 \times (1 - 0.42 \times 0.48) \times 20$$
$$= 2.76 \text{ MN/m}^2$$

$$d_{\text{required}} = \sqrt{\frac{11.36 \times 10^6}{2.76 \times 1000}}$$

$$= 64.15 \text{ mm.}$$

Step 4 Calculation for steel area.

$$A_{st} = \frac{M_{ue}}{0.87 f_y (d - 0.42 x_{u, \text{max}})}$$

$$= \frac{11.36 \times 10^6}{0.87 \times 415 (110 - 0.42 \times 0.48 \times 110)}$$

$$= 358 \text{ mm}^2$$

$$\therefore \text{Spacing} = \frac{\text{Area of one bar} \times 1000}{\text{Required area in mm}^2 \text{ per mt.}}$$

provide 8mm dia. bar

$$\text{Spacing of main bar} = \frac{\frac{\pi}{4} (\phi)^2 \times 100}{358}$$

$$= 140 \text{ mm.}$$

∴ Distribution bar Area = 0.15% bD

$$= \frac{0.15}{100} \times 1000 \times 130$$

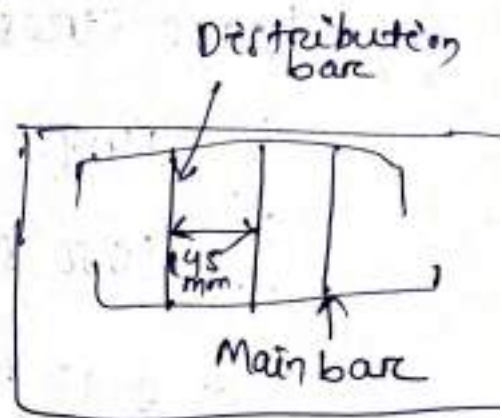
$$= 195 \text{ mm}^2$$

Spacing of distribution bar =

$$\frac{\text{Area of one bar} \times 1000}{\text{Required area in mm}^2 \text{ per m}}$$

$$\frac{\frac{\pi}{4} \times (6)^2 \times 1000}{1.95}$$

$$= 145 \text{ mm.}$$



Step-5

check for shear

$$d_{\text{eff}} = 110 \text{ mm.}$$

$$(P.E.) = \frac{100 A_{st}}{b d}$$

$$= \frac{100 \times 358}{1000 \times 110} = 0.32$$

Multiplication

Table-19

$Z_c = 0.42 \text{ N/mm}^2$	0.25	→	0.36
	0.50	→	0.48
	0.25	→	0.12
	1	→	0.12

$$\therefore K = 1.30$$

(Table - 19, pg no - 72)

$$0.13 \Rightarrow \frac{0.12}{0.25} \times 0.13$$

$$= 0.0624$$

$$0.32 = 0.36 + 0.0624$$

$$= 0.42$$

\therefore Design Shear strength = $K \times Z_c$

$$= 1.30 \times 0.42 = 0.546 \text{ N/mm}^2$$

\therefore Shear stress, $Z_v = \frac{V_u}{bd}$

$$= \frac{14.72 \times 10^3}{1000 \times 110}$$

$$= 0.13 \text{ N/mm}^2 < 0.546 \text{ (safe)}$$

Table-19 pg no-73

$$Z_c = 0.39 \text{ N/mm}^2$$

$$\therefore K = 1.30$$

(Table - 19 pg no - 72)

\therefore Design shear strength

$$= K \times Z_c$$

$$= 0.507 \text{ N/mm}^2$$

\therefore Shear stress, $Z_v = \frac{V_u}{bd}$

$$= \frac{14.72 \times 10^3}{1000 \times 110}$$

$$= 0.13 \text{ N/mm}^2 < 0.546$$

(safe)

