LECTURER NOTES ON STRUCTURAL MECHANICS

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Simple Stress and Strain

Stress:-
The Internal resistance per unit area to deformation is called stress.

Example:-
When a body of uniform section 'A' subjected to a load 'p' transverse to the cross section the Average stress 'σ' is given by P/A (N/m^2)

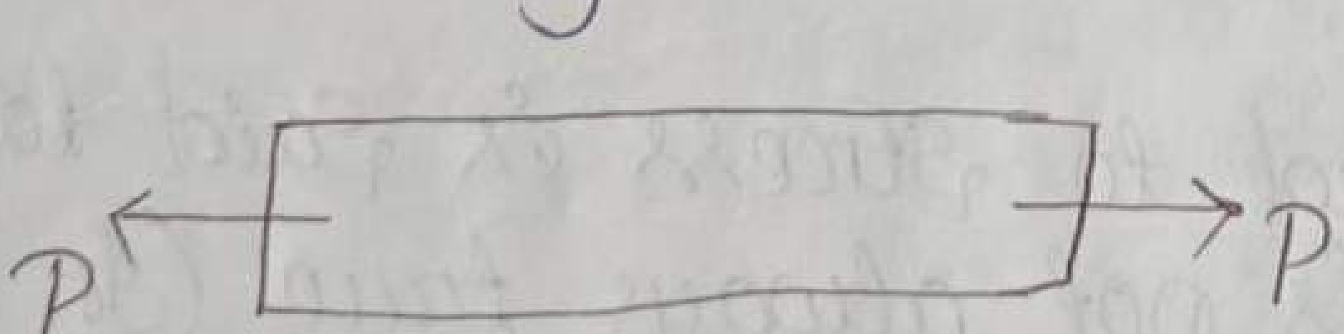
$\boxed{1\ N/m^2 = 1\ \text{Pascal}}$

$$\text{Stress } \sigma = \frac{\text{Force}}{\text{Cross-sectional Area}} = \frac{F}{A_0}$$

Imp
$1 \text{ mega pascal} = 1\ N/mm^2$
$1 \text{ gega pascal} = 10^9$
$1m = 1000\ mm$

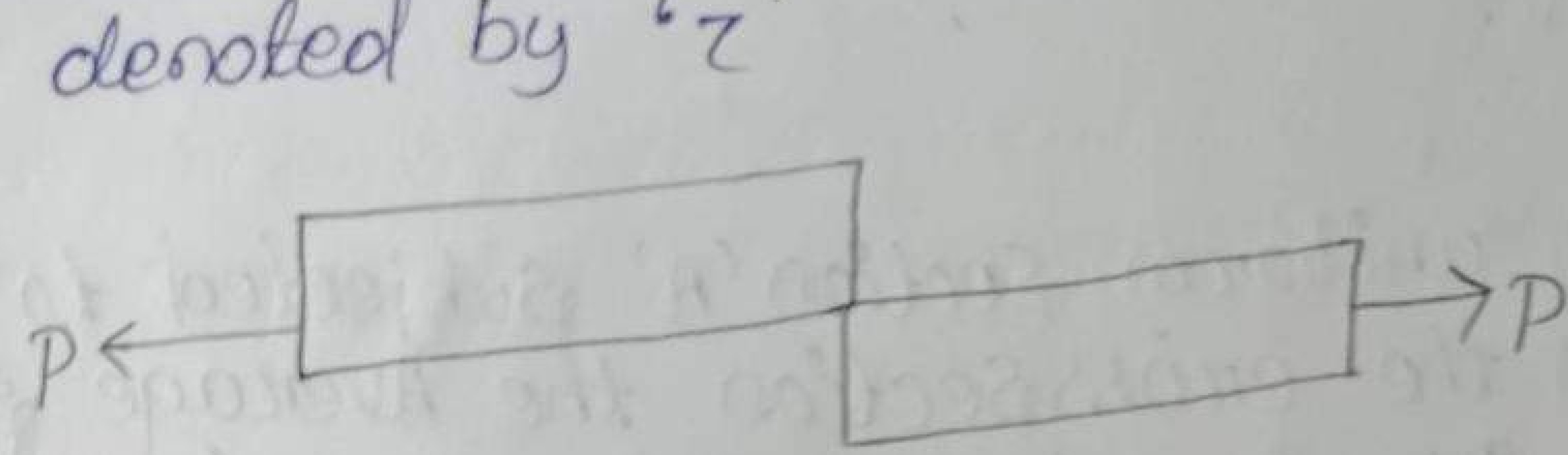
Type of stresses
1. Tensile stress
2. compressive stress
3. Shear stress
4. Bending stress
5. Torsional stress

1. Tensile stress (σ_t)
→ If the applied force tends to increase the length of the solid body the stress induced is called Tensile stress.
→ It is denoted by σ_t

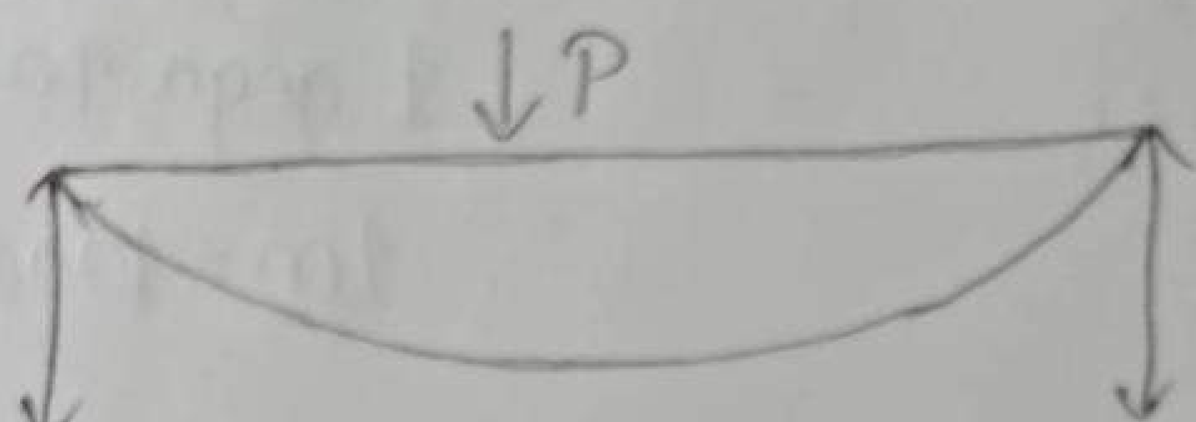


2. compressive stress: (σ_c)
If the applied force tends to decrease the length of the solid body, the stress induced is called compressive stress.
It is denoted by σ_c

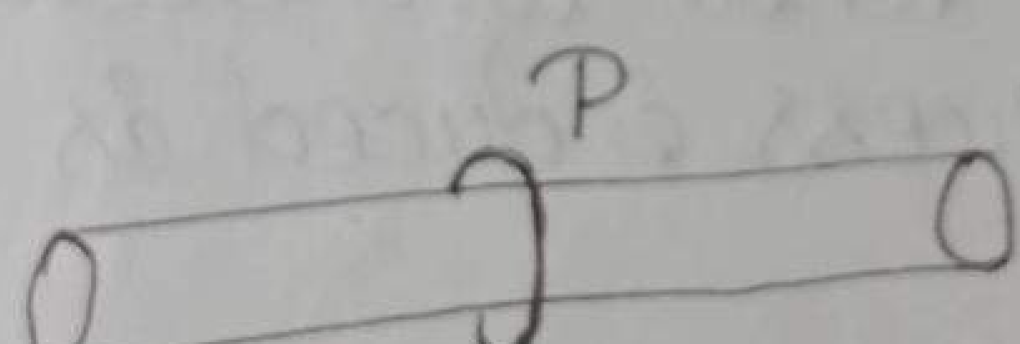
Shear Stress:-(τ)
t is the Stress caused by force action along or
arallel to the area resisting that force.
It is denoted by 'τ'



Bending Stress:-(σ_b)
The stress developed in a member due to
ding action of the transvers load is called
nding stress.



Torsional Stress:-(τ)
The stress developed against the deformation
a member due to torque is called Torsional
ess.



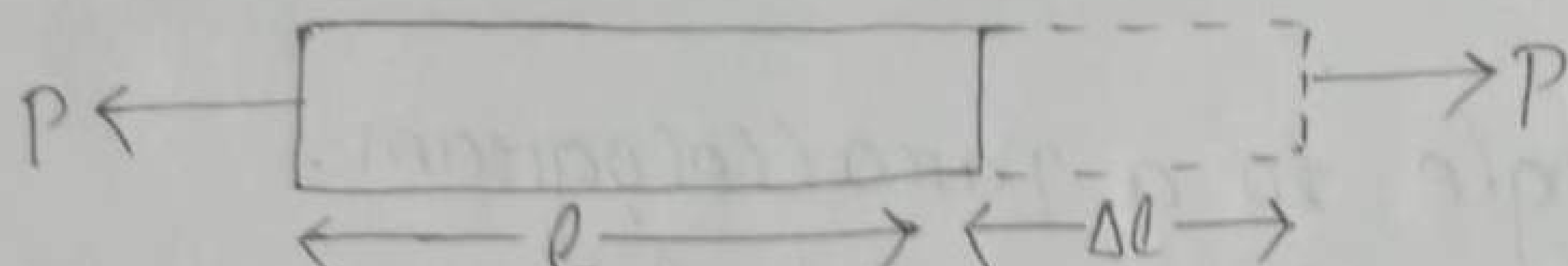
ain (ϵ)
When a body Subjected to stress is said to be
ained But reverse is not always true (In case of
thermal strain). $\boxed{\text{Strain} = \frac{\Delta l}{l}}$
e, the stress is the deformation produced by the
ain. [Strain has no unit]
s classified in 3 Parts:-
ensile strain

1. Te
→ Th
str

→
C
→

. Tensile Strain:-

The deformation due to the direct action of the tensile stress this is called Tensile strain.

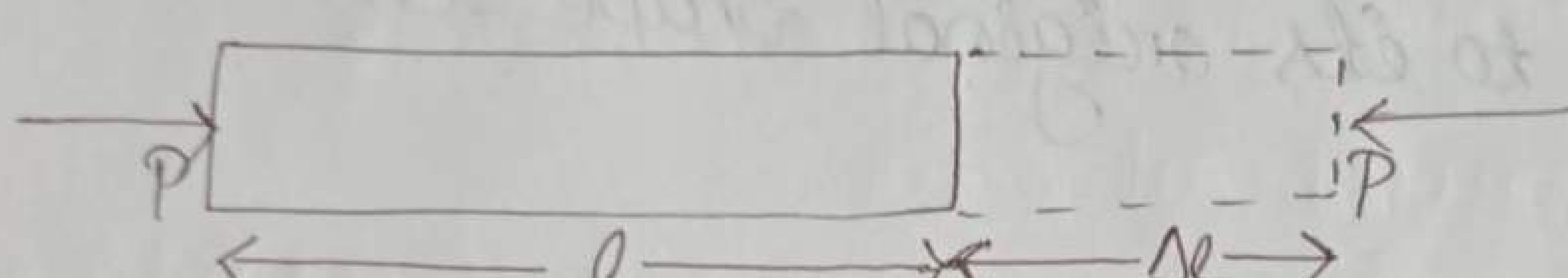


$$\boxed{E = \frac{\Delta l}{l}}$$

It has no unit or unitless.

ompressive Strain:-

The deformation due to the direct action of the compress
tress this is called compressive strain.



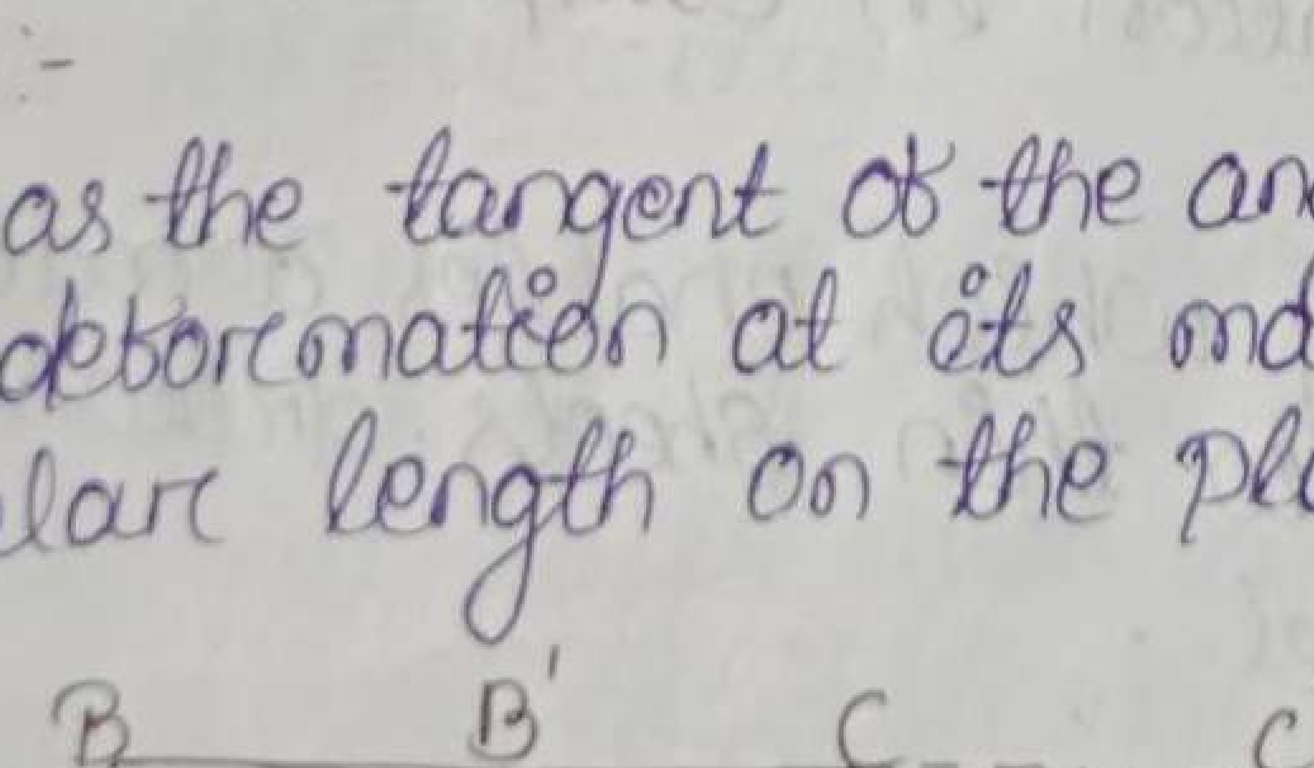
$$\boxed{E = \frac{\Delta l}{l}}$$

t has no unit or unitless.

Generally tensile strain is (+) strain and compress
rain is (−) strain.

ear Strain:-

is defined as the tangent of the angle and is equal to length of deformation at its maximum devided by perpendicular length on the plane of force
lication.



$\tan\phi = \frac{BB'}{...}$

nen a body is subjected to shear does not change it's
gth, breath of thickness but, it under goes a
nge in shape.

om a rectangle to a parallelogram.

hanical properties of material:-

igitity:-
f referes to the property of a solid to resist
ange in shape.

Elasticity:-
is defind as the ability of material to deform and
rern back to its original shape when the load is
nove.

lasticity:-
is defined as the ability of a material to get
nanent deformation under the action of load is
ed plasticity.

uctility:-
is the ability of a material to under go large
manent deformation in
ion:
-: The property which enables a materials to be
drawn into wire.

Ex:- Mild steel.

alleability:-
is the ability of the material to under go large
manent deformation in compression and allows
o expand in all direction with out failure.
Example:-

re rolled into thin sheets.

compressibility:-
It is the property of a material due to which its volume decreases when pressure is applyed.

Hardness:-
The resistance of a material to indentation including, scratching, or surface abrasion is called Hardness.

Toughness:-
It is the capacity of a structure to with stand and impact load.
n:- The capacity to absorb energy with out failure, s depends upon the ductility of a material and its ultima strength.

Stiffness:-
It is the ability of material to resist any deformatio under stress.

$$\text{Stiffness} = \frac{\text{Resisting force}(P)}{\text{long action}(\Delta)} = \frac{N}{mm}$$

Brittleness:-
It is opposite to ductility.
r Example: When a material can not be drawn ou tension to smaller section.
brittle material fails instantly under the load ith out showing any significant to deformation.
x:- concrete or cast iron.

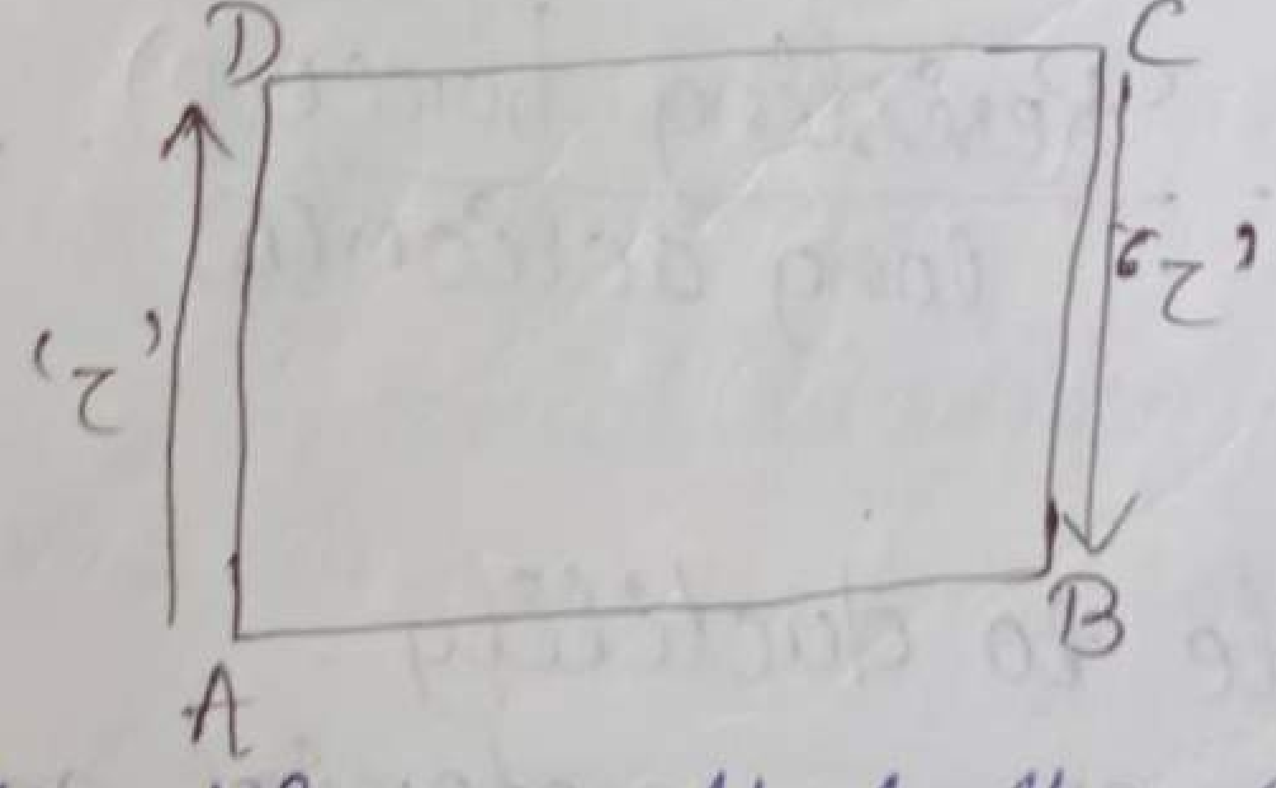
Fatigue:-
When loading are repeated thousands or millio

Creep:-
If the stress exceed the yeild point, the strain caused in the material by the application of load does not disappear totally of the removal of load. The plastic deformation caused to the material is known as creep.

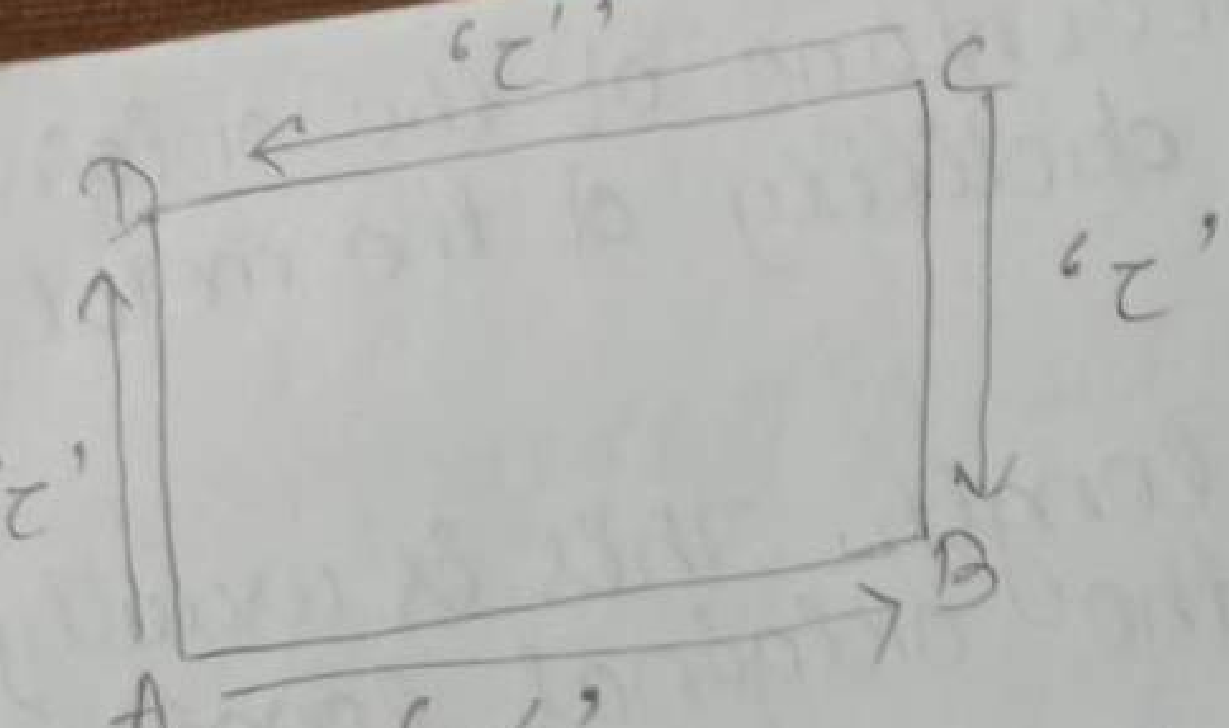
Tenacity:-
Tenacity is the resistance of material to breaking.

Durability:-
It is the property of a material in which it can with stand the wear or tear due to environmental effects such as :- wind, rain, hot, cold etc.

Complimentary Shear Stress:-
Consider an infinitely small rectangular ABCD under shear stress of intensity 'τ' acting in plane AD and BC.



It is clear from the figure that the shear stress acting on the element will tend to rotate in clockwise direction.
As there is no other forces acting on the element, static equilibrium of the element can only be attained if another couple of the same magnitude is applied in the anti-clockwise direction. This can be achieved by having shear stress of intensity



Assume 'x' and 'y' to be the length of side AB and BC nd a unit thickness perpendicular to the figure.

Force of the given couple $= \tau \times A$
$= \tau \times (y \times 1)$

moment of the given couple $= F \times$ perpendicular distan
$= \tau \times y \times 1 \times x$

orce of the balancing couple $= \tau' \times$ Area
$= \tau' \times (x \times 1)$

oment of the balancing couple $= F \times \perp d$
$= \tau' \times x \times 1 \times y$

or equilibrium:-
moment of the given couple = moment of the balancing couple

$$\tau \times y \times 1 \times x = \tau' \times x \times 1 \times y$$

$$\boxed{\tau = \tau'}$$

This shows that the magnitude of balancing shear stress is same as the applied shear stress. The shear stress on the transverse pair of tac re known as complimentary shear stress. Thus very shear stress is always accompained by an equ omplimentary shear stresses.

longation:-

ies is usually expressed as percentage of the original
gth and is a measure of the ductility of the metal.

traction:-

ntraction mean decrease in length. This is usually
pressed as a percentage of the original length.

ngitudinal strain:-

Strain of a body in the direction of force is
lled longitudinal strain or linear strain.
is expressed as $\frac{\Delta L}{L}$

eral strain:-

strain of a body opposite to that of force and act
ght angle to its is called lateral strain.
is expressed as $\frac{\Delta \text{diameter}}{\text{diameter}}$

sson's ratio:-

he ratio of the lateral strain to the longitudinal
ain of a material when it is subjected to a
gitudinal stress is known as poisson's ratio.
ithin the elastic limit lateral strain is
ectly proportional to longitudinal strain.

teral strain = μ x (longitudinal strain)

$$\frac{\text{lateral strain}}{\text{longitudinal strain}} = \mu$$

* Poisson's ratio is unit less.

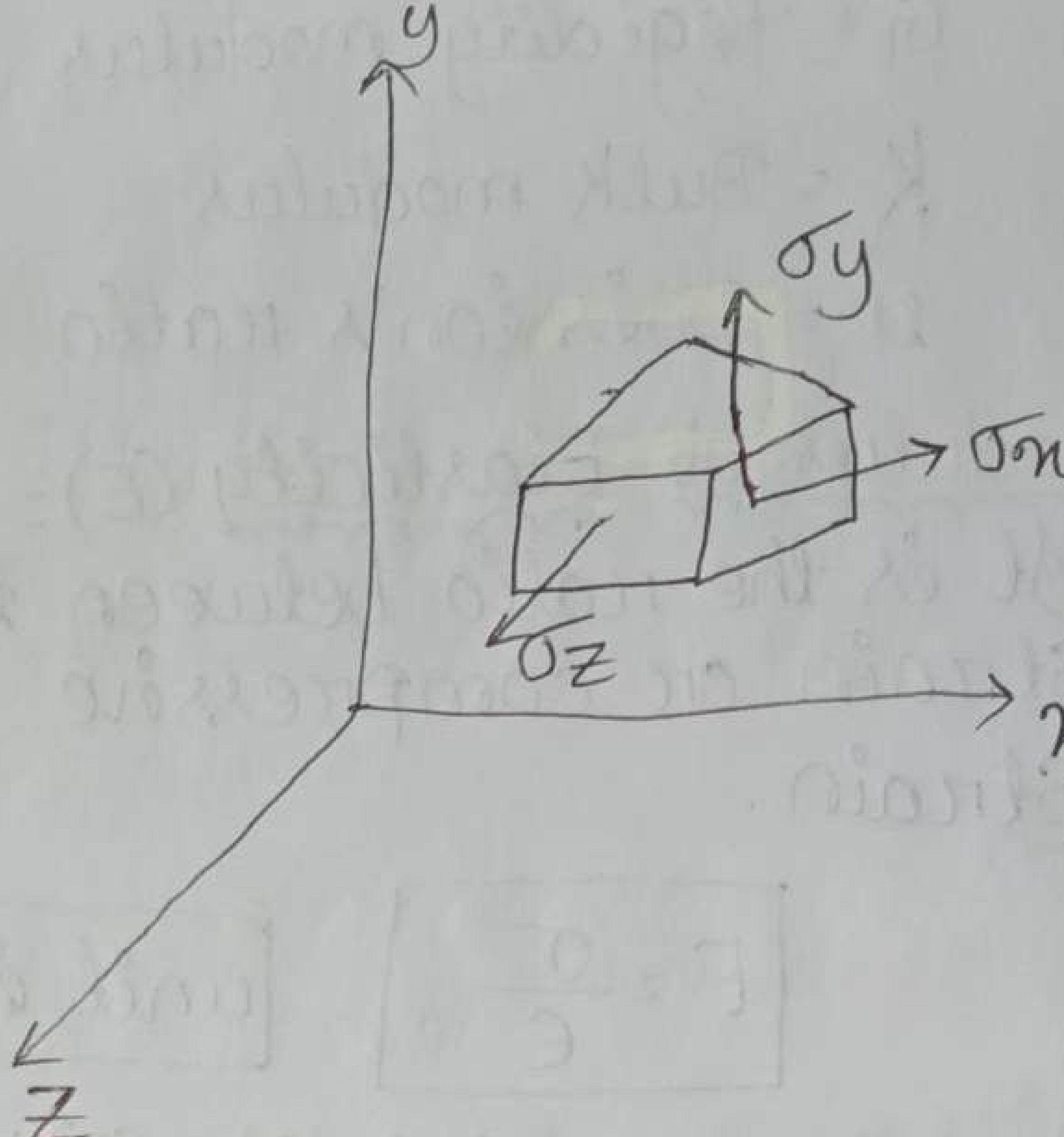
$$\mu = \frac{\Delta d/d}{\Delta l/l}$$

$\boxed{\text{* Poisson's ratio range} \rightarrow -0.5 \text{ to } 0.5}$

ke's law:- (1 dimension)

t states that when a body is loaded with in elastic (proportionality)
it, the stress is proportional to strain.

Change in dimension and volume :-
(For uniaxial stress)



$$\epsilon_x = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E}\right)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right)$$

$$\epsilon_z = \frac{\sigma_z}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right)$$

$$\boxed{\sigma_y = \sigma_z = 0}$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = -\mu\frac{\sigma_x}{E}$$

$$\epsilon_z = -\mu\frac{\sigma_x}{E}$$

Volumetric strain :-

It is defined as the change in volume by original when the body is subjected to stress.

$$\boxed{\epsilon_V = \frac{\Delta V}{V}} \quad \frac{\text{Change in volume } (\Delta V)}{\text{original vome } (V)}$$

$$\epsilon_V = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E}$$

$$\boxed{\epsilon_V = \frac{\sigma_x}{E}(1-2\mu)} \text{ for 1D}$$

For 3D

$$\epsilon_V = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1-2\mu)$$

If $\sigma_x = \sigma_y = \sigma_z = \sigma$

$$\epsilon_V = \frac{3\sigma}{E}(1-2\mu)$$

astic constant :-

E = modulus of elasticity

G = Rigidity modulus

K = Bulk modulus

μ = Poisson's ratio

odulus of Elasticity (E) :-

It is the ratio between tensile stress and tensile train or compressive stress and compressive train.

$\boxed{E = \frac{\sigma}{\epsilon}}$ $\boxed{\text{unit is } N/mm^2}$

odulus of Rigidity (G) :-

It is the ratio between shear stress and shear train, $\tau \propto \phi$

$\boxed{\tau = G\phi}$ $\boxed{\text{unit is } N/mm^2}$

lk modulus :- (K - (kapa))

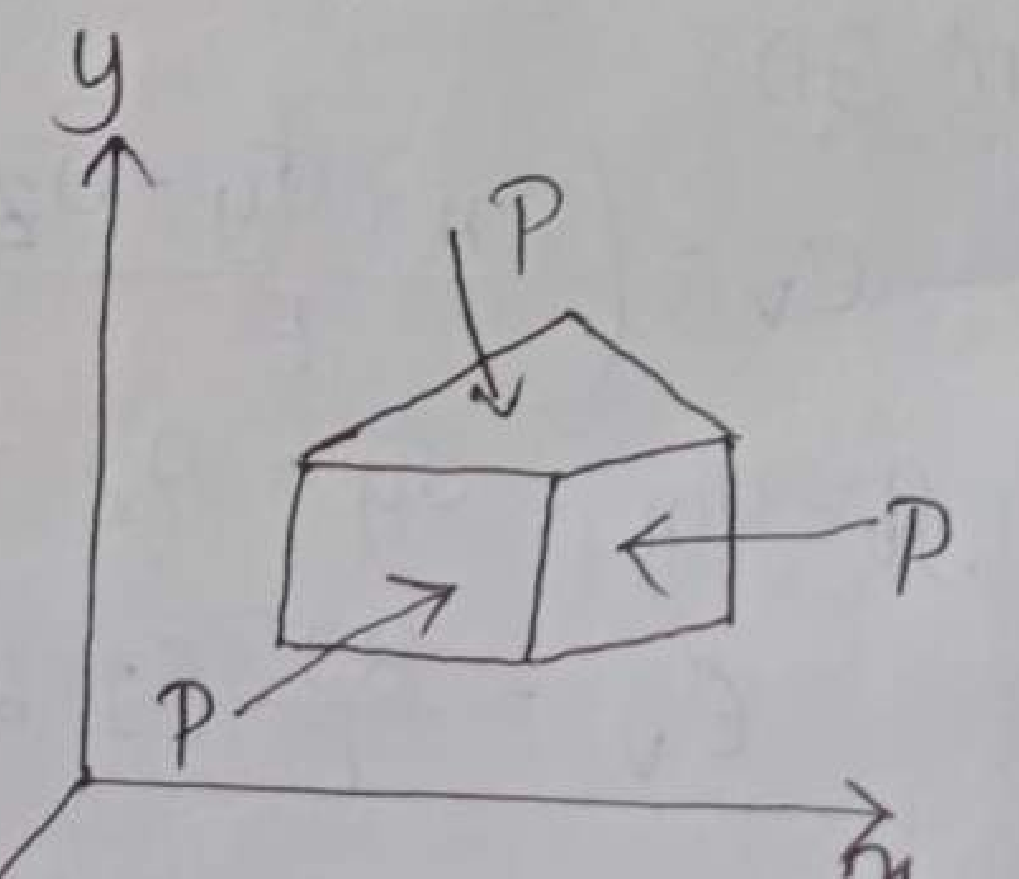
It is the ratio between normal stress and olumetric strain.

$\boxed{K = \frac{\sigma}{\epsilon_V}}$ $\boxed{K = \frac{-P}{\epsilon_V}}$ $\boxed{\text{unit is } N/mm^2}$

lation between E and K :-

$$K = \frac{\sigma}{\epsilon_V}$$

$$= \frac{-P}{\epsilon_V}$$



$$\epsilon_V = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1-2\mu)$$

$$= \frac{-P-P-P}{E}(1-2\mu)$$

$$= \frac{-3P}{E}(1-2\mu)$$

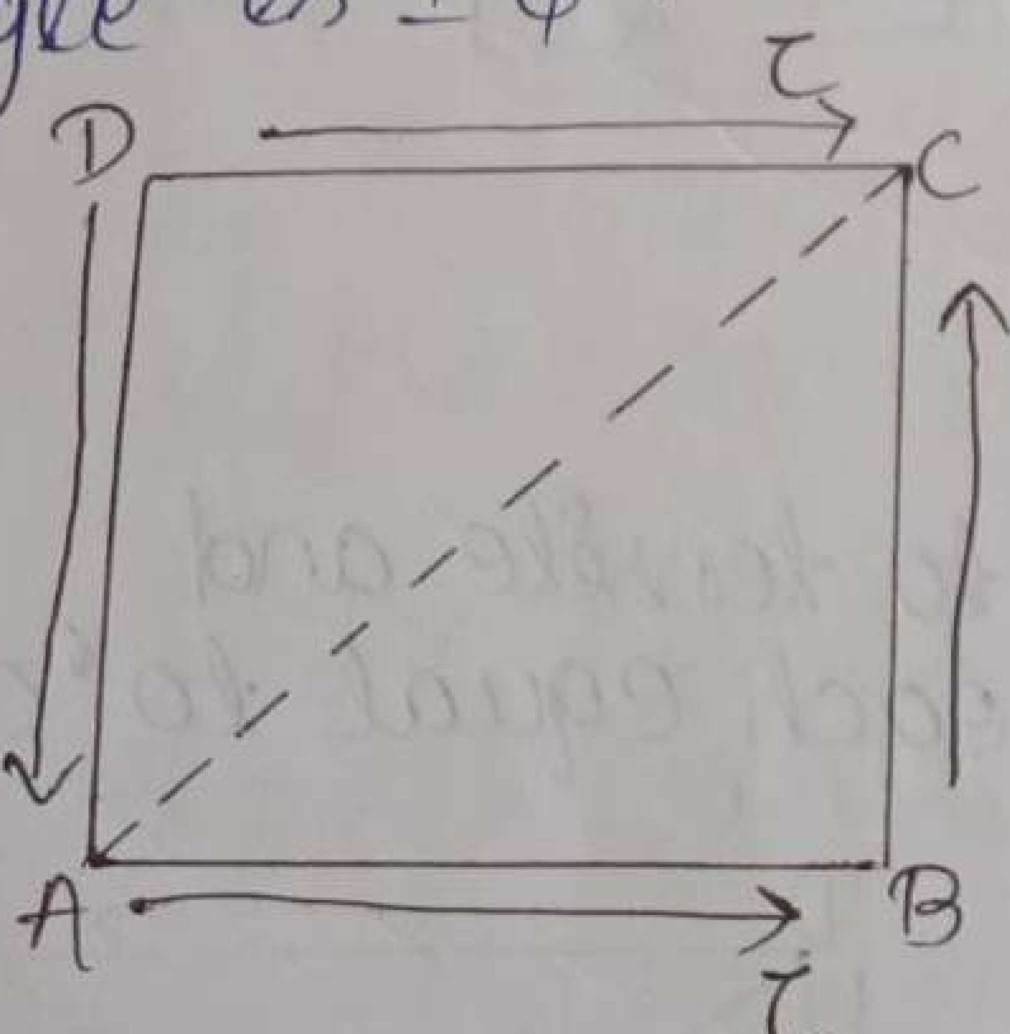
$$\epsilon_V = \frac{-3P}{E}(1-2\mu)$$

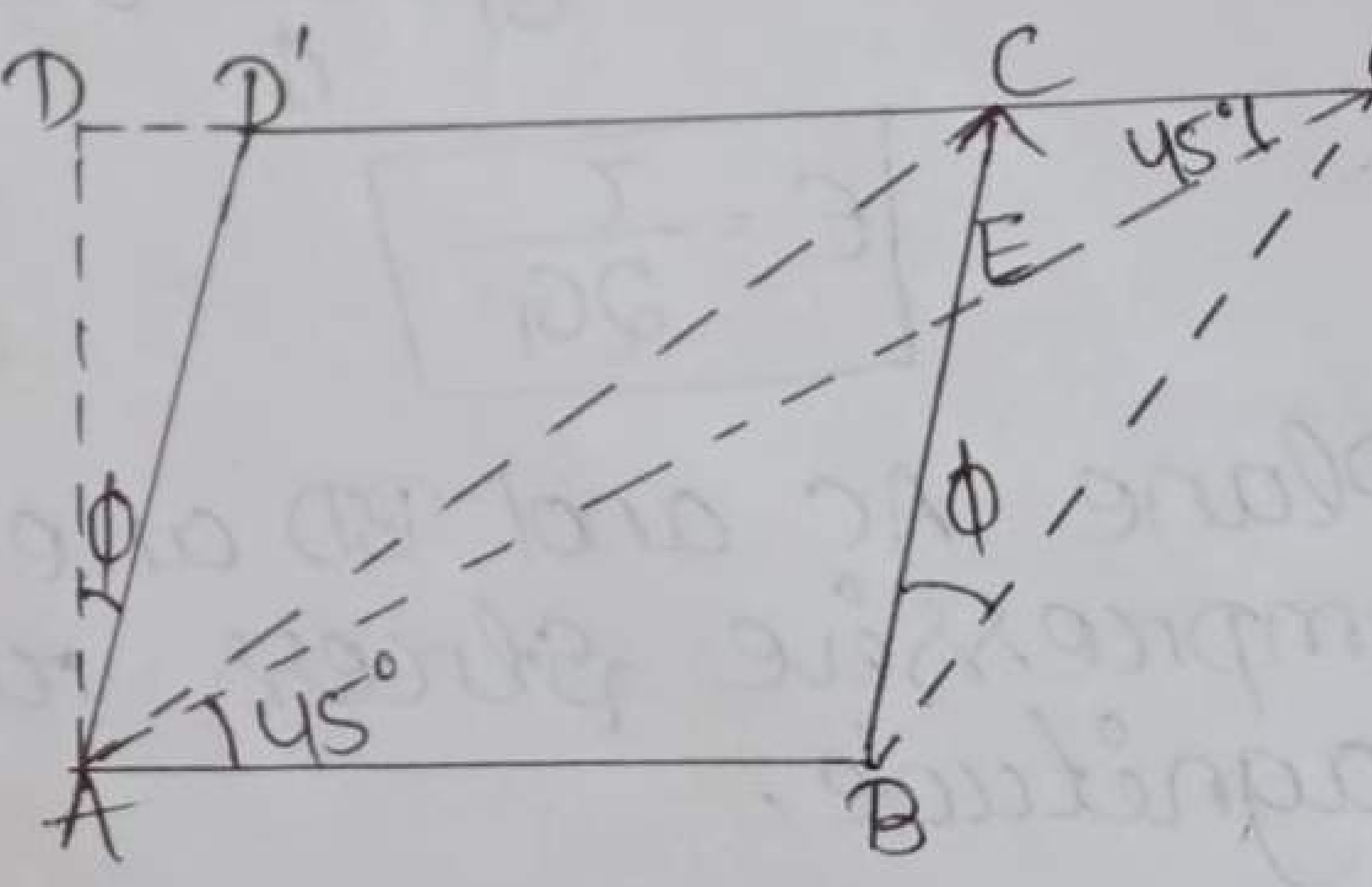
$$E = \frac{-3P}{\epsilon_V}(1-2\mu)$$

$$\boxed{E = 3K(1-2\mu)}$$

elation between E and G :-

consider a square element ABCD, under the actior a simple shear stress (τ). The total change in gle is $\pm\phi$.