Manipulation

$$0.25 \rightarrow 0.36$$

$$0.50 \rightarrow 0.48$$

$$\frac{0.12}{0.25} \rightarrow 0.12$$

$$1 = \frac{0.12}{0.25}$$

$$0.07 = \frac{0.12}{0.25} \times 0.07$$

$$= 0.0336$$

$$0.32 \rightarrow 0.0336 + 0.36$$

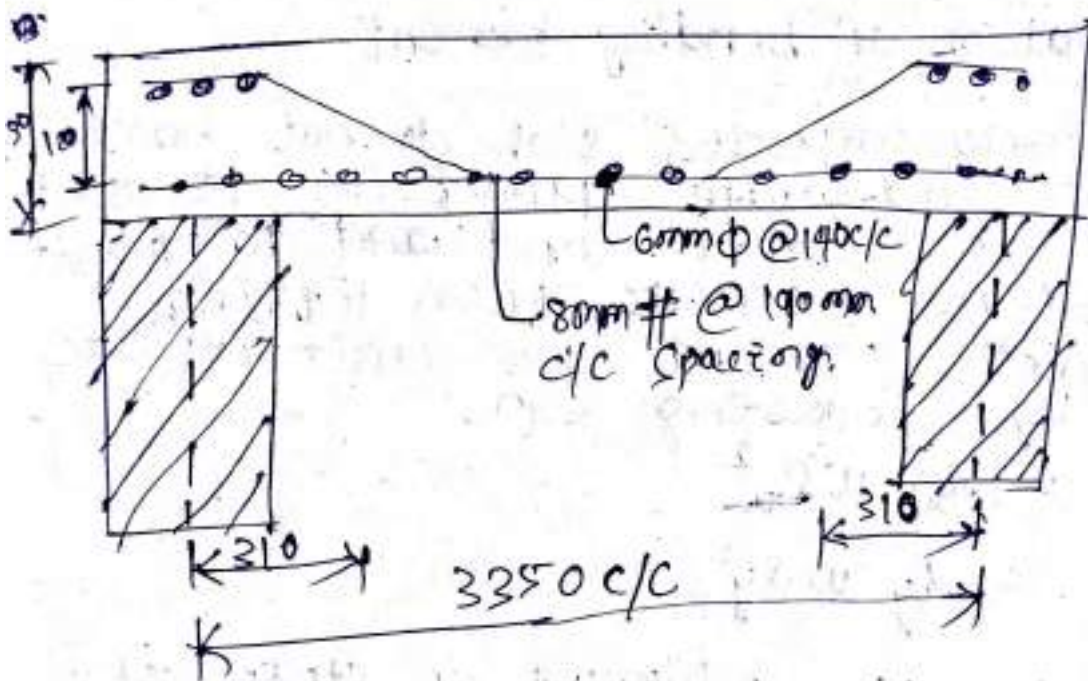
$$= 0.39$$

Step-6

check for development length

Step-7

Deflection



Two way slab

↳ Generally when a two way slab is compared to one way slab the deflection & bending of slab are reduced.

↳ In this type of slab the bending moment is distributed in both the dirⁿ. This results increase in load carrying capacity of the section.

DT: 22/05/23

Analysis of simply supported two way slab

(i) Computation of bending moment

When simply supported slab do not have adequate provision to resist torsion at corner end ~~and~~ to prevent the corner from lifting the max^m moment per unit wt. are given by following eqⁿ.

$$M_x = d_x \cdot w \cdot l_x^2$$

$$M_y = d_y \cdot w \cdot l_y^2$$

Where M_x & M_y = Moment on strips of unit width spanning l_x & l_y respective values.