ear strain of diagonal AC $= \frac{\Delta L}{L}$

$$= \frac{AC' - AC}{AC}$$

$$\epsilon = \frac{EC'}{AC} \quad \text{— (i)}$$

$$\cos 45^\circ = \frac{EC'}{CC'}$$

$$EC' = CC' \cos 45^\circ \quad \text{— (ii)}$$

$$AC = \frac{AB}{\cos 45^\circ} \quad \text{—— (iii)}$$

Put eq^n (ii) and (iii) in eq^n (i)

$$\epsilon = \frac{CC' \cos 45^\circ}{AB / \cos 45^\circ} \quad \text{—— (iv)}$$

$$\tan\phi = \frac{CC'}{BC} \qquad \tan\phi \simeq \phi \text{ (For small angle)}$$

$$\phi BC = CC' \quad \text{—— (v)}$$

$$AB = BC$$

$$\epsilon = \frac{\phi \cancel{BC} \cos^2 45^\circ}{\cancel{BC}}$$

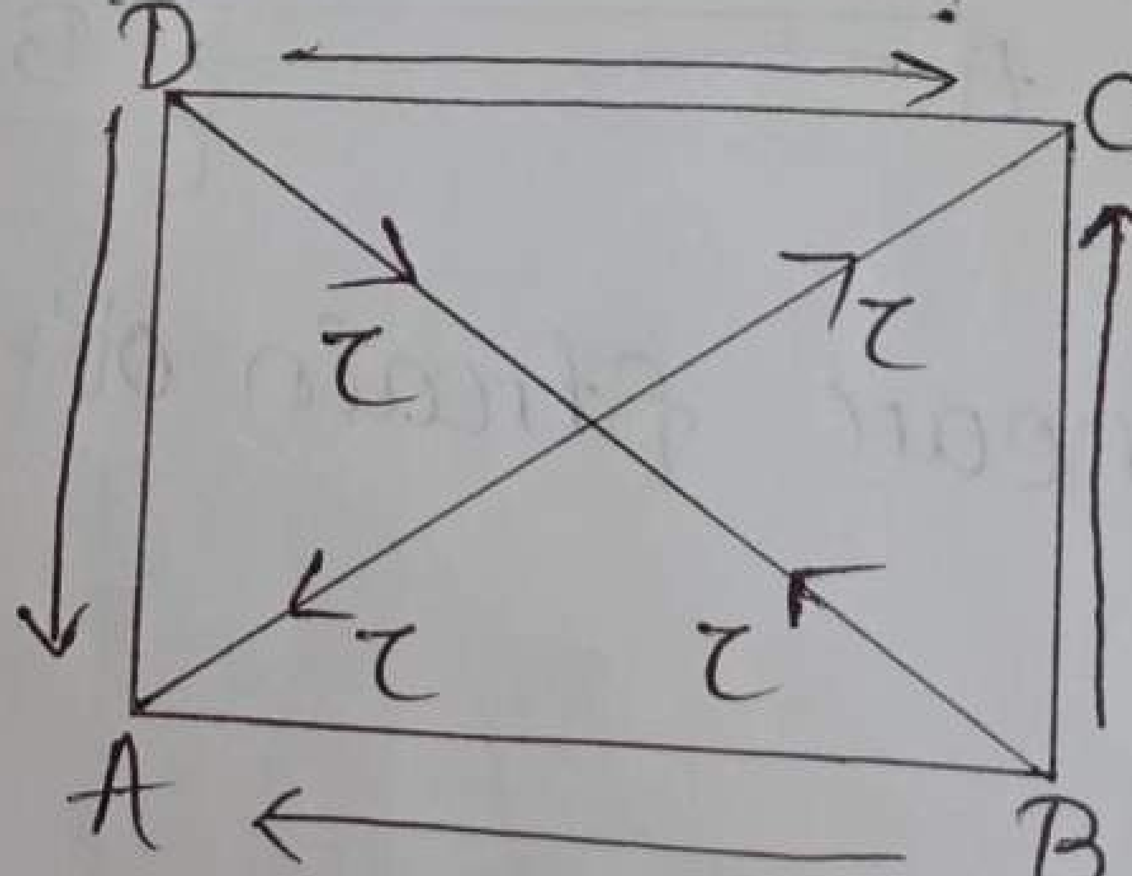
$$\boxed{\epsilon = \phi/2}$$

But modulus rigidity

$$G = \frac{\tau}{\phi} \text{ or } \phi = \frac{\tau}{G} \quad \text{as } \boxed{\epsilon = \frac{\phi}{2}}$$

$$\boxed{\epsilon = \frac{\tau}{2G}}$$

ane AC and BD are subjected to tensile and
pressive stress respectively each equal to 'τ'
nitude.



$$\epsilon = \frac{\tau}{E} - \left(-\mu \frac{\tau}{E}\right)$$

$$\epsilon = \frac{\tau}{E}(1+\mu)$$

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

$$\boxed{E = 2G(1+\mu)}$$

Elastic constant formula :-

$$\boxed{\begin{aligned} E &= 2G(1+\mu) \\ E &= 3K(1-2\mu) \\ E &= \frac{9KG}{3K+G} \end{aligned}}$$

A material as E of 2×10^5 N/mm² and a poission's
tio of 0.25. Calculate modulus of rigidity and
lk modulus.

$E = 2\times10^5$ N/mm²

$\mu = 0.25$

$E = 2G(1+\mu)$

$2\times10^5 = 2G(1+0.25)$

$2G = \frac{2\times10^5}{1.25}$

$2G = 160000$

$G = 80,000$ N/mm² $= 80$ GPa

$E = 3K(1-2\mu)$

$\Rightarrow 3K = \frac{2\times10^5}{1-2\times0.25}$

$3K = 400000$

$K = \frac{400000}{3}$

$K = 133333.33$ N/mm²

$\boxed{K = 133.33 \text{ GPa}}$

A bar 24mm diameter and 400 mm length is
ted upon by an uniaxial load of 38 kN, the elongation
the bar and change in diameter are measured
0.165 mm and 0.0 31 respectively determined
sson's ratio and value of 3 elastic modulus.

Given:- $d = 24mm$
$l = 400 mm$
$\Delta l = 0.165$ mm
$\Delta d = 0.0\,31$
$P = 38$ kN

oisson's ratio $= \dfrac{\text{lateral strain}}{\text{longitudinal strain}}$

$$\mu = \frac{\Delta d/d}{\Delta l/l}$$

$$\mu = \frac{\frac{0.0\,31}{24}}{\frac{0.165}{40}}$$

$$\mu = 0.313$$

$$\sigma = P/A$$

$$\sigma = \frac{38 \times 10^3 \text{ N}}{\frac{\pi}{4} \times (24)^2 \text{ mm}^2} \qquad \boxed{d = 24mm}$$

$\sigma = 84$ N/mm^2 or 84 MPa.

So. we know =

$$\sigma = E\epsilon$$

$$E = \frac{\sigma}{\epsilon}$$

$$E = \frac{84}{\Delta l/l}$$

$$E = 2G(1+\mu)$$

$$\Rightarrow 203.636 = 2G(1+0.313)$$

$$\Rightarrow 2G = \frac{203.636}{1+0.313}$$

$$\Rightarrow 2G = 155.09$$

$$\Rightarrow G = \frac{155.09}{2}$$

$$\Rightarrow G = 77.546 \text{ GPa or } 77546 \text{ MPa}$$

$$E = 3K(1-2\mu)$$

$$\Rightarrow 203636 = 3K(1-2\times 0.313)$$

$$\Rightarrow 3K = \frac{203636}{1-2\times 0.313}$$

$$\Rightarrow 3K = 544481.283$$

$$\Rightarrow K = \frac{54481.283}{3}$$

$$\Rightarrow K = 181493.76 \text{ MPa}$$

A bar 12 mm in diameter is acted upon by a axial
d 20KN change in diameter is measured 0.003
termine the poission's ratio, the modulus of
asticity and the bulk modulus. The value of
odulus of rigidity is 80GPa.

eq

$$\text{Area} = \frac{\pi}{4} \times (12)^2 = 36\pi \, mm^2$$

$$\sigma = \frac{20,000}{36\pi} = 176.84 MPa$$

Poission's ratio = ?

$$\mu = \frac{\text{lateral strain}}{\text{linear strain}}$$

$$\Rightarrow \text{Lateral strain} = \mu \times \epsilon (\text{linear strain})$$

$$\Rightarrow \frac{\Delta d}{d} = \mu \times \epsilon$$

$$\Rightarrow \frac{0.003}{12} = \mu \times \epsilon$$

$$\Rightarrow \epsilon = \frac{0.00025}{\mu}$$

$$E = 2G(1+\mu)$$

$$= 2 \times 80,000 (1+\mu)$$

$$E = 160000 + 160000\mu \quad \text{—— (i)}$$

$$E = \frac{\sigma}{\epsilon}$$

$$= \frac{176.84}{\frac{0.00025}{\mu}}$$

$$E = 707360\mu \quad \text{—— (ii)}$$

equate (i) & (ii)

$707360\mu = 160000 + 160000\mu$

$\Rightarrow 547360\mu = 160000$

$\Rightarrow \mu = \dfrac{160000}{547360}$

$= 0.2923$

$E = 707360\mu$

$= 707360 \times 0.2923$

$= 206761$ MPa

or 206.761 GPa

$K = \dfrac{E}{3(1-2\mu)}$

$= \dfrac{206761}{3(1-2\times0.2923)}$

$= 165913.176$ MPa

What will be percentage change in the volume of a teel bar of 20 mm diameter and 600 mm length when tensile stress of 180 MPa is applied to it along s longitudinal axis?

$$E_S = 205\ GPa,\ \mu = 0.3$$

Given,

$d = 20\ mm$ $\quad L = 600\ mm$

$E_S = 205\ GPa$ $\quad \mu = 0.3$

$\sigma = 180\ MPa$

Volume of the bar $= \frac{\pi}{4} \times d^2 \times h$

$= \frac{\pi}{4} \times (20)^2 \times 600$

$= 60{,}000\pi\ mm^3$

Change in volume $= V \times \frac{\sigma(1-2\mu)}{E}$

$= 60{,}000\pi \times \frac{180(1-2\times 0.3)}{205{,}000}$

$= 66.2\ mm^3$

Percentage change in volume $= \frac{66.2}{60{,}000\pi}$

$= 0.035$

$\boxed{\text{Volume of cylinder} = \pi r^2 h}$

Q: A bare of 240 mm long and 40×30 mm² crosssection is subjected to an axile tensile force of 100 kN. Find the change in length, breadth, thickness and volume. Take $E = 200\,GPa$, $\mu = 0.3$

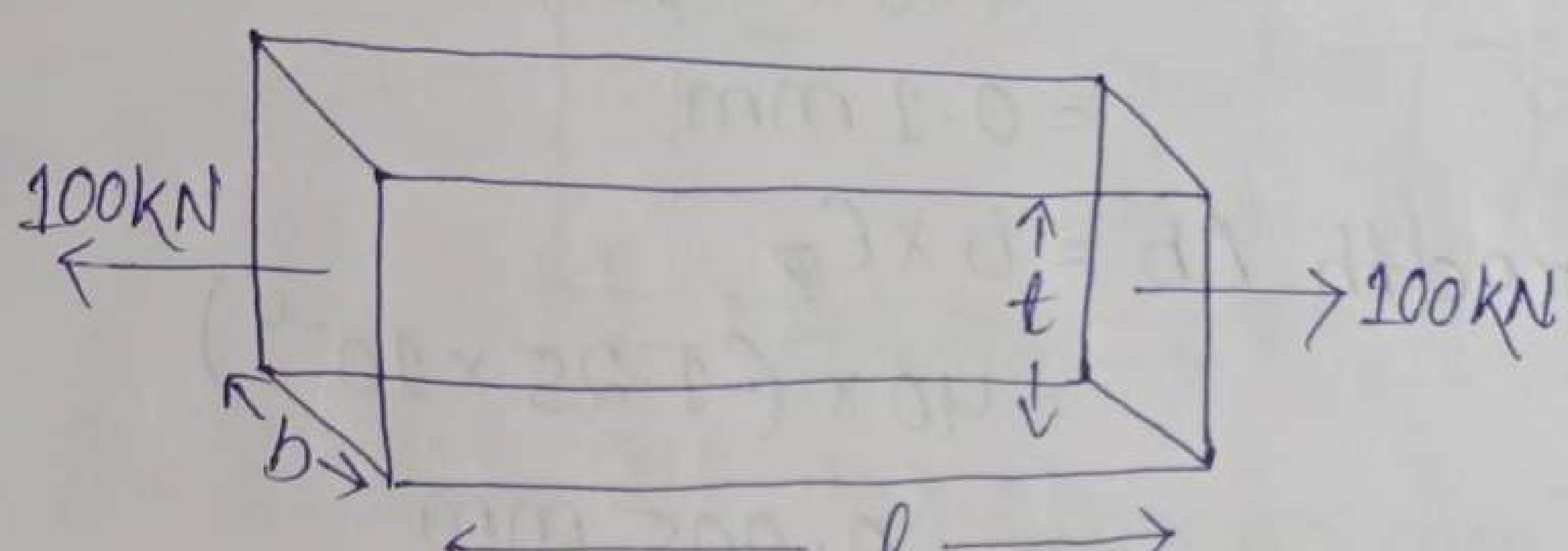
Ans

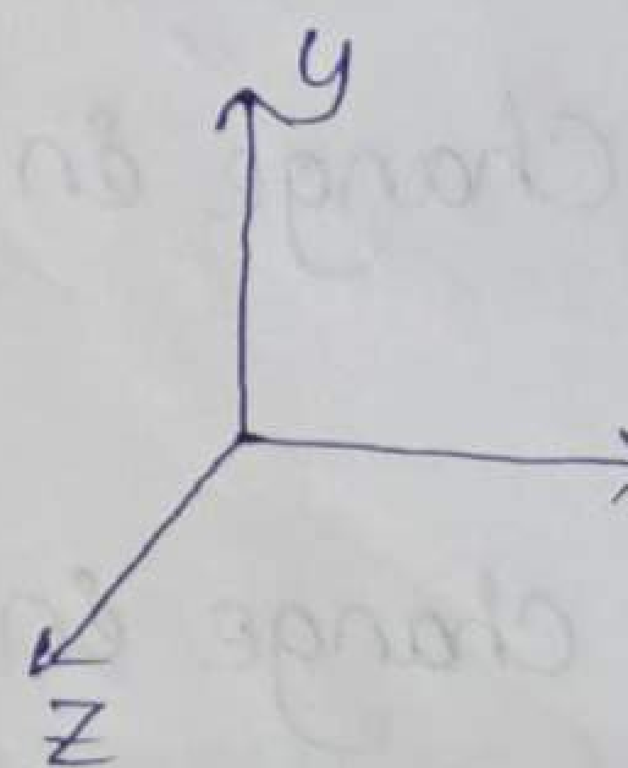
Given data:-

$l = 240\,mm$ $\qquad \mu = 0.3$

$b = 40\,mm$ $\qquad E = 200\,GPa$

$t = 30\,mm$ $\qquad P = 100\,kN$





$$\sigma_x = \frac{P}{A} = \frac{100\times10^3}{40\times30} = 83.34\ N/mm^2$$

$$\sigma_y = 0, \quad \sigma_z = 0$$

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \mu(\cancel{\sigma_y} + \cancel{\sigma_z})\right)$$

$$= \frac{1}{E}(\sigma_x)$$

$$= \frac{83.34}{200\times10^3}$$

$$= 4.167\times10^{-4}$$

$$\epsilon_y = \frac{1}{E}\left(\cancel{\sigma_y} - \mu(\sigma_x + \cancel{\sigma_z})\right)$$

$$= \frac{-\mu\sigma_x}{E}$$

$$= \frac{-0.3\times83.34}{200\times10^3}$$

$$= -1.25\times10^{-4}$$

$$\epsilon_z = \frac{1}{E}\left(\cancel{\sigma_z}^{0} - \mu\left(\sigma_x + \cancel{\sigma_y}^{0}\right)\right)$$

$$= \frac{-\mu \sigma_x}{E}$$

$$= \frac{-0.3 \times 83.34}{200 \times 10^3}$$

$$= -1.25 \times 10^{-4}$$

Change in length $\Delta l = l \times \epsilon_x$

$$= 240 \times 4.167 \times 10^{-4}$$

$$= 0.1\,mm$$

Change in breadth $\Delta b = b \times \epsilon_y$

$$= 40 \times (-1.25 \times 10^{-4})$$

$$= -0.005\,mm$$

change in thickness $\Delta t = t \times \epsilon_z$

$$= 30 \times (-1.25 \times 10^{-4})$$

$$= -0.0037\,mm$$

volumetric strain, $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$$\epsilon_v = (4.167 - 1.25 - 1.25) \times 10$$

$$= 1.667 \times 10^{-4}$$

change in volume $\Delta v = v \times \epsilon_v$

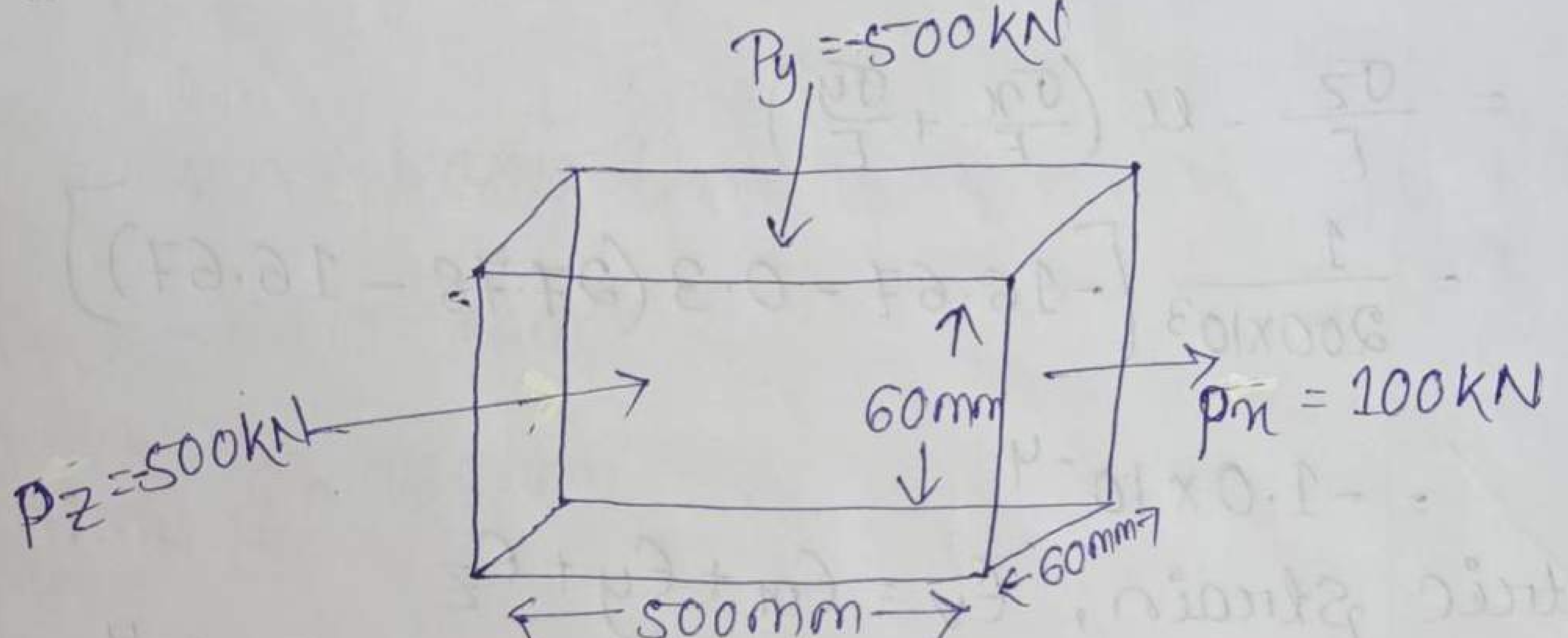
$$= 240 \times 40 \times 30 \times 1.667 \times 10^{-}$$

$$= 48\,mm^3$$

$$\boxed{\Delta v = 48\,mm^3}$$

A bar 500mm long is having squeral crosssection of size 60mm x 60mm. If the bar is subjected to an axial load of 100KN and a lateral compression of 500KN in face of size 60mm, 500mm. Find the change in size and volume. Take $E = 200\,GPa$, $\mu = 0.3$

Ans



$l = 500\,mm$ $A = 60 \times 60\ mm^2$

$P_x = 100\ KN$ (Tensile) $P_y = P_z = -500\ KN$ (compressive)

$E = 200 \times 10^3\ N/mm^2$, $\mu = 0.3$

Now, $\sigma_x = \frac{P_x}{A} = \frac{100 \times 10^3}{60 \times 60} = 27.78\ N/mm^2$

$\sigma_y = \sigma_z = P/A = \frac{-500 \times 10^3}{60 \times 500} = -16.67\ N/mm^2$

Strain in the direction of 'x'

$$\epsilon_x = \frac{\sigma_x}{E} - \mu(\sigma_y + \sigma_z)$$

$$= \frac{1}{E}\left(27.78 - 0.3(-16.67 - 16.67)\right)$$

$$= \frac{1}{200 \times 10^3}\left(27.78 + 0.3(2 \times 16.67)\right)$$

$$= 1.889 \times 10^{-4}$$

$\epsilon_y = \frac{\sigma_y}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_z}{E}\right)$

$= \frac{1}{200\times10^3}\left[-16.67 - 0.3\,(-16.67 + 27.78)\right]$

$= -1.0\times10^{-4}$

$\epsilon_z = \frac{\sigma_z}{E} - \mu\left(\frac{\sigma_x}{E} + \frac{\sigma_y}{E}\right)$

$= \frac{1}{200\times10^3}\left[-16.67 - 0.3\,(27.78 - 16.67)\right]$

$= -1.0\times10^{-4}$

lumetric strain, $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$

$\epsilon_v = (1.889 - 1.0 - 1.0)\times10^{-4}$

$= -0.111\times10^{-4}$

hange in volume $= \frac{\Delta V}{V}$ $\quad (\because \Delta V = V\times\epsilon_v)$

$\frac{\Delta V}{V} = \epsilon_v$

$\Rightarrow \Delta V = -0.111\times10^{-4}\times V$

$\Rightarrow \Delta V = -0.\underline{111}\times10^{-4}\times500\times60\times60$

$\Rightarrow \boxed{\Delta V = -19.98\ mm^3}$

change in size ?

$\Delta L = L\times\epsilon_x \quad \Delta t = t\times\epsilon_t$

$\Delta b = b\times\epsilon_y$

$$b = 60 \times (-1.0 \times 10^{-4})$$

$$= -6 \cdot \times 10^{-3}\ mm$$

$$= -0.006\ mm$$

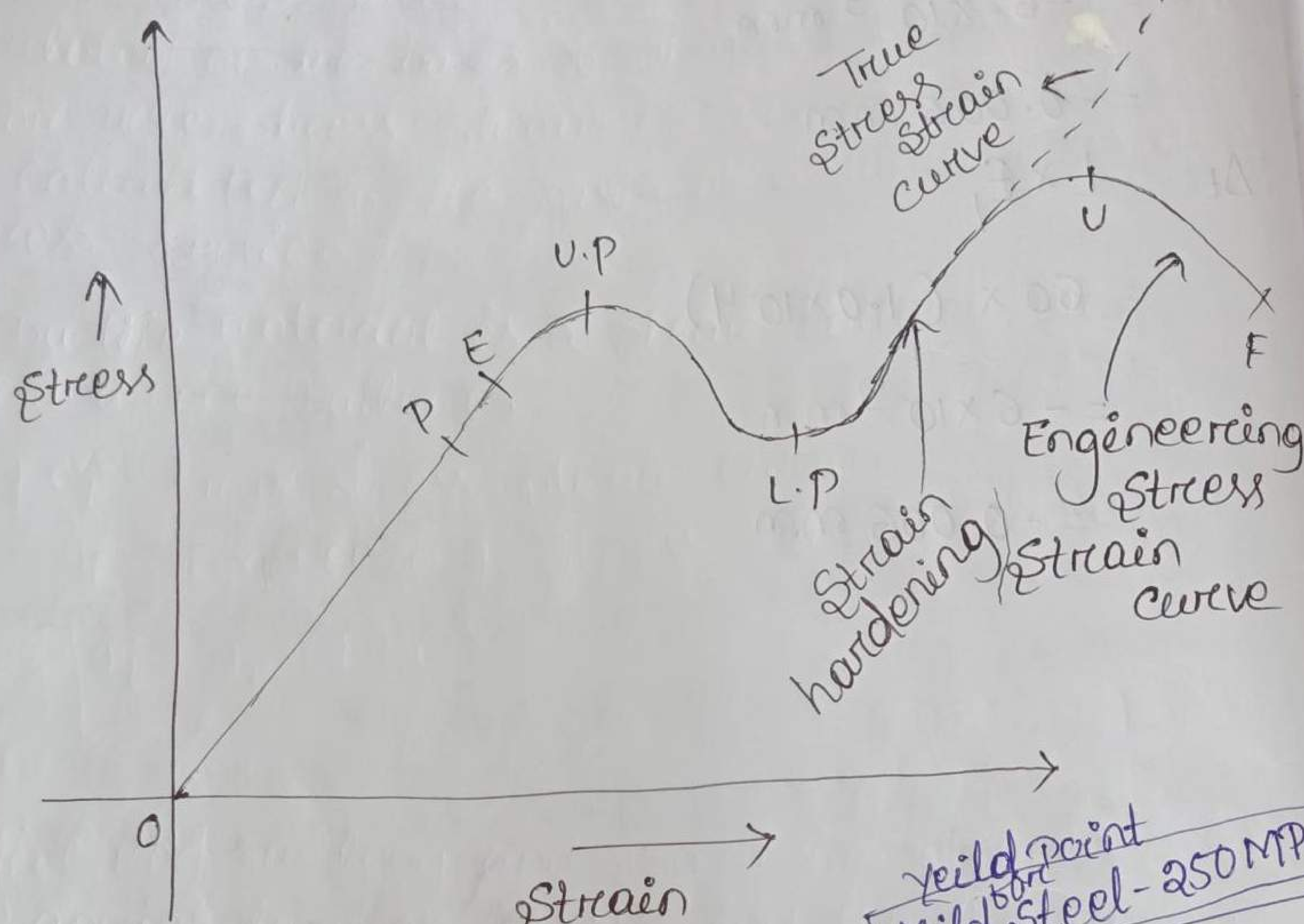
$$= t \times \epsilon_t$$

$$= 60 \times (-1.0 \times 10^{-4})$$

$$= -6 \times 10^{-3}\ mm$$

$$= -0.006\ mm$$

Stress Strain curve of Ductile material



Yeild point for Mild Steel - 250 MPa

Mild Steel Strain - 22% - 25%

P = proportionality limit
E = Elastic limit
U.P = upper yeild point
L.P = Lower yeild Point
U = Ultimate Point
F = Breaking Point or Fracture point

lient feature of diagram:-
he Stress Strain curve is obtained by tensile
t using ductile material as the standard
ecimen. The load is hydraulically applied
measurement is done. The elongation of the
cimen is noted simultaneously...

Portion O-P :-

It is straight i.e. stress, is proportional to strain. The point 'P' is known as limit of proportionality. In other word this is the limit of linear elasticity.

Portion P-E :-

In this portion the curve departs from linearity, but the material is still elastic i.e. in this portion the stress is no longer proportional to strain. The point 'E' is known as elastic limit. It is the point of greatest stress that the material can withstand without giving a permanent deformation when the load is removed.

Portion E-UP-LP :-

After Elastic limit, yeilding start (Plastic flow of materials). The yeild point is the point at which there is an appreciable change in length without any corresponding increase of load, even it decreases. The structural members have uppear yeild point and lower yeild point.

Portion LP-U :-

After the lower yeild point, the curve becomes smoo and much flatter. It rises till a point U, known ultimate point. It is the point of maximum stress that the specimen is capable of sustaining its original area of cross-section.

Portion U-F :-

After the point U, the area of cross-section is reduced appreciably and this phenomenon is call necking simultaneously the apparent stress ominal stress strain curve ⇒ all the stresses are alculated on the bases of original cross-section.

t for True stress strain curve ⇒ all the stresses e calculated on the bases of instantaneous area cross-section.

Ductile material is a shear failure or up and cone failure.

* σ_t = True stress strain curve

σ_E = Engineering stress strain curve

$$\boxed{\sigma_t = \sigma_E (1+\epsilon)}$$

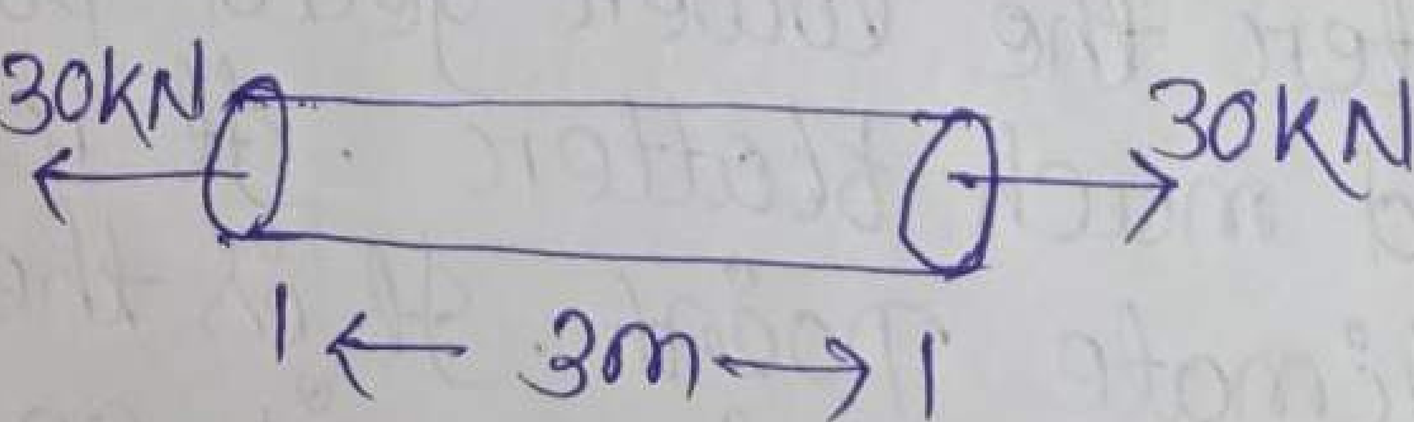
steel rod 3m long and 33mm diameter is subjected n axial load of 30 kN. Find the change in th, diameter and volume of the rod. Take E 200 GPa and Poission's ratio as 0.32.

~~St~~ Given,

3m = 3000mm

= 30mm , P = 30kN

= 0.32 , E = 200 GPa



ind :- Δl, Δd, ΔV

$$= \frac{\sigma_x}{E} - \mu \left(\frac{\sigma_y}{E} + \frac{\sigma_z}{E} \right) \quad \boxed{\sigma_y = \sigma_z = 0}$$

$$= \frac{\sigma_x}{E}$$

$$\sigma_x = P/A = \frac{30 \times 10^3}{\pi}$$

$= 0.2123 \times 10^{-3}$

$= 0.2123 \times \cancel{10^{-3}} \times 3 \times \cancel{10^{3}}$

$l = 0.6369\ mm$

$y = -\mu \frac{\sigma_x}{E}$

$y = -0.32 \times \frac{42.46}{200 \times 10^3}$

$\epsilon_y = -0.0679 \times 10^{-3}\ mm$

$\frac{d}{d} = -0.0679 \times 10^{-3}$

$d = -0.0679 \times 10^{-3} \times 30$

$d = -2.037 \times 10^{-3}\ mm$

* negative is for direction

$z = \frac{\cancel{\sigma_z}^{0}}{E} - \mu \left(\frac{\sigma_x}{E} + \frac{\cancel{\sigma_y}^{0}}{E} \right)$

$= -\mu \frac{\sigma_x}{E}$

$\boxed{z = \epsilon_y = -0.0679 \times 10^{-3}\ mm}$

$= \epsilon_x + \epsilon_y + \epsilon_z$

$= 0.2123 \times 10^{-3} - 0.0679 \times 10^{-3} - 0.0679 \times 10^{-3}$

$= 0.0765 \times 10^{-3}$

$= 0.0765 \times 10^{-3}$

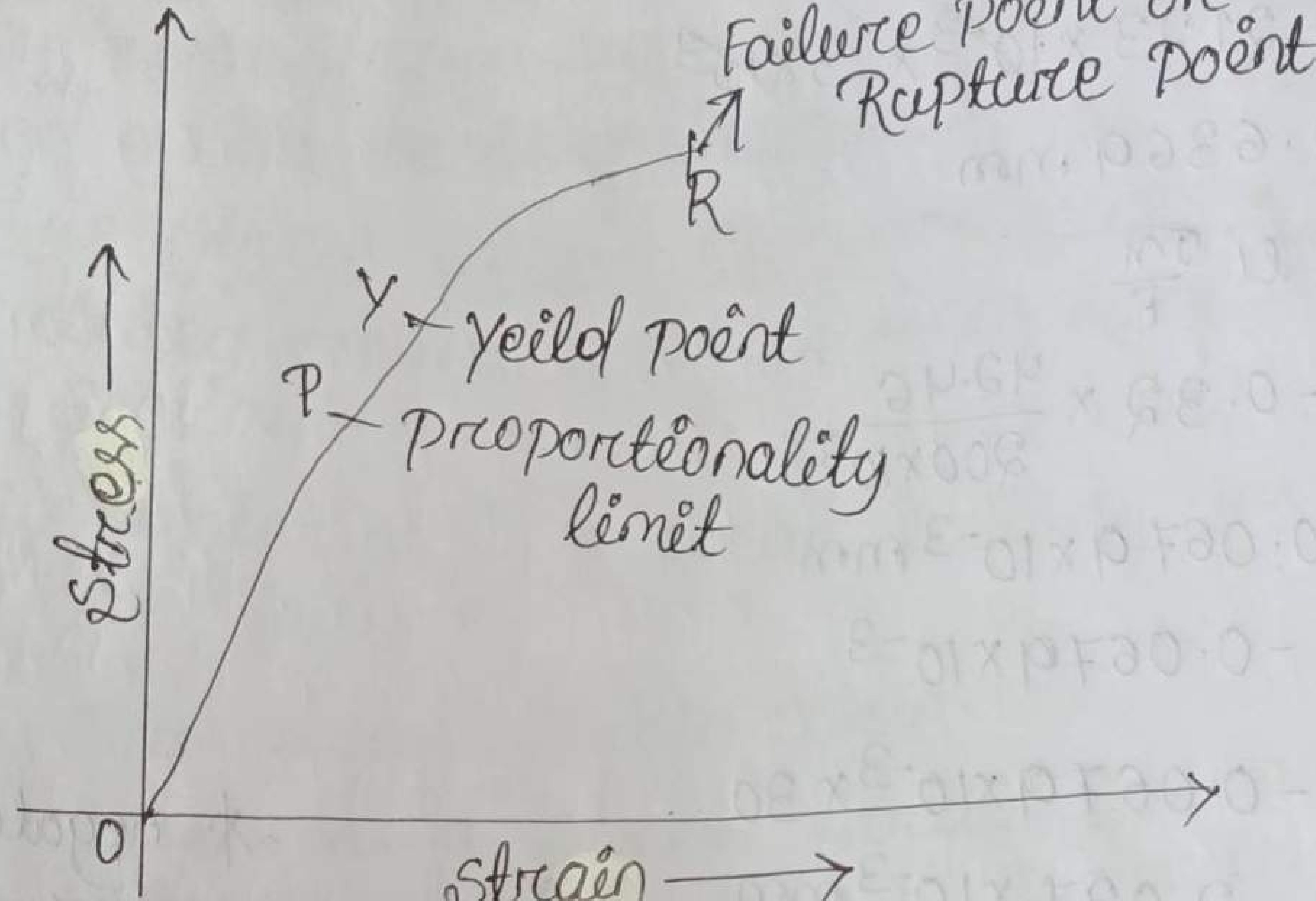
$= \pi \frac{d^2}{4} \times h$

$= \pi \times \frac{(30)^2}{4} \times 3 \times 10^3$

$= 2120.18 \times 10^3$

$= 2120.18 \times \cancel{10^{3}} \times 0.0765 \times \cancel{10^{-3}}$

tress Strain curve of Brittle material:



Example of Brittle material:- castiron and concrete.
Brittle material have a very low strain and fails
immediately after yeilding.
Ductile materials have a large signficant
ermanent deformation and does not fail
nmediately.

roportionality limit:-
This is called as limit of linear elasticity and to
his point stress is directly proportional to
train.

astic limit:-
is the point of greatest stress that the material
n with stand without giving a permanent
formation when the load is removed.

ild stress:-

corresponding to this stress is called yeild point.

Ultimate Stress :-

It is the point of maximum stress that the material capable of substaining in its original area of cross - section.

Rupture Stress :-

The stress at which material fails is called Breaking stress and the point corresponding to this stress is called Rupture point or Breaking point.

Percentage of elongation :-

It is ratio between change in length at rupture and the original length expressed in percentage

if l' = Final change in length.

l = original length.

$$\% \text{ Elongation} = \frac{l' - l}{l} \times 100$$

Percentage elongation is the measure of ductility. the higher the value of percentage elongation, the more ductile is the material.

Percentage reduction in area :-

The reduction in area of the specimen or material at the neck divided by the original area of specimen or material expressed as percentage is known as percentage reduction in area.

A' = Reduced area at the neck.

A_0 = Original area.

$$\boxed{\text{Percentage Reduction} = \frac{A_0 - A'}{A_0} \times 100}$$

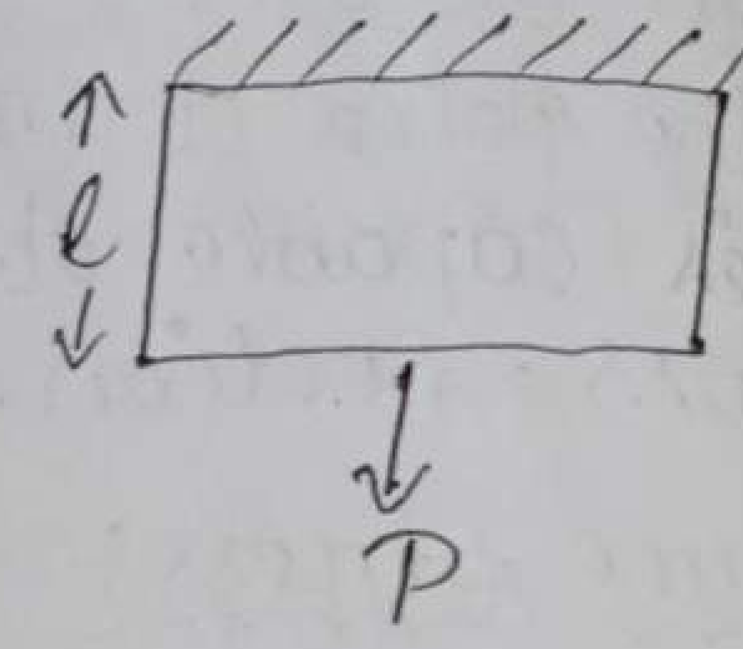
Deformation of Prismatic bar in uniaxial loading:

$\sigma = P/A$

$\sigma = E\epsilon$

$P/A = E\epsilon$

$P/A = E \times \frac{\Delta l}{l}$



$$\boxed{\Delta l = \frac{Pl}{AE}}$$

Principle of Superposition:-

The net elongation of the body is equal to the algebraic sum of elongation of the individual section. This principle of finding elongation is known as principle of superposition.

$$\boxed{\Delta l_{net} = \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} + \frac{P_3 l_3}{A_3 E_3}}$$

Elongation of body due to self weight:-

Consider a bar AB hanging freely under the action of its own weight.

Let, A = cross-sectional Area

l = original length

E = young's modulus

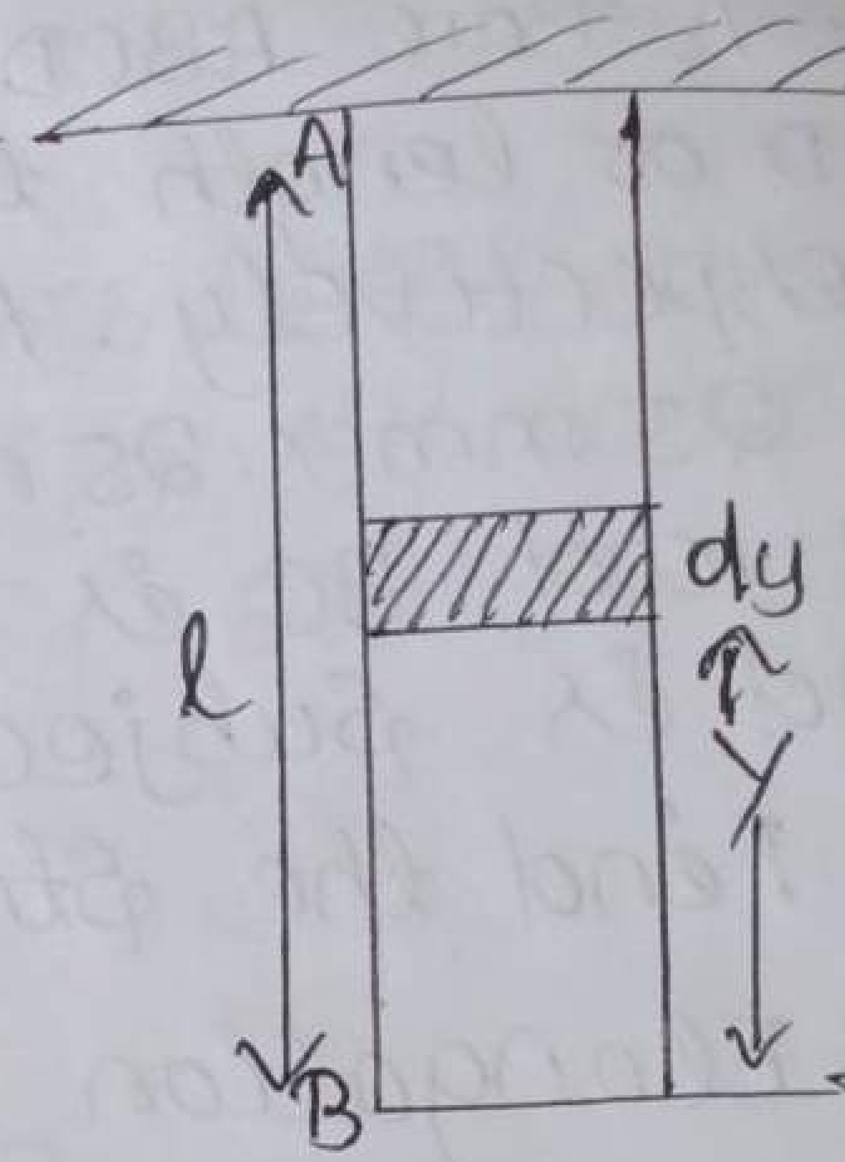
w = weight density of the bar material

Let us consider an elementary length dy at a distance y from free end. The total pull acting on this section dy is the weight of the length y

Hence total pull on elemantary length $P = WAY$

Elongation of elematary length

$$\Delta y = \frac{P\,dy}{AE} = \frac{W\cancel{A}y\,dy}{\cancel{A}E} = \frac{wy\,dy}{E}$$



Therefore total elongation of body due to self weight

$$\Delta l = \int \Delta y = \int_0^l \frac{wy\,dy}{E}$$

or $\boxed{\delta l = \frac{wl^2}{2E}}$ Total weight $= WAl$

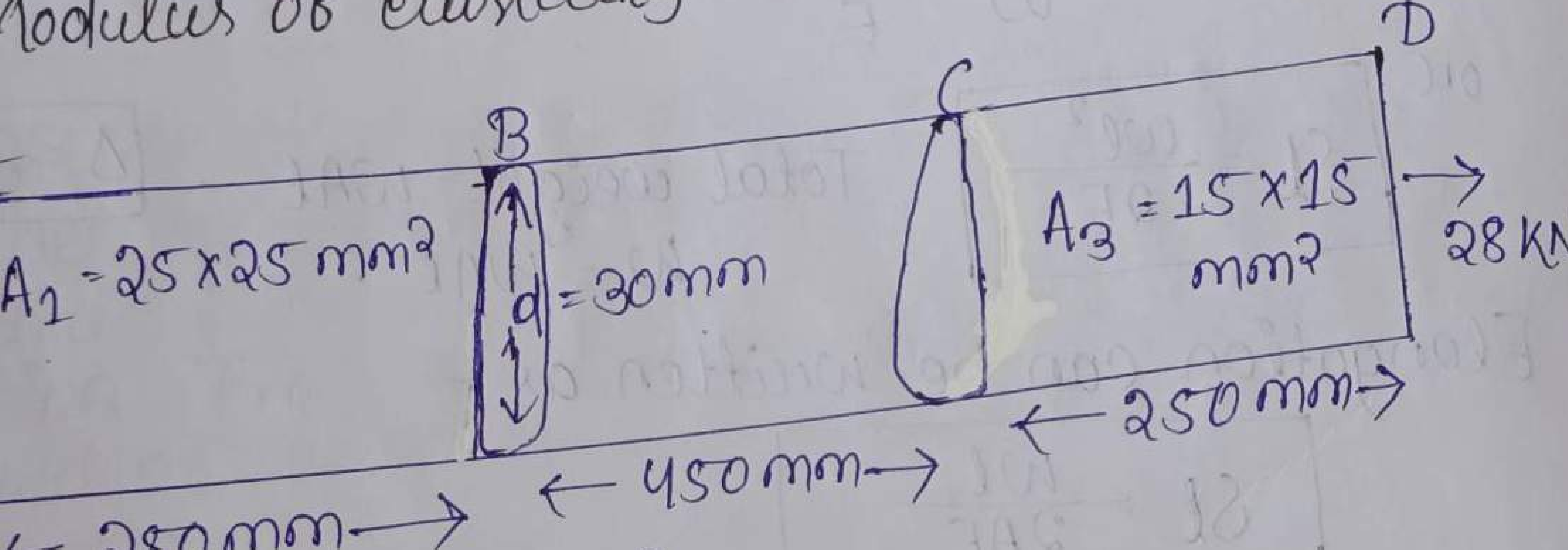
$W = WAl$.