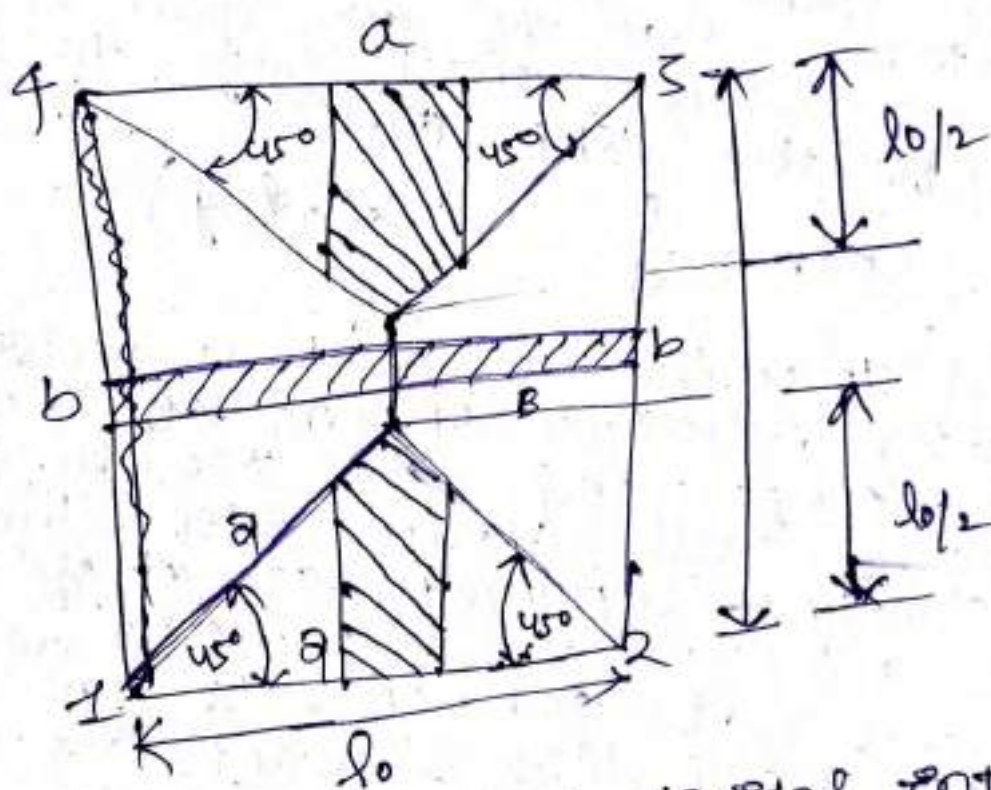
d_x & d_y = Coefficient given in table no 26 Pg-90

l_x & l_y = lengths of the shorter span & longer span respectively

w = Total design load per unit area.

As per IS 456 at least 50% of the tension reinforcement provided at mid span should extend to the supports. The remaining 50% should extend to $0.1 l_x$ or $0.1 l_y$ of the support, as appropriate. CPG no-91

Computation of shear force
 Shear force are computed following the procedure started below with ~~the~~ reference to the figure given below



The two way slab is divided into two triangular & two trapezoidal zones by drawing lines from each corner at an angle of 45° . The loads of triangular segment a will be transfer to beams 1 & 2 and the loads of trapezoidal segment B will be transfer to b-beam 2 & 3.

The shear force in both strips are equal & we can write $V_u =$

$$w \times \left(\frac{l_x}{2}\right)$$

$$\therefore Z_v = \frac{V_u}{bd}$$

Check for deflection

The deflection of two way slab shall be checked as per one way slab. For slabs spanning in two direction the shorter of the two span should be use for calculating the span to effective depth ratio.

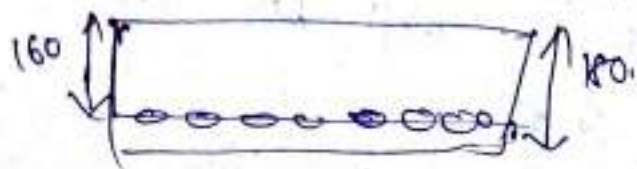
$$\text{Span} = \frac{l}{d} \quad (l_x, l_y)$$

Q. A drawing room of a residential building measures $4.3 \text{ m} \times 6.55 \text{ m}$. It is supported on 350 mm thick walls on all four sides. The slab is simply supported at ~~at~~ edges with a provision to resist torsion at corners. Design the slab using M20 concrete & HYSD reinforcement of grade Fe415.