$\boxed{\Delta = \delta}$

Elongation can be written as :-

$$\boxed{\delta l = \frac{Wl}{2AE}}$$

Bar ABCD 950 mm is made up 3 parts AB, BC &
f length to 250 mm, 450 mm and 250 mm
ectively. AB and CD are square cross-section
5 mm x 25 mm and 15 mm x 15 mm respectively.
od BC is spherical of diameter 30 mm. The
is subjected to a pull of 28 KN.
ind the stress in 3 parts of the rod?
longation of the rod?
odulus of elasticity, $E = 2 \times 10^5\ N/mm^2$



$= 25 \times 25 = 625\ mm^2$

$= \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (30)^2 = 706.73\ mm^2$

$= 15 \times 15 = 225\ mm^2$

$\sigma_{AB} = \frac{28 \times 10^3\ N}{25 \times 25} = 44.8\ N/mm^2$

$\sigma_{BC} = \frac{28 \times 10^3}{\pi/4 \times d^2} = \frac{28 \times 10^3}{706.73} = 39.62\ N/mm^2$

$\sigma_{CD} = \frac{28 \times 10^3}{15 \times 15} = 124.44\ N/mm^2$

$\delta l = \delta l_1 + \delta l_2 + \delta l_3$

$\delta l = \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E} + \frac{P l_3}{A_3 E}$

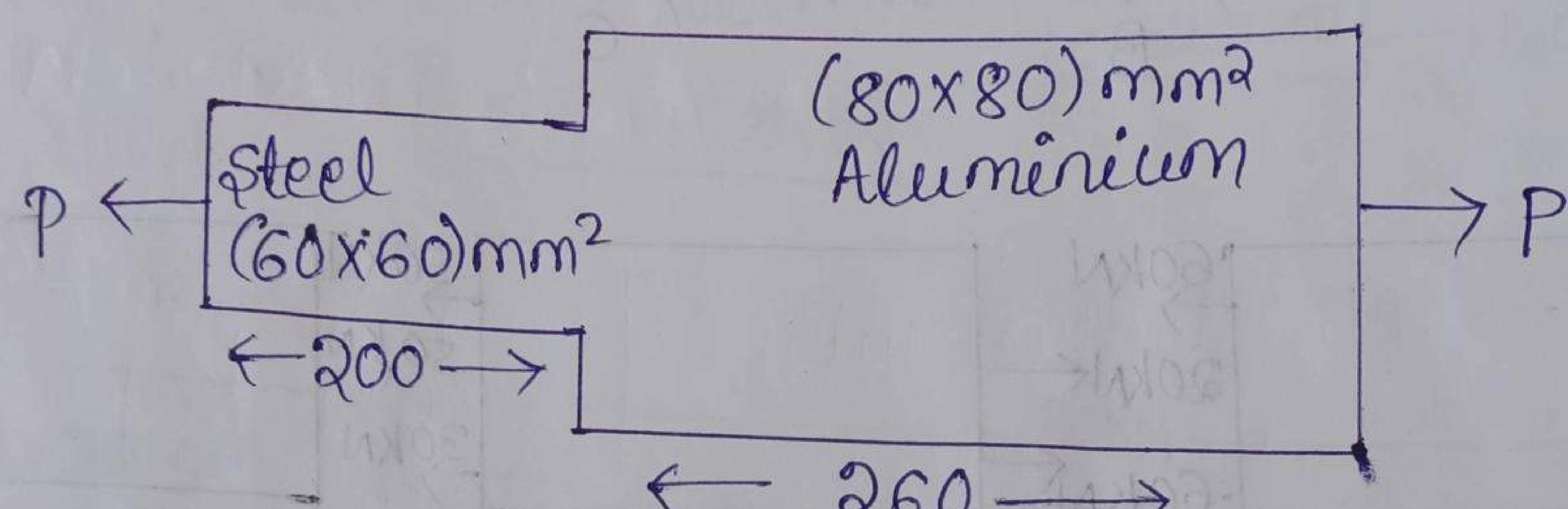
$28 \times 10^3 \times 450$ $\quad$ $28 \times 10^3 \times 250$

Q: A
Alu
ente
EA
Ans

= 0.056 + 0.0891 + 0.155

= 0.3 mm.

member formed by 2 different bars (steel & ...minium) is subjected to a load of 'p'. If total ...ension is 0.3 mm. Find 'p'. take E_s = 200 GPa a... = 70 GPa.



$$\delta l = \delta l_1 + \delta l_2$$

$$\delta l = \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E}$$

$$0.3 = \frac{P \times 200}{60 \times 60 \times 2 \times 10^5 \, N/mm^2} + \frac{P \times 260}{80 \times 80 \times 70 \times 10^3}$$

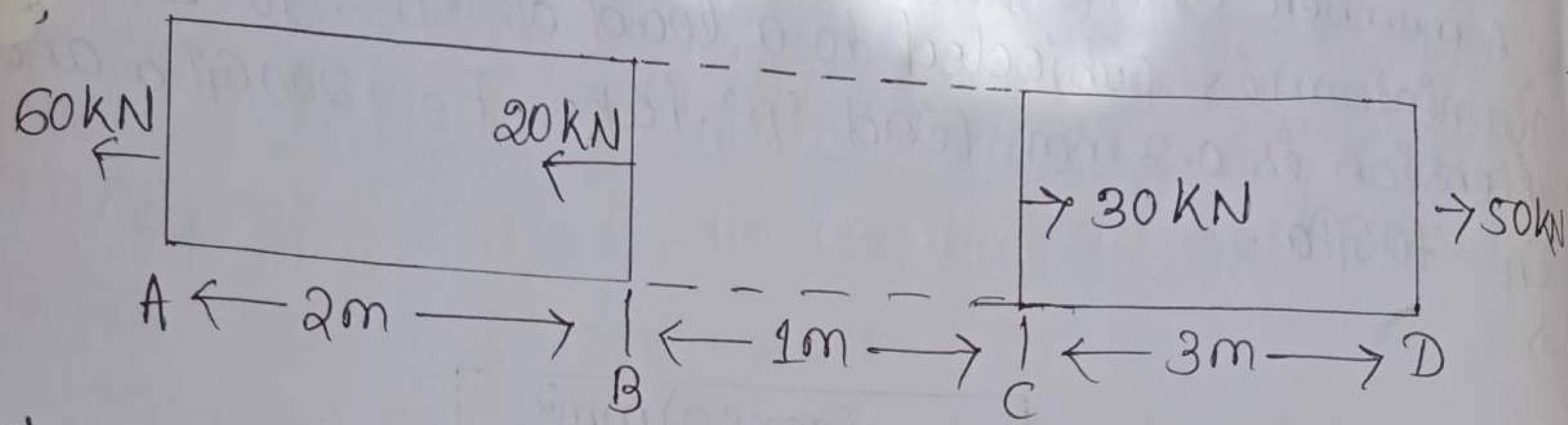
$$0.3 = 0.0277 \times 10^{-5} P + 0.058 \times 10^{-5} P$$

$$0.3 = 0.857 \times 10^{-5} P$$

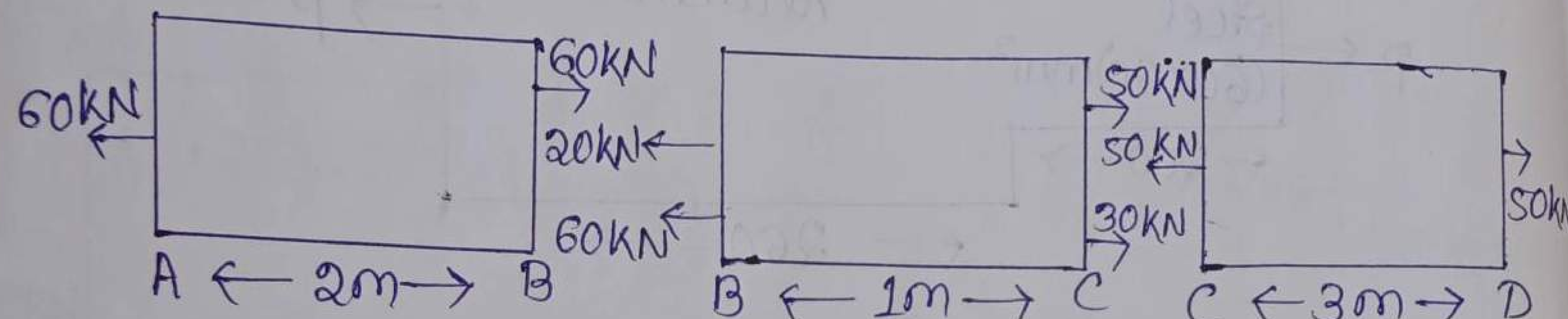
$$\boxed{P = 350.058 \, kN}$$

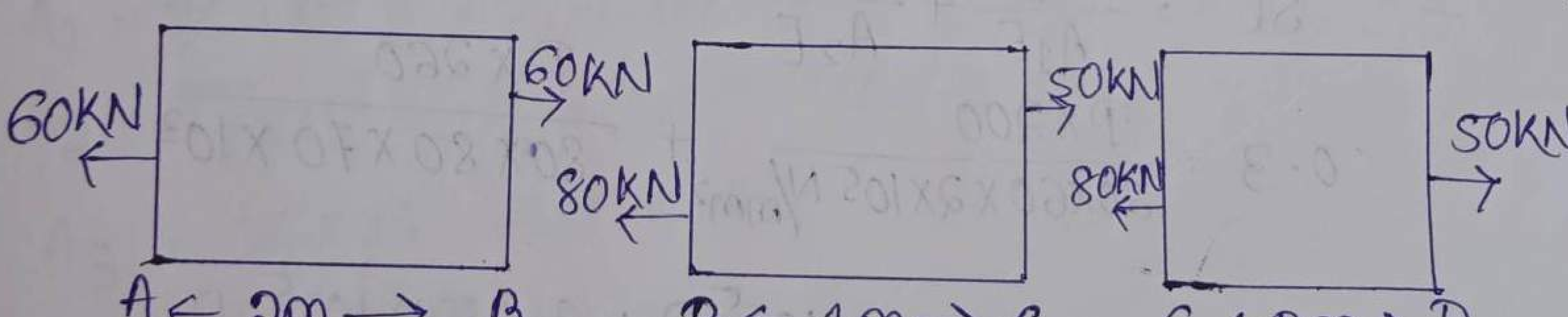
Imp

A steel bar of 25 mm diameter is acted upon by forces as shown in fig. What is the elongation of the bar, E = 190 GPa.



Ans





$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \delta_{AB} + \delta_{BC} + \delta_{CD}$$

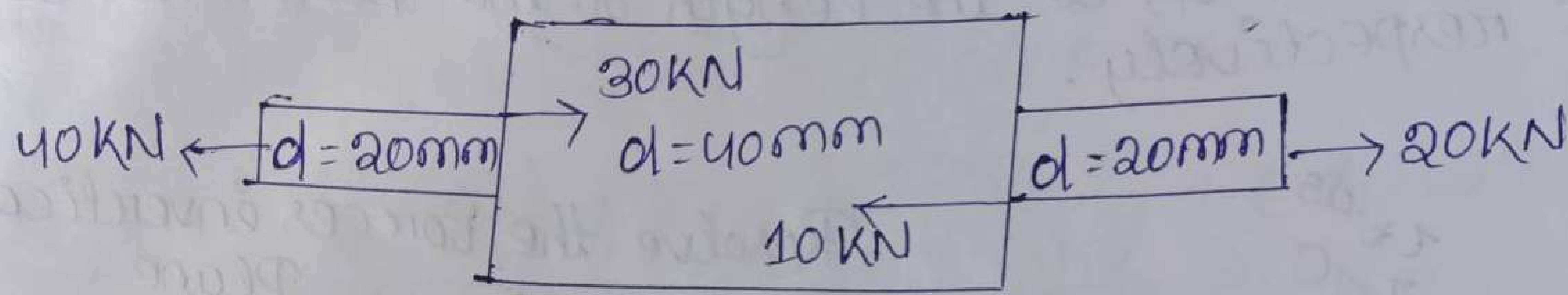
$$= \frac{P l_{AB}}{A_{AB} \times E} + \frac{P l_{BC}}{A_{BC} \times E} + \frac{P l_{CD}}{A_{CD} \times E}$$

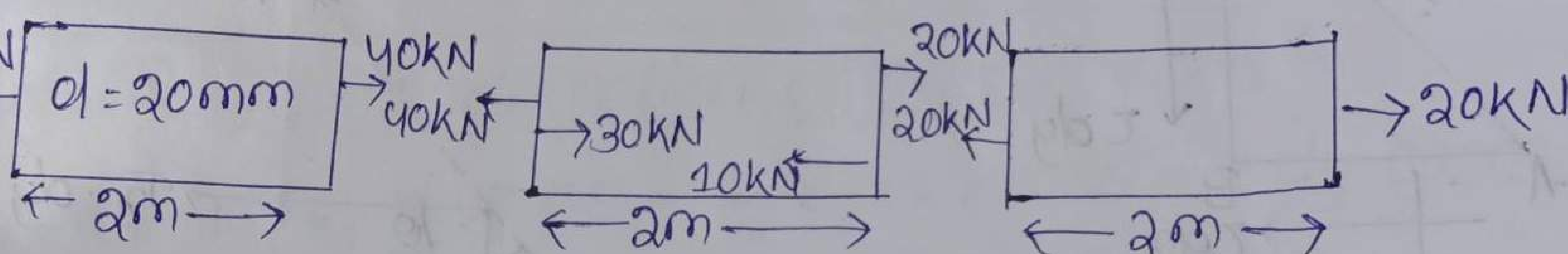
$$= \frac{60 \times 10^3 \times 2000}{\frac{\pi}{4} \times (25)^2 \times 190 \times 10^3} + \frac{80 \times 10^3 \times 1000}{\frac{\pi}{4} \times (25)^2 \times 190 \times 10^3}$$

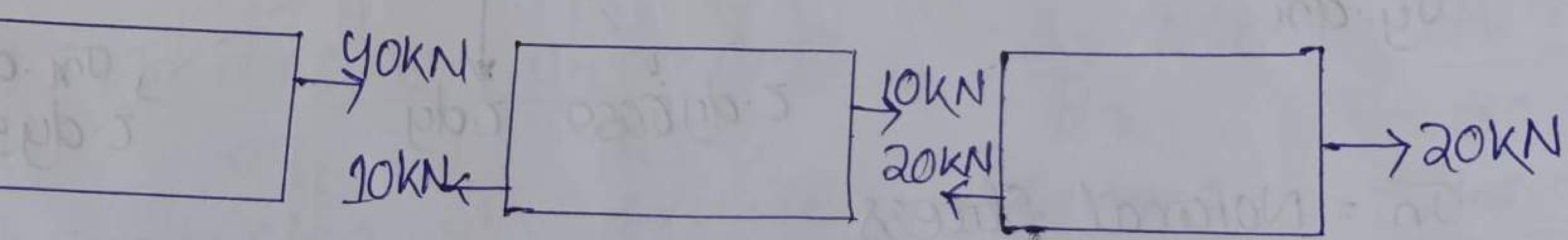
$$+ \frac{50 \times 10^3 \times 3000}{\frac{\pi}{4} \times (25)^2 \times 190 \times 10^3}$$

$= 3.75 mm$

A steel bar ABCD 6m. long consisting of different section is subjected to force as soon as figure. find the elongation of the bar. Take $E = 200$ GPa.







$$\delta l = \frac{P l_1}{A_1 E} + \frac{P l_2}{A_2 E} + \frac{P l_3}{A_3 E}$$

$$= \frac{40 \times 10^3 \times 2000}{\frac{\pi}{4} \times (20)^2 \times 200 \times 10^3} + \frac{10 \times 10^3 \times 2000}{\frac{\pi}{4} \times (40)^2 \times 200 \times 10^3}$$

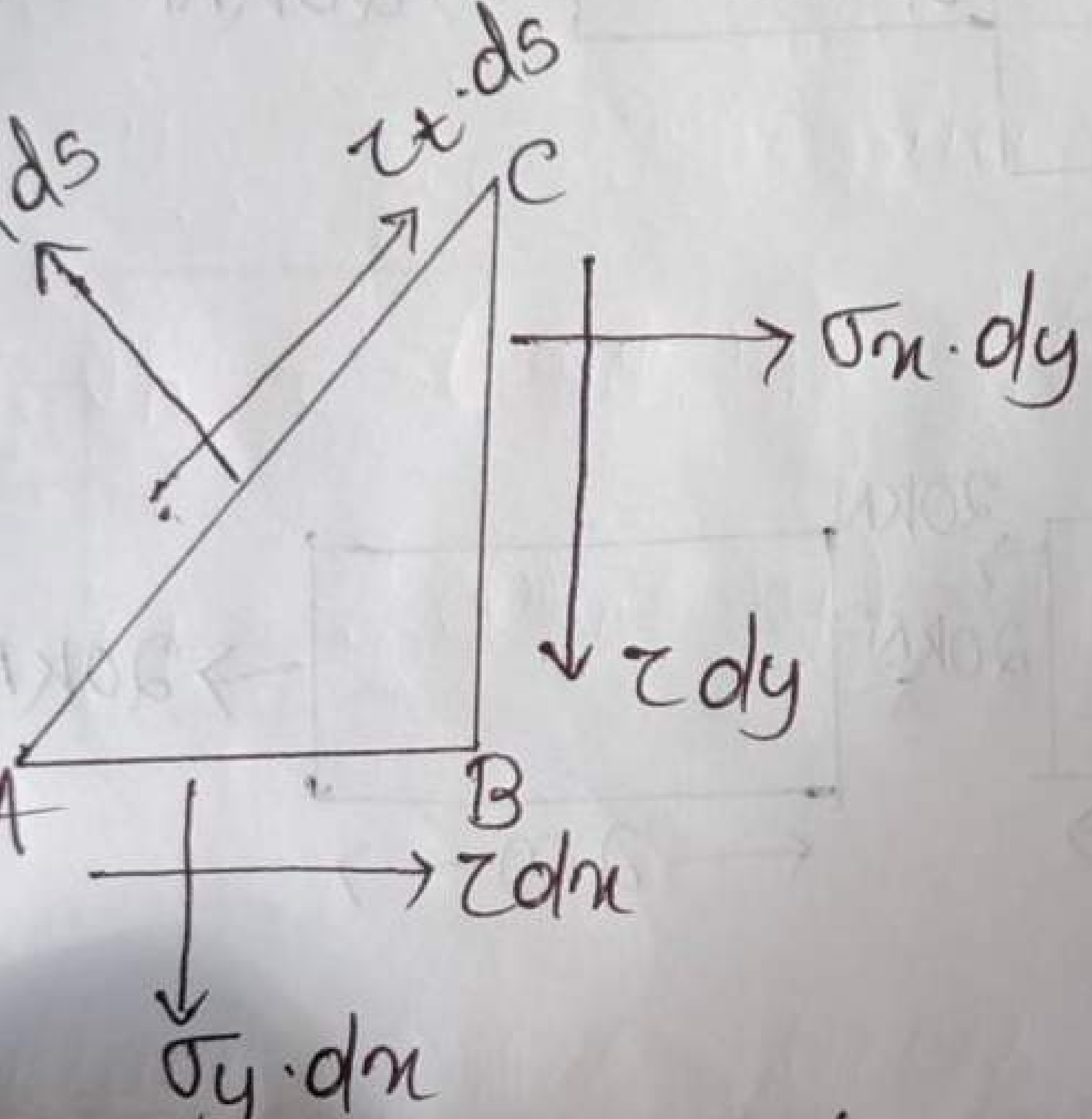
$$+ \frac{20 \times 10^3 \times 2000}{\frac{\pi}{4} \times 20^2 \times 200 \times 10^3}$$

$$= 1.273 + 0.079 + 0.636$$

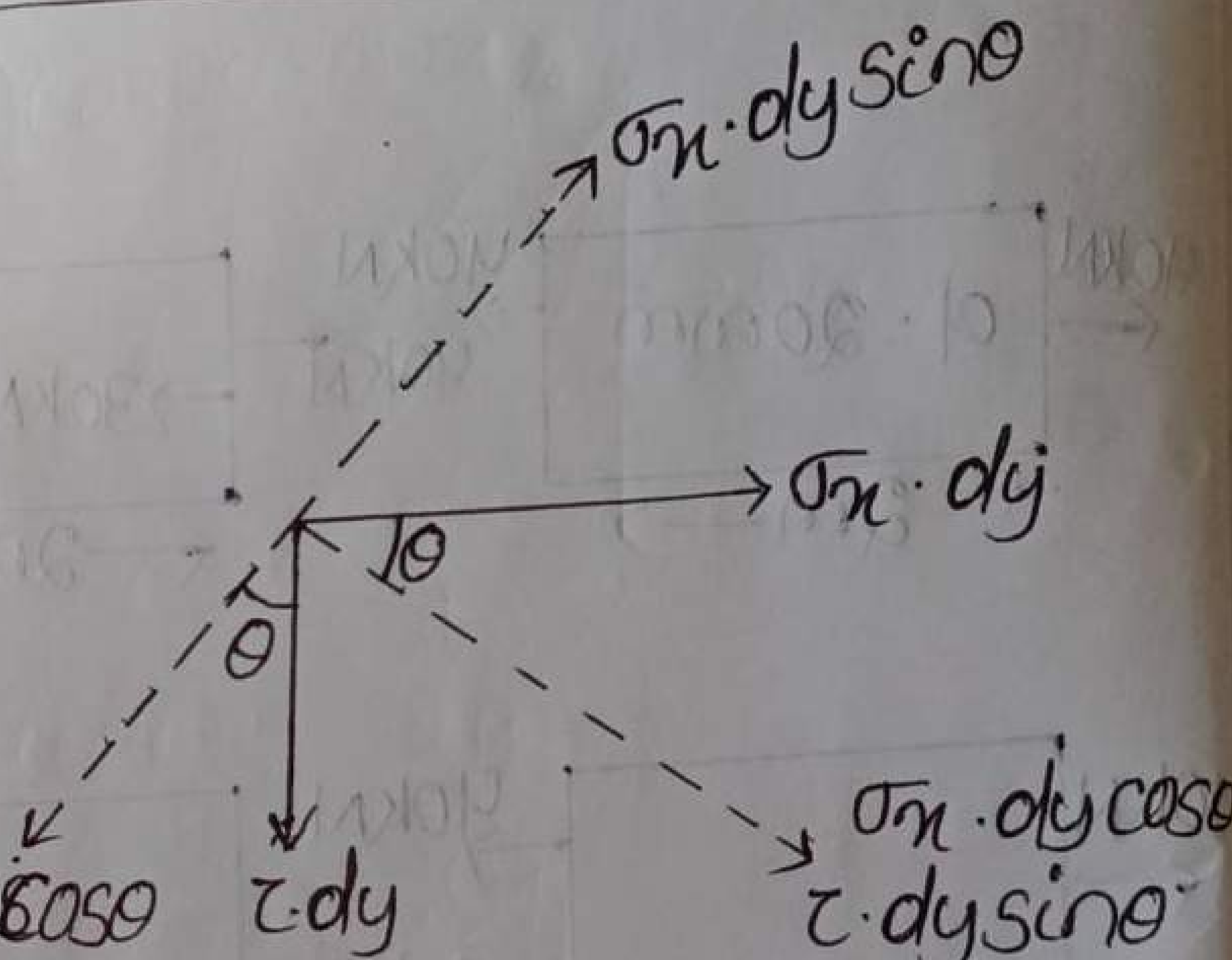
...en Stress and Strain

...xial and Shear Stress condition

... an element of a body acted upon by ... two tensile ...esses along with shear stresses acting on two ...pendicular planes of the body as shown in figure. ... dx, dy and ds be the length of the sides AB, BC ... AC respectively.



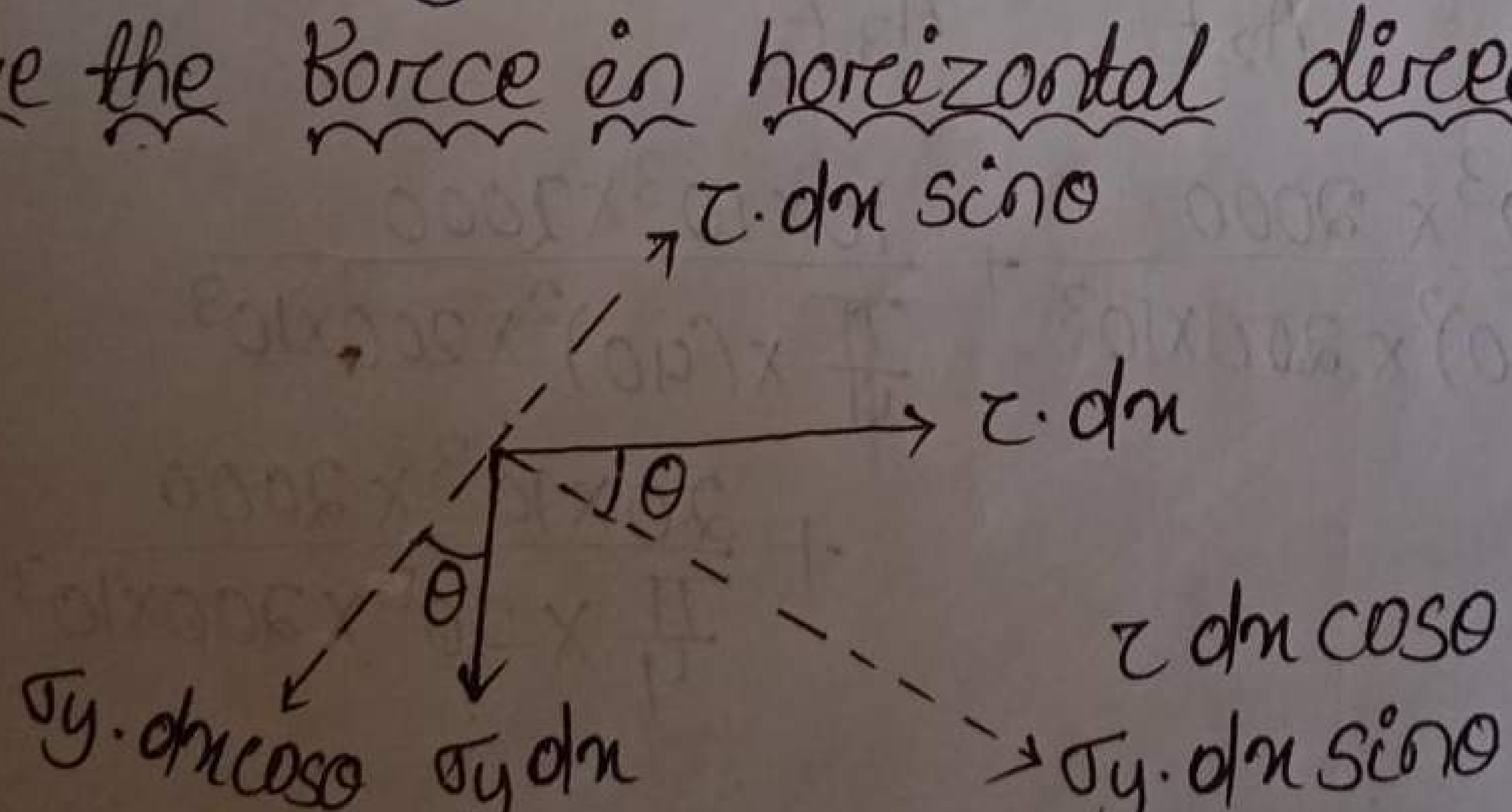
Resolve the forces in vertical plane



σ_n = Normal stress

τ_t = Tangential stress

...lve the force in horizontal direction:



...sidering unit thickness of the body and resolving ... forces in the direction of σ_n.

$$\sigma_n = \frac{\sigma_x \cdot dy\cos\theta}{ds} + \frac{\sigma_y dx\sin\theta}{ds} + \frac{\tau dy\sin\theta}{ds} + \frac{\tau \cdot dx \cdot \cos}{ds}$$

$$\sigma_n = \frac{\sigma_x \cdot \cancel{dy}\cos\theta}{\cancel{dy}/\cos\theta} + \frac{\sigma_y \cancel{dx}\sin\theta}{\cancel{dx}/\sin\theta} + \frac{\tau \cancel{dy}\sin\theta}{\cancel{dy}/\cos\theta} + \frac{\tau \cancel{dx}\cos}{\cancel{dx}/\sin}$$

$$\sigma_n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + \tau\sin\theta\cos\theta + \tau\ \sin\theta c$$

$$n = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau\sin\theta\cdot\cos\theta$$

$$= \sigma_x\left(\frac{1+\cos^2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) + \tau\sin 2\theta$$

$$\boxed{= \frac{1}{2}(\sigma_x+\sigma_y) + \frac{1}{2}(\sigma_x-\sigma_y)\cos 2\theta + \tau\sin 2\theta}$$

solving in the direction of τ_t :

$$t \cdot ds + \sigma_x \cdot dy\sin\theta - \sigma_y \times dx \times \cos\theta - \tau\, dy\cos\theta + \tau\, dx\sin\theta = 0$$

$$= -\frac{\sigma_x dy\sin\theta}{ds} + \frac{\sigma_y dx\cos\theta}{ds} + \frac{\tau dy\cos\theta}{ds} - \frac{\tau dx\, s}{ds}$$

$$\frac{-\sigma_x \cancel{dy}\sin\theta}{\cancel{dy}/\cos\theta} + \frac{\sigma_y \cancel{dx}\cos\theta}{\cancel{dx}/\sin\theta} + \frac{\tau\cancel{dy}\cos\theta}{\cancel{dy}/\cos\theta} - \frac{\tau\cancel{dx}\, s}{\cancel{dx}/s}$$

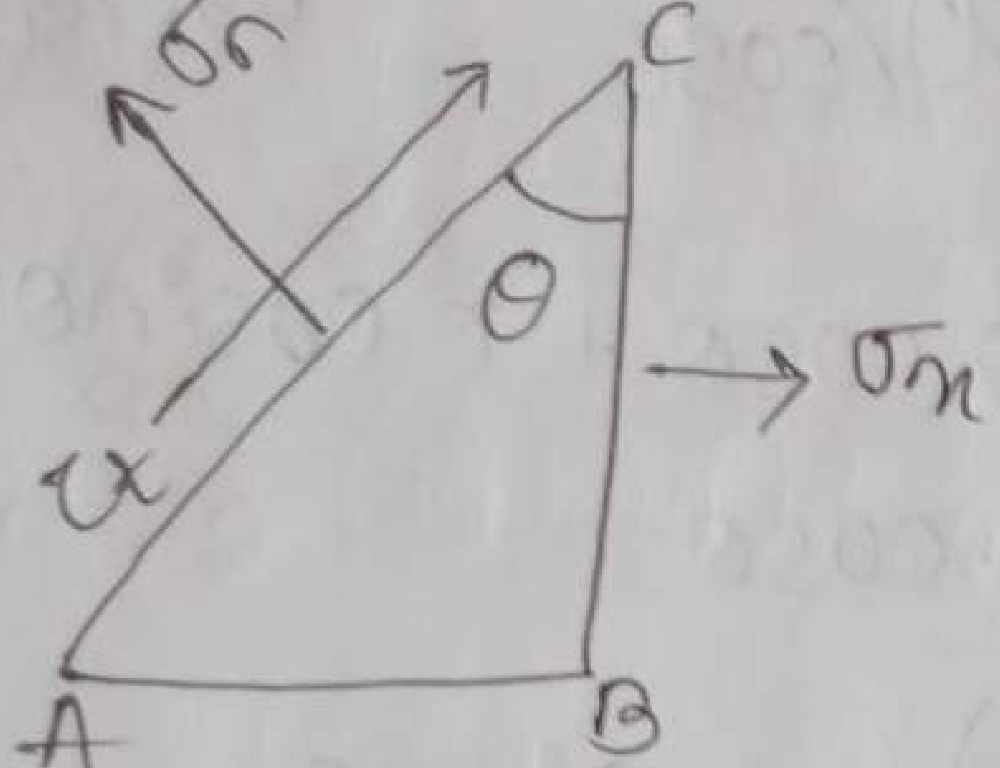
$$-\sigma_x\sin\theta\cos\theta + \sigma_y\sin\theta\cos\theta + \tau\cos^2\theta - \tau\sin^2\theta$$

$$-\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\theta + \tau\left[\left(\frac{1+\cos 2\theta}{2}\right) - \left(\frac{1-\cos^2\theta}{2}\right)\right]$$

$$\boxed{-\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\theta + \tau\cos 2\theta}$$

rious cases in Plane stress condition:

ase-I :- Direct stress condition



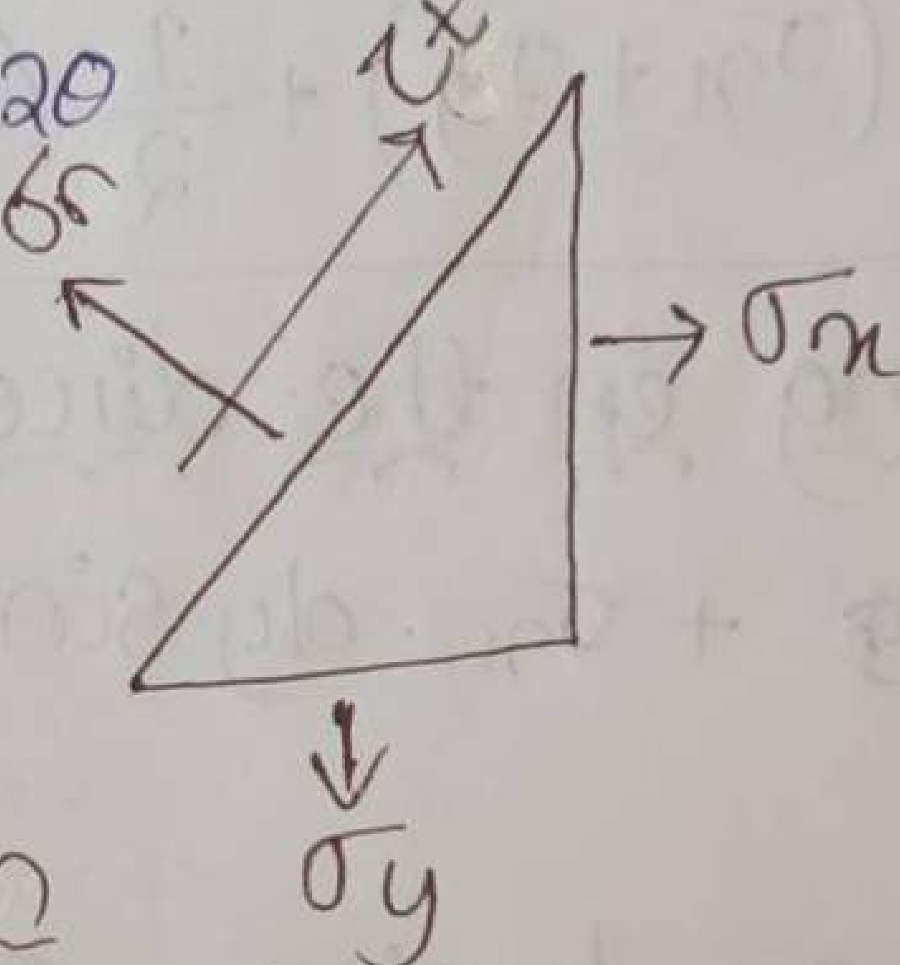
$$\sigma_n = \sigma_x \cos^2\theta$$

$$\tau_t = -\frac{1}{2}\sigma_x \sin 2\theta$$

se-II :- Biaxial stress condition

$$\sigma_n = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta$$

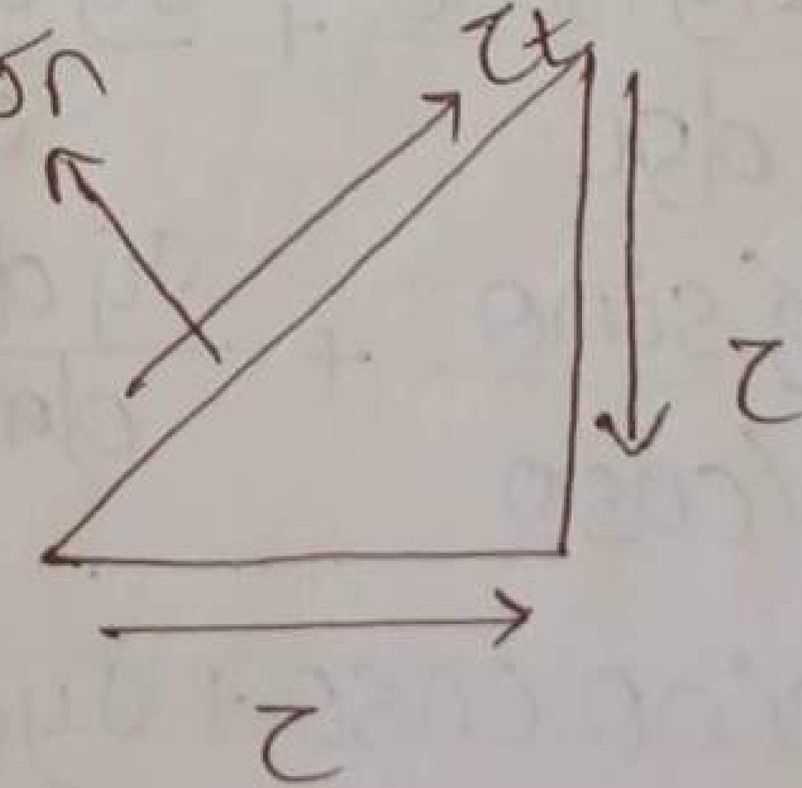
$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$$



se-III :- Pure stress condition

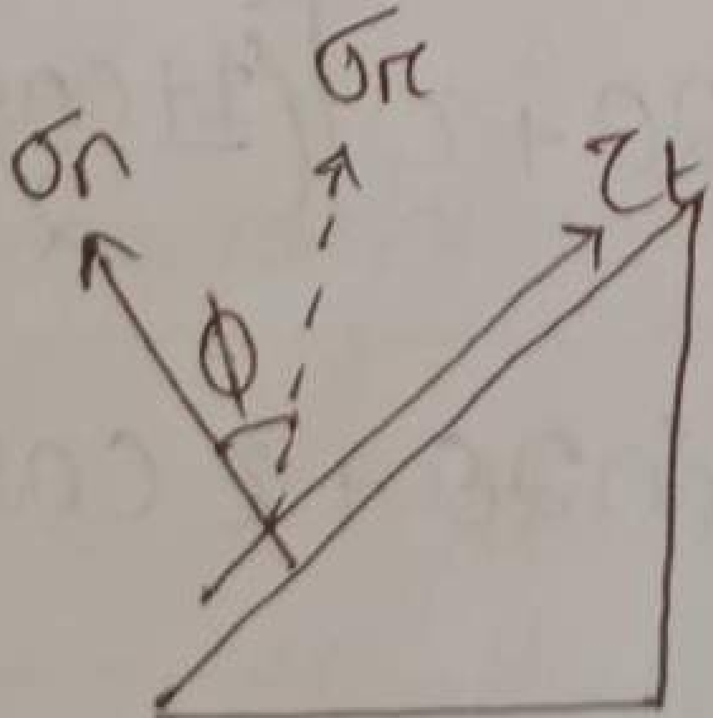
$$\sigma_n = \tau \sin 2\theta$$

$$\tau_t = \tau \cos 2\theta$$



ultant stress :-

$$= \sqrt{\sigma_n^2 + \tau_t^2}$$



e of inclination of the resultant with σ_n :

Principal stress and principal plane:

Principal planes are those plane on which shear stress (τ_t) is zero. These planes are mutually perpendicular.

Principal stresses are the maximum and minimum normal stresses.

The maximum normal stress is called major principal stress.

The minimum normal stress is called minor principal stress.

The planes on which the maximum normal stress acts, called major principal plane.

The plane on which the minimum normal stress acts, called minor principal plane.

As shear stress is zero in principal plane.