Soln:-

Consider 1 m wide strip

\therefore Assume 180 mm thick slab with 160 mm effective depth.



$$L_x = 4.3 \text{ m} + 0.160 \text{ m} = 4.46 \approx 4.5 \text{ m}$$

$$L_y = 6.55 + 0.160 \text{ m} = 6.71 \approx 6.75 \text{ m}$$

Step 1

Load calculation

$$\text{Dead load} = 0.180 \times 2.5 = 4.5 \text{ kN/m}^2$$

$$\text{Floor load} = 1.0 \text{ kN/m}^2 \text{ (Assume)}$$

$$\text{Live load} = 2.0 \text{ kN/m}^2$$

$$\text{Total load} = 7.5 \text{ kN/m}^2$$

WSM \rightarrow LSW
Factored
load

$$\begin{aligned} \text{Factored load} &= 1.5 \times 7.5 \\ &= 11.25 \text{ kN/m}^2 \\ &= 11.25 \text{ kN/m}^2 \end{aligned}$$

$$\frac{L_y}{L_x} = \frac{6.75}{4.5} = 1.5 \text{ (Two way slab)}$$

$$M_x = \alpha_x w l_x^2$$

$$\begin{aligned} &= 0.104 \times 11.25 \times (4.5)^2 \\ &= 23.7 \text{ kN}\cdot\text{m} \end{aligned}$$

Table - 27
Pg - 91

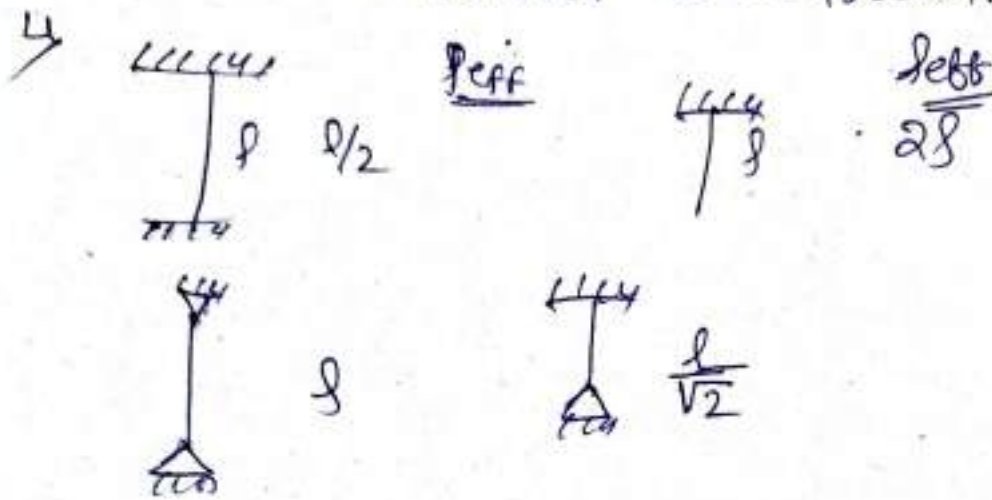
$$M_y = \alpha_y w l_y^2$$

$$\begin{aligned} &= 0.046 \times 11.25 \times (6.75)^2 \\ &= 10.45 \text{ kN}\cdot\text{m} \end{aligned}$$

Column

Dt of 23/08/23

↳ A compression member whose effective length exceeds three times its least lateral dimension is termed as column.



↳ If the effective length is less than ~~3~~ 3 times its least lateral dimension is known as pedestal.

↳ The shape of column may be square, rectangular, circular depending upon requirements.

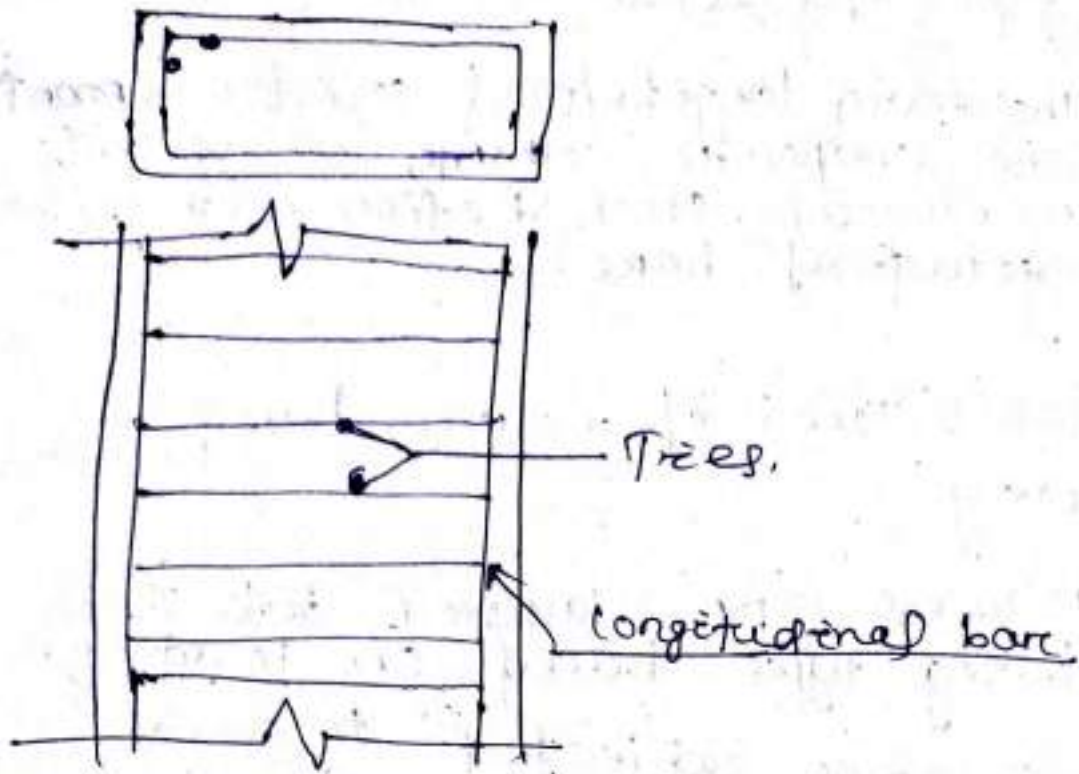
Classification of column based on reinforcement

The reinforced concrete column are classified into three group.

- (1) Tied column
- (2) Column with helical reinforcement
- (3) Composite column.

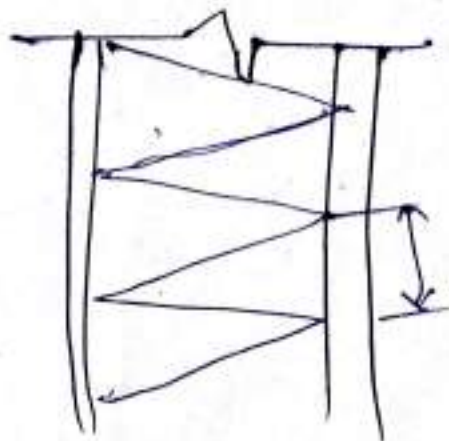
(1) Tied column

The main longitudinal reinforcement bars are enclosed within closely spaced lateral ties.



Tied Column

(2) Column with helical reinforcement
 The main longitudinal reinforcement bars are enclosed within closely spaced spiral & continuously wound circular reinforcement. Columns of this type are mostly of octagonal type.



(3) Composite Column

The main longitudinal reinforcement of the composite column consist of structural steel section with without longitudinal bars.

Classification of column based on loading

Columns are classified into three following types based on loading:

- (1) Columns subjected to axial loads only. (concentric)
- (2) Columns subjected to axial & uniaxial loading bending.
- (3) " " " " combined Axial & Bi-axial bending.



Q. A short RCC column is to carry a braced load of 1900 kN. If the column is to be a square then design the column assume $e_{min} < 0.05 D$. The materials are M20 grade concrete & mild steel (Fe25)

Soln:

Min^m percentage of steel = 0.8% of gross sectional area (A_g)

∴ Area of longitudinal reinforcement, A_{sc}
 $= 0.008 A_g$

Area of concrete, $A_c = A_g - A_{sc}$

$$= A_g - 0.008 A_g$$

$$= A_g (1 - 0.008)$$

$$A_c = 0.992 A_g$$

short column

$$l = 4m$$

$$A_{sc} = 0.008 A_g$$



T -