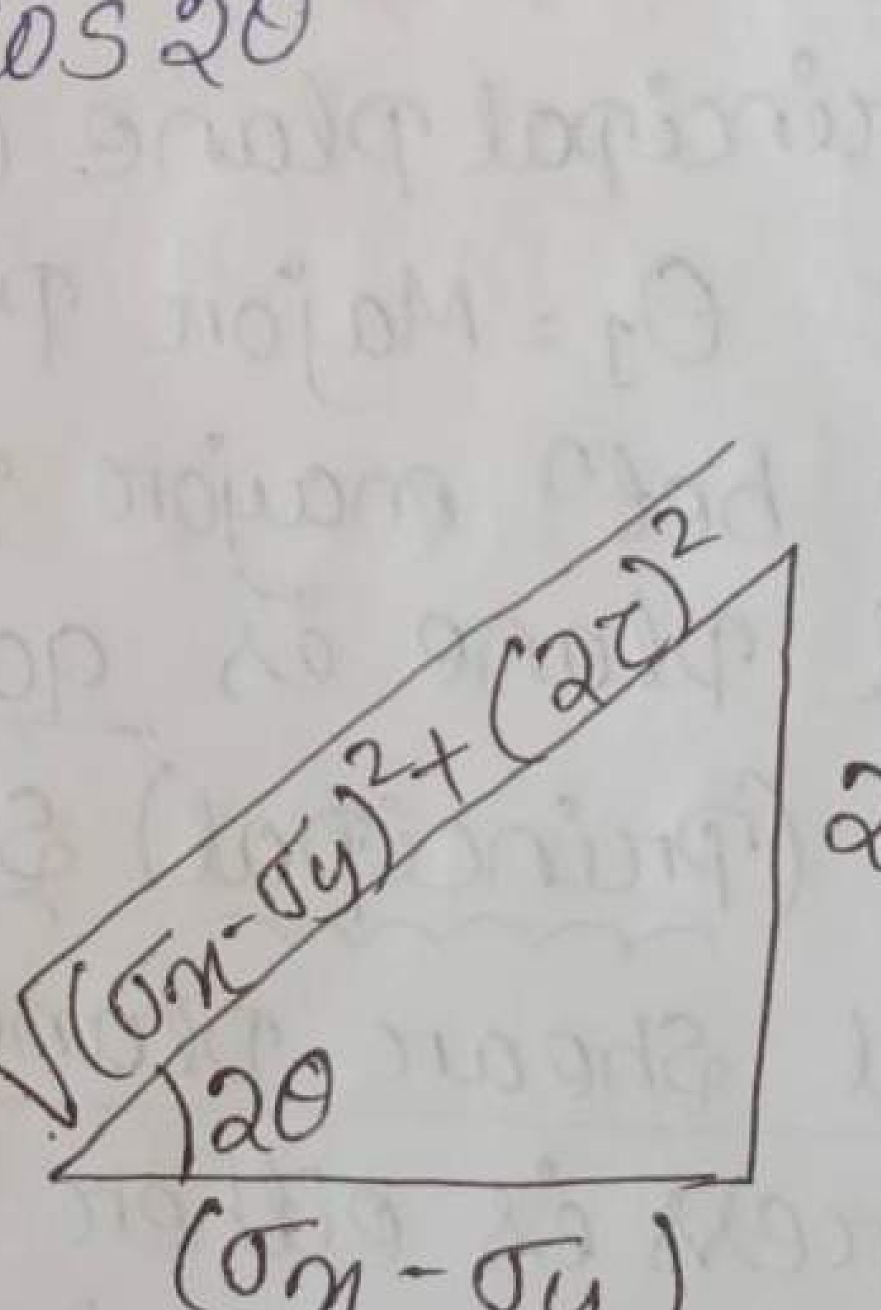
$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \tau\cos 2\theta$$

$$\theta = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta + \cos 2\theta$$

$$\tan 2\theta = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

$$\sin 2\theta = \pm\frac{2\tau}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$\cos 2\theta = \pm\frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$



Put the value of $\sin 2\theta$ and $\cos 2\theta$ in 'σ_n'

$$\sigma_{1,2} = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\theta + \tau\sin 2\theta$$

$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\frac{(\sigma_x - \sigma_y)^2 + 4\tau^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$\boxed{\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

→ major principal stress

$$\boxed{\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}}$$

→ minor principal stress

Direction of principal plane :-

$$\tan 2\theta = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

inor principal plane $(\theta_2) = (\theta_1) + 90^\circ$

θ_1 = Major principal plane.

e angle betn major principal plane and minor
incipal plane is 90°.

ximum (Principal) Shear stress:

incipal shear planes are those planes on which
ar stress is either maximum or minimum.
e planes are mutually perpendicular to each
er.

normal stress is ...

$$\tau_1 = +\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

↳ maximum shear stress

$$\tau_2 = -\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

↳ minimum shear stress.

The angle between maximum shear stress plane and minimum shear stress plane is 90°.

The angle between principal maxm shear plane and principal plane is 45°.

$$\boxed{\theta_s = \theta_p + 45^\circ}$$

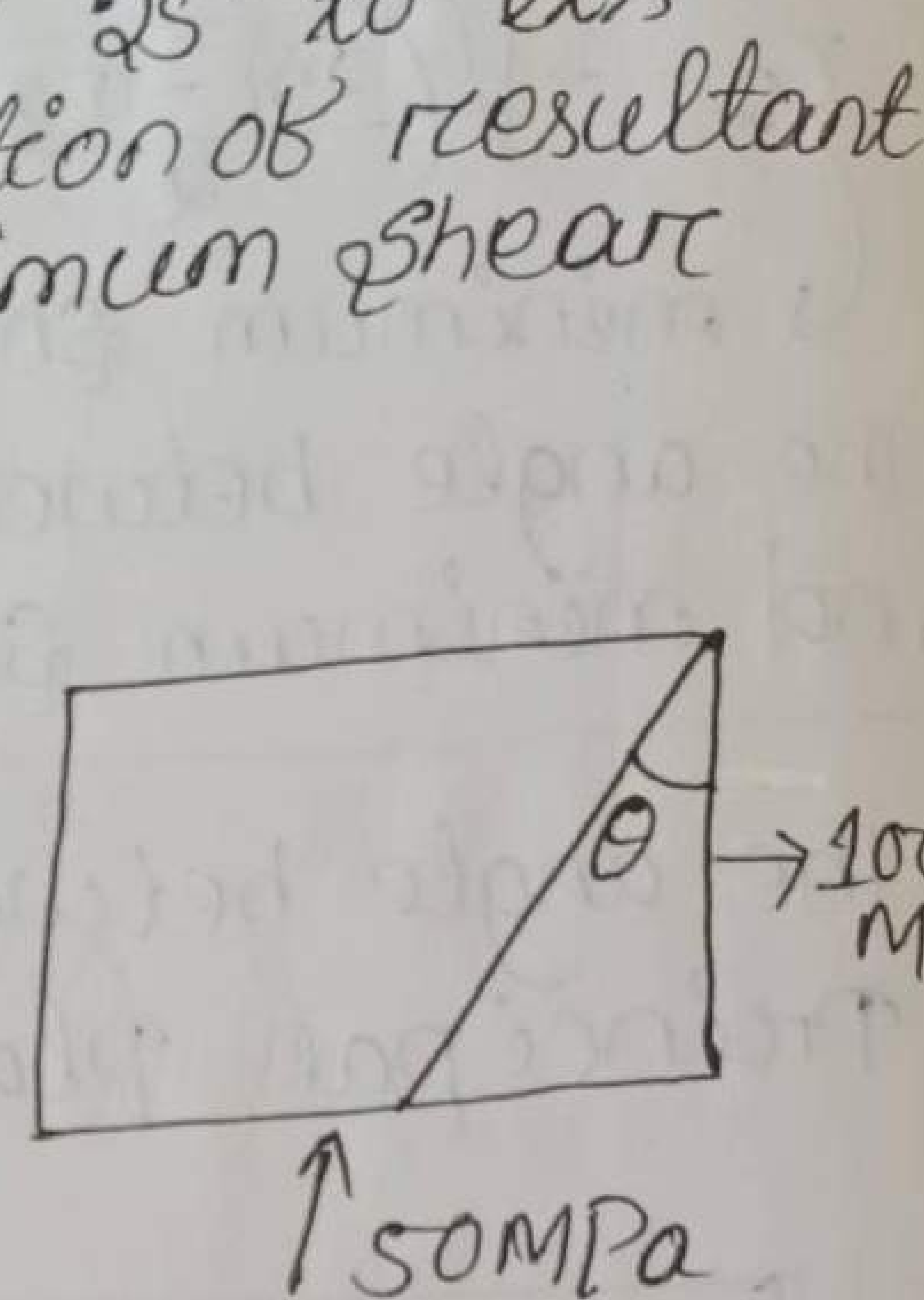
$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}}$$

↓

maximum shear stress.

Q. The stress at a point in a components are 100 MPa tensile and 50 MPa compressive. Determine the magnitude of the normal stress and shear stress on a plane inclined at an angle of 25° to its tensile, Also determine the direction of resultant stress and the magnitude of maximum shear stress.

Ans



$\sigma_x = 100\ MPa$

$\sigma_y = -50\ MPa$

$\theta = 25°$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{100 - 50}{2}\right) + \left(\frac{100 + 50}{2}\right)\cos 2 \times 25°$$

$$= 50 + 75 \cos 50°$$

$$= 73.209\ MPa$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$$

$$= -\frac{1}{2}(100 + 50)\sin 2 \times 25°$$

$$= -\frac{1}{2} \times 150 \times \sin 50°$$

$$= -57.4533\ MPa$$

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$$

$$= \sqrt{(73.209)^2 + (-57.4533)^2}$$

$$\tan\phi = \frac{\tau_t}{\sigma_n}$$

$$\tan\phi = \frac{-57.45}{73.209} = -0.7847$$

$$\phi = \tan^{-1}(-0.7847) = -38^\circ 12'$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \qquad \boxed{\tau = 0}$$

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$= \frac{\sigma_x - \sigma_y}{2}$$

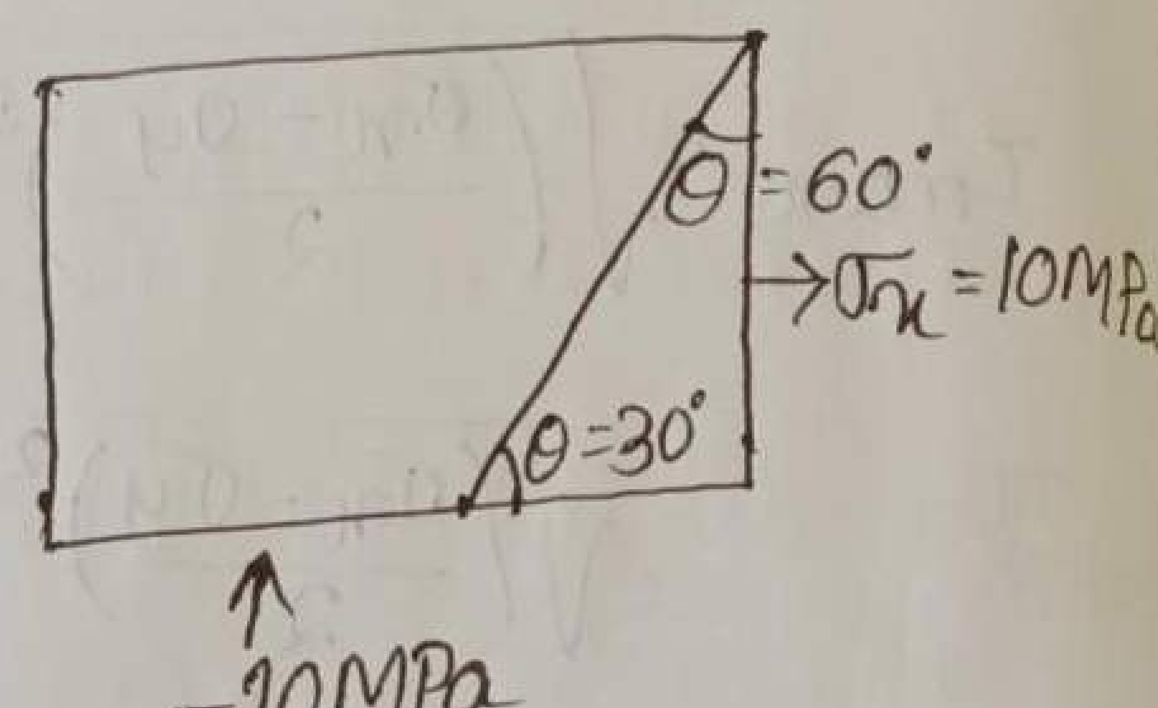
$$= \frac{100 + 50}{2}$$

$$\boxed{\tau_{max} = \pm 75 \text{ MPa}}$$

Q. A rectangular block is subjected to two Perpendicular stress of 10 MPa (tensile) and 10MPa (compressive). Determine stresses on plane inclined at 30°, 45° and 60° with the plane of compressive stress.

Ans
$\sigma_x = 10 \, MPa$

$\sigma_y = -10 \, MPa$



When $\theta = 30°$

Let's calculate 'θ' with σ_x

$$\theta = 90° - 30° = 60°$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta$$

$$= \cancel{\left(\frac{10-10}{2}\right)}^{0} + \left(\frac{10+10}{2}\right) \cos 2 \times 60°$$

$$= \frac{20}{2} \times \cos 120°$$

$$= -5 \, MPa$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y) \sin 2\theta$$

$$= -\frac{1}{2}(10+10) \sin 2 \times 60°$$

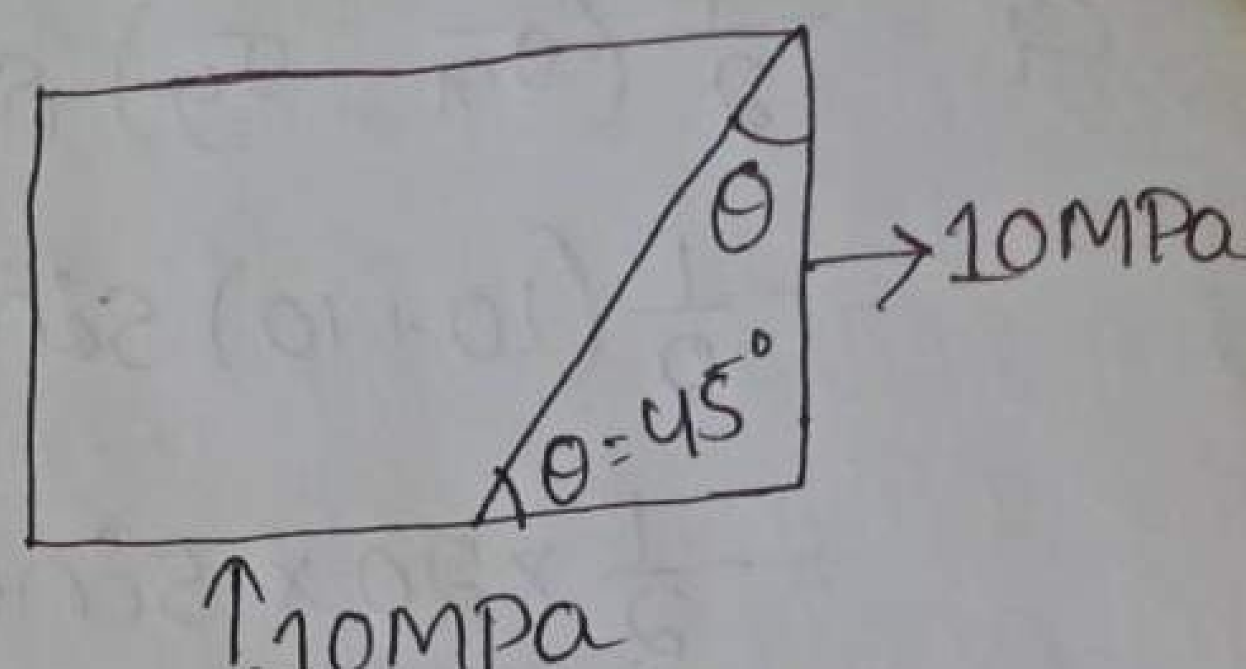
$$= -\frac{1}{2} \times 20 \times \sin 120°$$

$$= -5\sqrt{3} \, MPa$$

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$$

When $\theta = 45°$

calculate 'θ' with 'σ_x' axis



$$\theta = 90° - 45° = 45°$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{10-10}{2}\right) + \left(\frac{10+10}{2}\right)\cos 2\times 45°$$

$$= 0 + 0$$

$$= 0 \text{ MPa.}$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$$

$$= -\frac{1}{2}(10+10)\sin 2\times 45°$$

$$= -\frac{1}{2}\times 20 \times \sin 90°$$

$$= -10 \text{ MPa}$$

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_t^2}$$

$$= \sqrt{0^2 + (-10)^2}$$

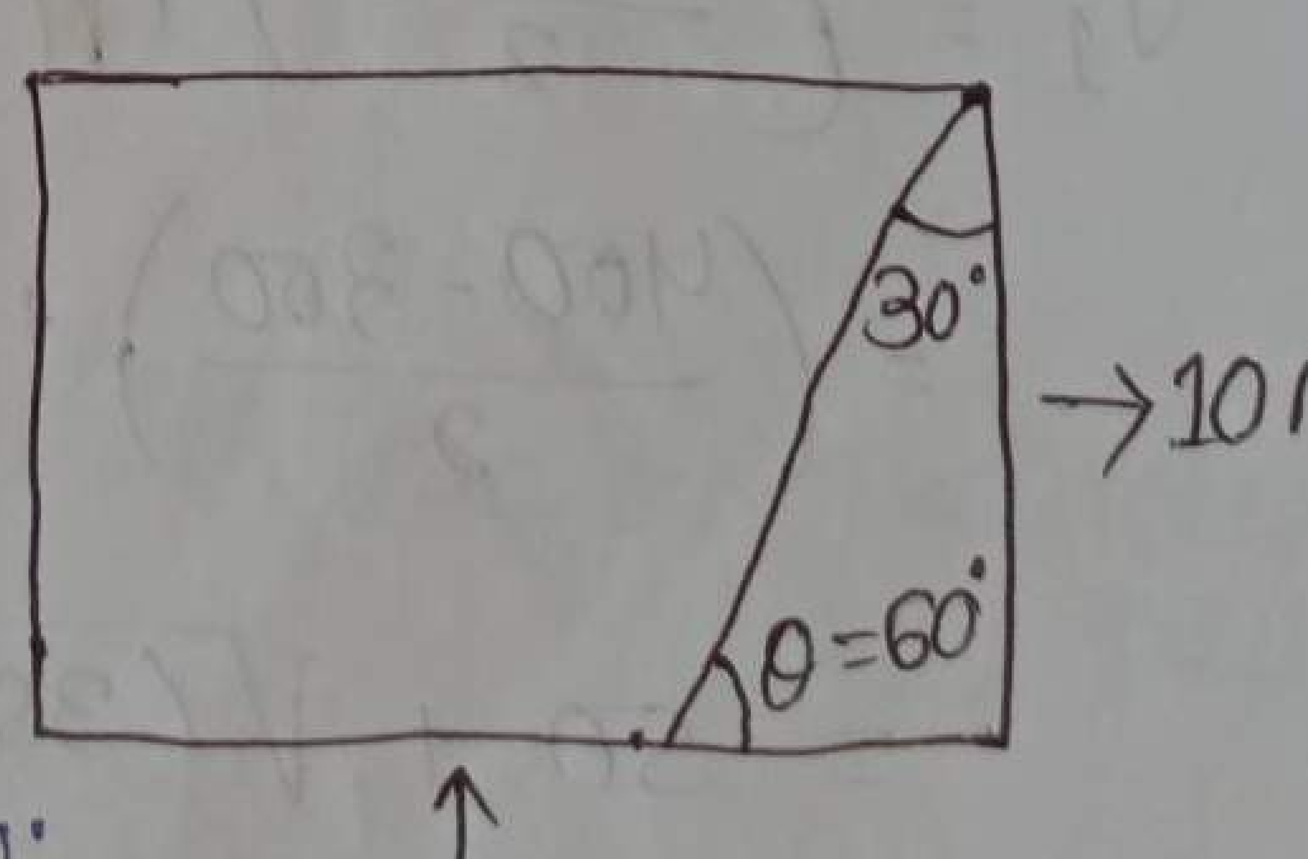
$$= 10 \text{ MPa}$$

When $\theta = 60°$

calculate 'θ' with 'σ_x' axis

$$\theta = 90° - 60°$$

$$= 30°$$



$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{10-10}{2}\right) + \left(\frac{10+10}{2}\right)\cos 2\times 30°$$

$$\tau_t = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta$$

$$= -\frac{1}{2}(10+10)\sin 2\times 30^\circ$$

$$= -\frac{1}{2}\times 20 \times \sin 60^\circ$$

$$= -5\sqrt{3}\ \text{MPa}$$

$$\sigma_R = \sqrt{(\sigma_n)^2 + (\tau_t)^2}$$

$$= \sqrt{(5)^2 + (-5\sqrt{3})^2}$$

$$= 10\ \text{MPa}$$

n an experiment it was found that the tensile
tress of 400 N/mm² and a compressive stress
f 300 N/mm² acting on mutually plane and
qual shear stress of 100 N/mm² on this plane.
nd the principal stress and the position of
rincipal plane. Find also maximum shear stress.

$\sigma_x = 400\ \text{N/mm}^2$ (Tensile)

$\sigma_y = -300\ \text{N/mm}^2$ (Compressive)

$\tau = 100\ \text{N/mm}^2$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \left(\frac{400-300}{2}\right) + \sqrt{\left(\frac{400+300}{2}\right)^2 + (100)^2}$$

$$= 50 + \sqrt{(350)^2 + (100)^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \frac{400-300}{2} - \sqrt{\left(\frac{400+300}{2}\right)^2 + (100)^2}$$

$$= 50 - \sqrt{(350)^2 + (100)^2}$$

$$= 50 - 364.005$$

$$= -314.005 \text{ N/mm}^2$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{400+300}{2}\right)^2 + (100)^2}$$

$$= 364.005 \text{ N/mm}^2$$

he Position of principal plane :-

$$\tan 2\theta_1 = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

$$= \frac{2 \times 100}{400+300}$$

$$= 0.285$$

$$2\theta_1 = \tan^{-1}(0.285)$$

$$= 15.907^\circ$$

$$\theta_1 = \frac{15.907^\circ}{2}$$

$= 7.953^\circ \rightarrow$ Angle with major principal plane.

$$\theta_2 = 90^\circ + \theta_1$$

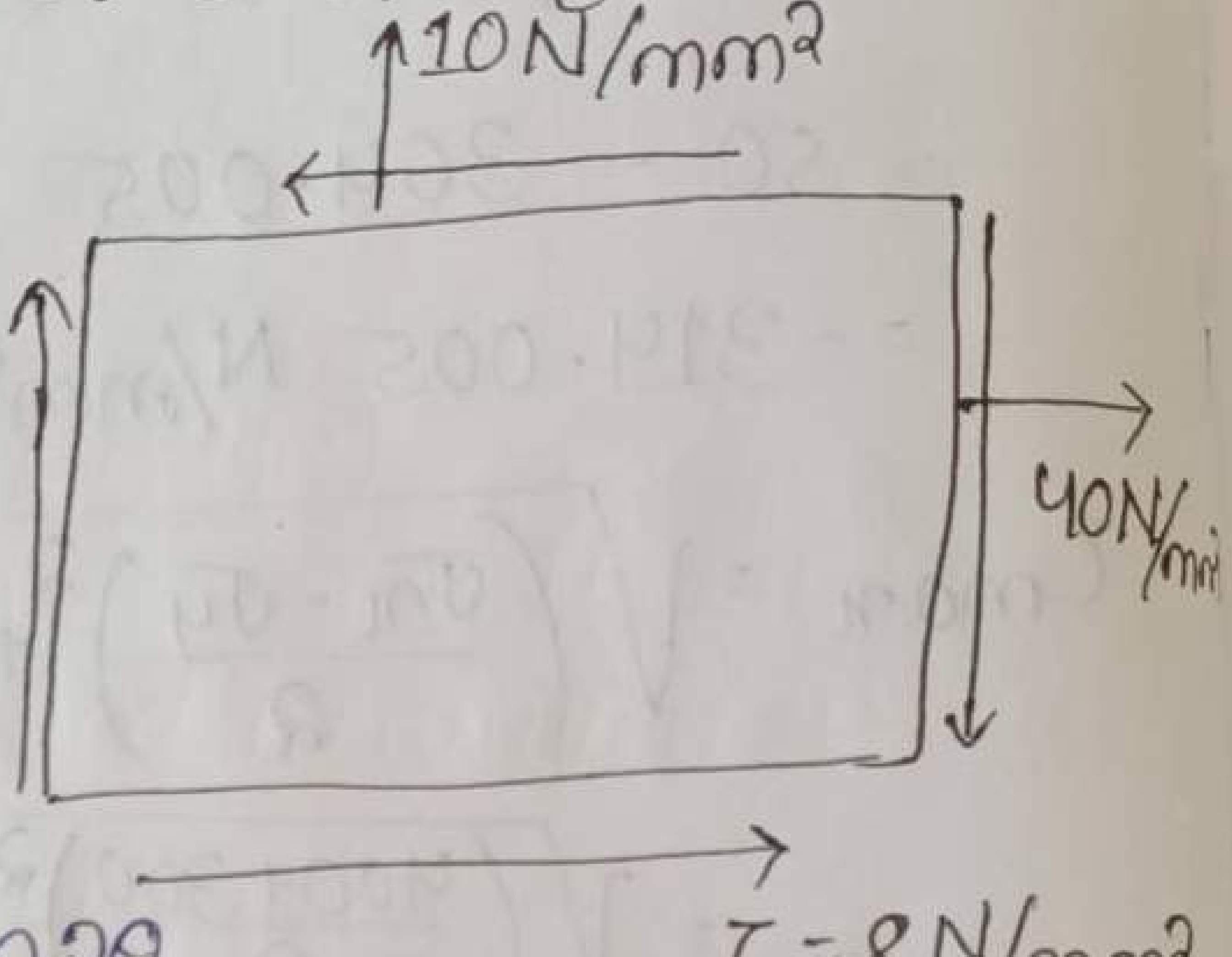
$$= 90^\circ + 7.953^\circ$$

At a point in a rectangular block the stress on [mu]tually perpendicular plane are 40 N/mm² (tensile) [an]d 10 N/mm² (tensile). The shear stress across [th]e plane is 8 N/mm². Find the magnitude and [dir]ection of resultant stress on a plane making [an] angle 30° with the plane of first stress.

$\sigma_x = 40\ N/mm^2$

$\sigma_y = 10\ N/mm^2$

$\tau = 8\ N/mm^2$



$$\left(\frac{\sigma_x+\sigma_y}{2}\right)+\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta+\tau\sin 2\theta$$

$$\left(\frac{40+10}{2}\right)+\left(\frac{40-10}{2}\right)\cos 2\times 30^\circ+8\sin 2\times 30^\circ$$

$$25+15\times\cos 60^\circ+8\sin 60^\circ$$

$$= 39.428\ N/mm^2$$

$$-\frac{1}{2}(\sigma_x-\sigma_y)\sin 2\theta+\tau\cos 2\theta$$

$$= -\frac{1}{2}(40-10)\sin 2\times 30+8\cos 2\times 30^\circ$$

$$= -\frac{1}{2}\times 30\times\sin 60^\circ+8\cos 60^\circ$$

$$= -8.990\ N/mm^2$$

$$\sqrt{\sigma_n^2+\tau_t^2}$$

$$= \sqrt{(39.428)^2+(-8.990)^2}$$

$$= 40.439\ N/mm^2$$

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(40-10)^2 + 4\times 8^2}$$

$$= 17 \text{ MPa}$$

$$\tan\phi = \frac{\tau_t}{\sigma_n}$$

$$= \frac{-8.990}{39.428}$$

$$= -0.228$$

$$\phi = \tan^{-1}(-0.228)$$

$$= -12^\circ . 84'$$

Note:

clockwise ⇒ +ve

Anti-clockwise direction ⇒ -ve

Tensile ⇒ +ve

compressive ⇒ -ve

Types of loads and beam:

Chapter-5

Types of load on a beam are :-

1. Point load or concentrated load →
It is assumed to act at a point

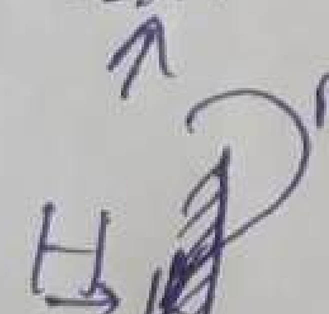
. uniformly distributed load (U.D.L.) :-
- It is distributed uniformly over some length. The intensity of loads is constant and measured as load per unit length.

v.g.

ypes of Support :

1. Roller  ⟶ Reaction → (1)

2. Hinge =  ⟶ Reaction → (2)

. Fixed →  ⟶ Reaction → (3)

Roller Support :-
hen beam rests on a sliding surface such as roller r any flat surface like masonary wall, the upport is known as roller support.
This support offers normal reaction or vertical eaction.

Hinge Support :-
n a Hinge Support no translational displacement of

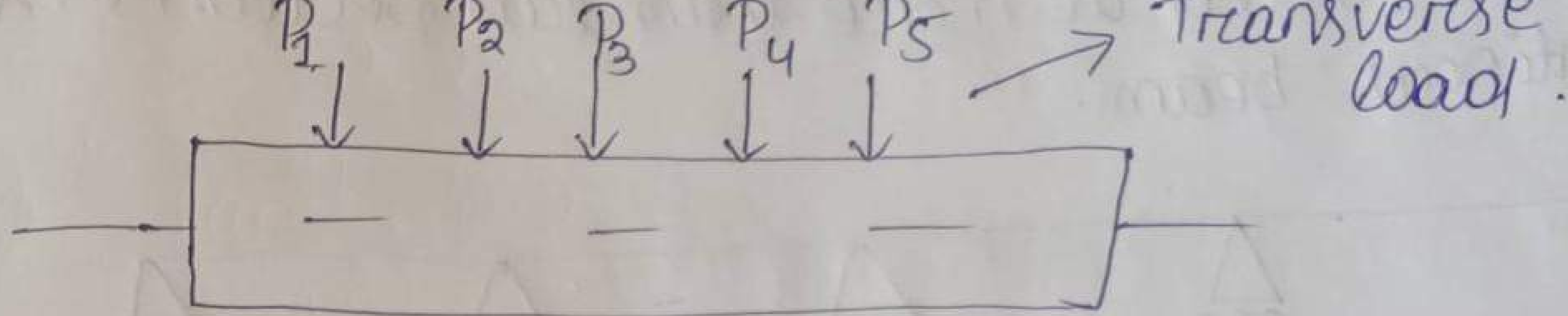
3. Fixed Support:

This is also called Built in or clamped Support whi
does not allow any type of movement or rotation. It
offers 3 reaction (Horizontal, vertical and moment
reaction)

Types of Beam:

Definition of Beam :-

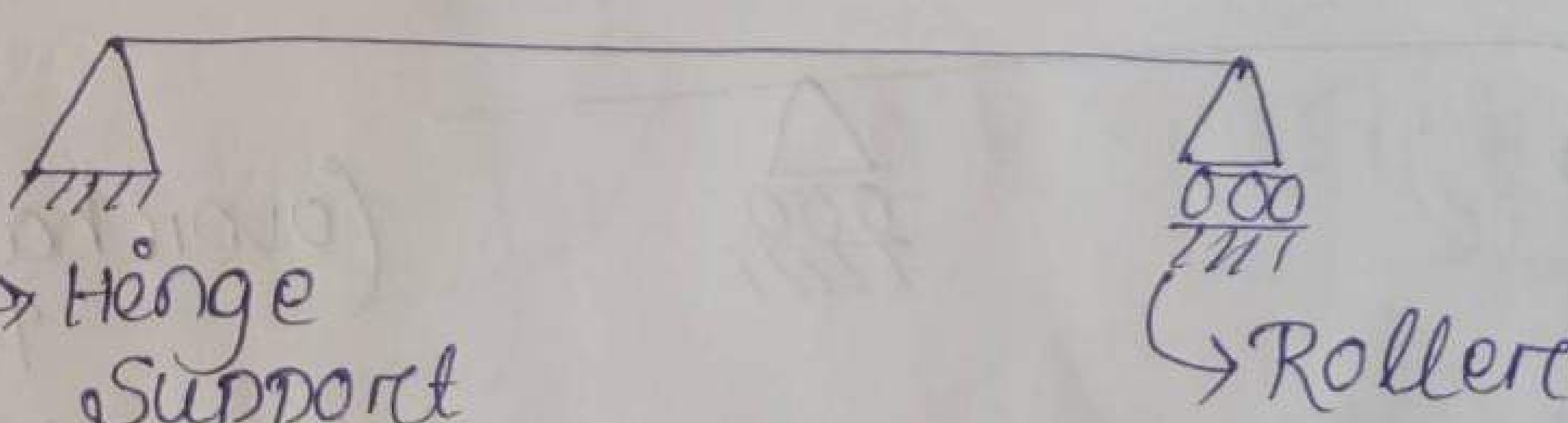
It is a structural element which is subjected to load transverse to its axis is known as beam.



Types of beam:

1. Simply Supported beam:- (S.S.B)

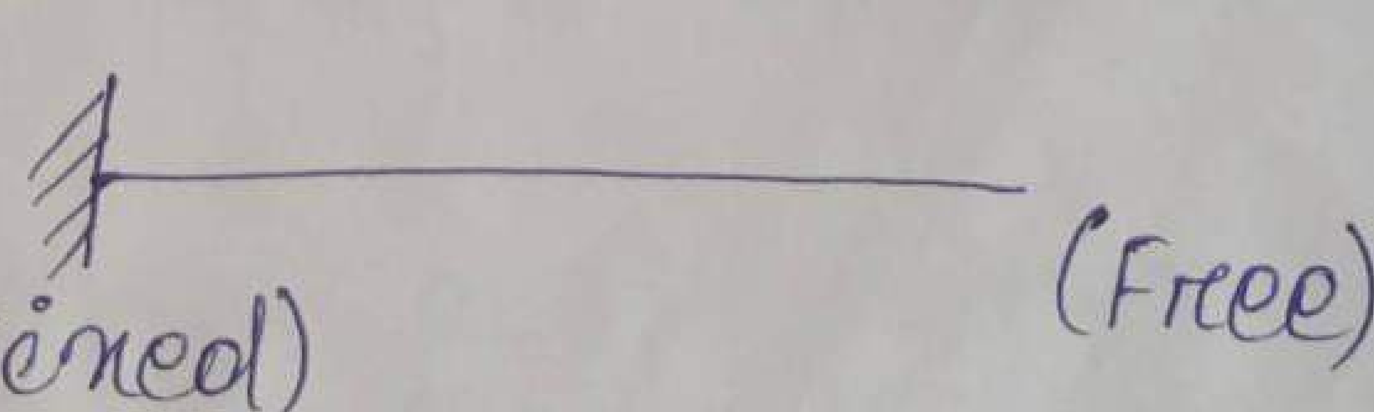
A beam having its both ends freely resting on supports is known as simply supported beam.



2. cantilever beam:-

A beam with one end fixed and other free is cal
cantilever beam.

* Free end d
no reac

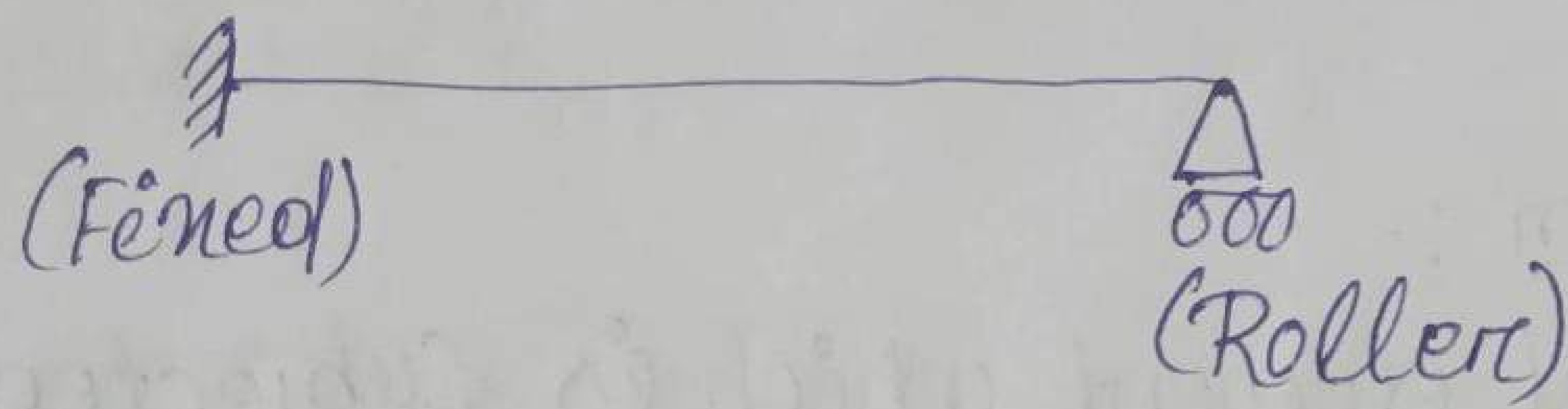


3. Fixed beam:

When both ends are fixed is called fixed beam

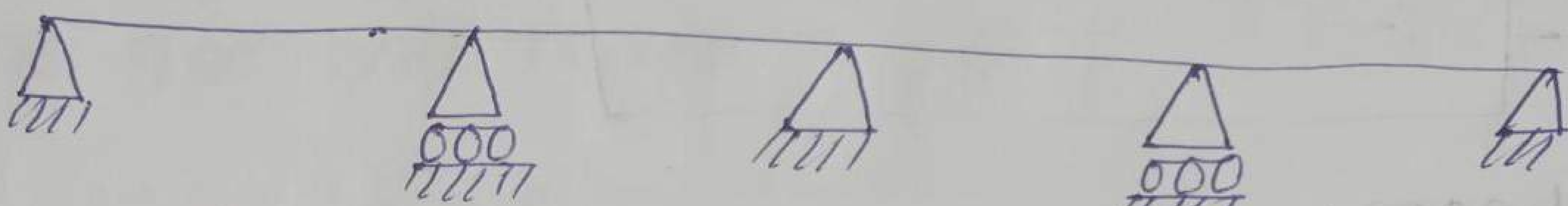
Propped cantilever :-

ams with one end fixed and the other simply
pported (roller, hinge) are known as propped
ntilever.



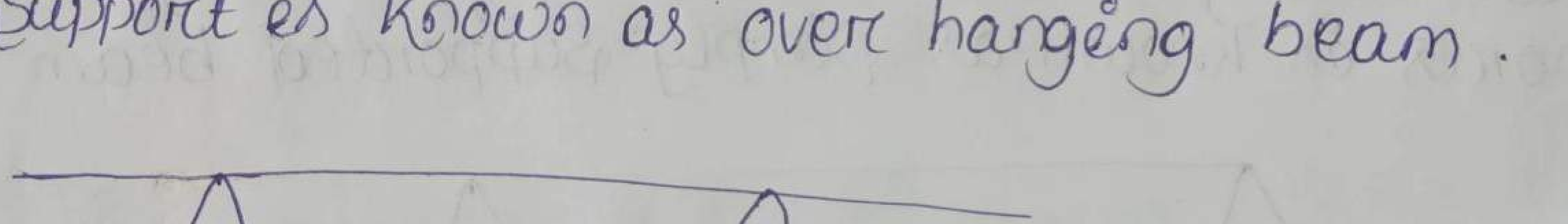
continous Beam :-

eams supported at more than two section is known
s continous beam.



over hanging beam :-

beam having its end portion extended beyond
e support is known as over hanging beam.



Sheare Force and Bending Moment

ear Force:-

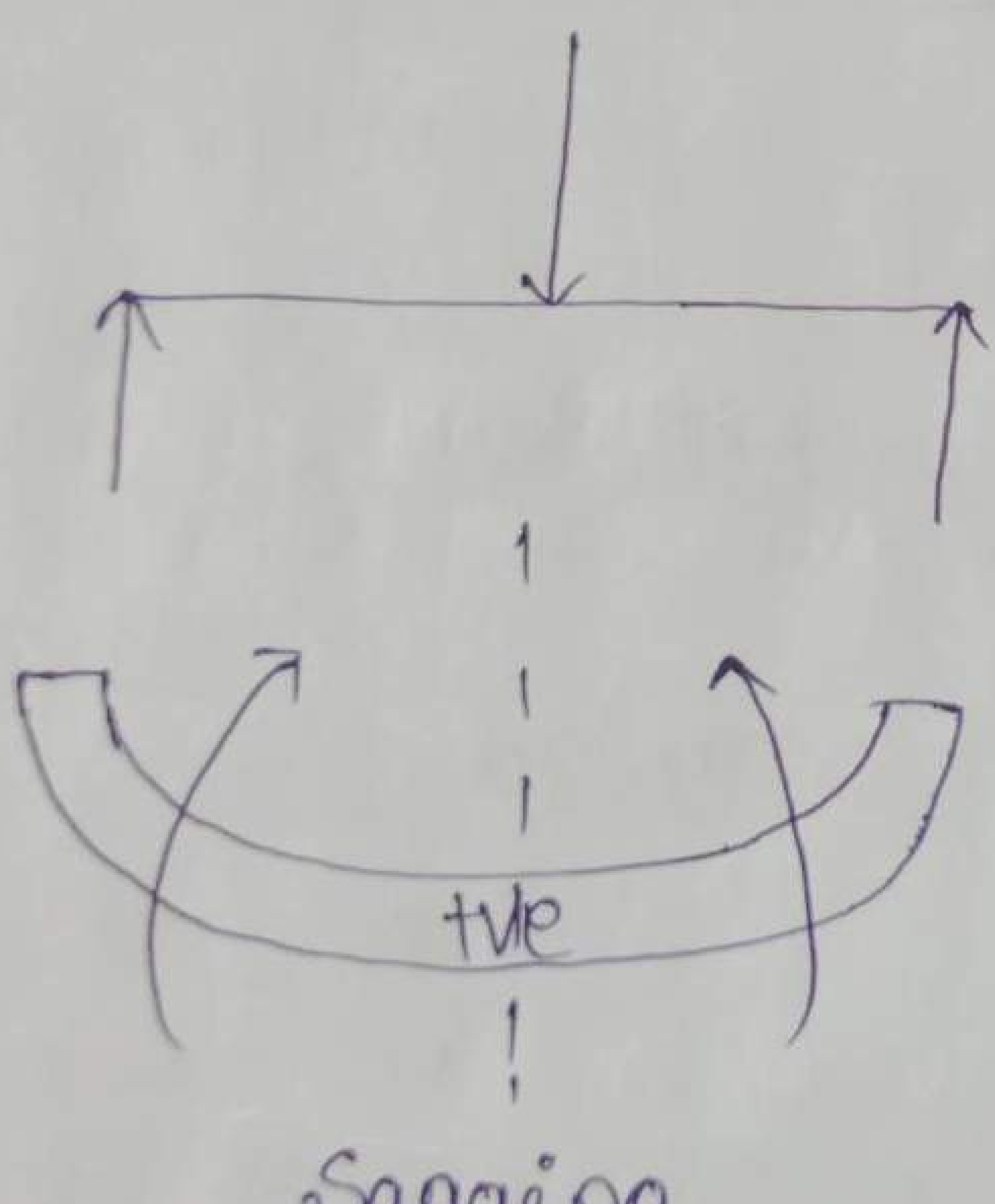
e Sheare force at the cross-section of a beam may be efined as the algebraic sum of unbalanced vertical orces to right or left of the section.

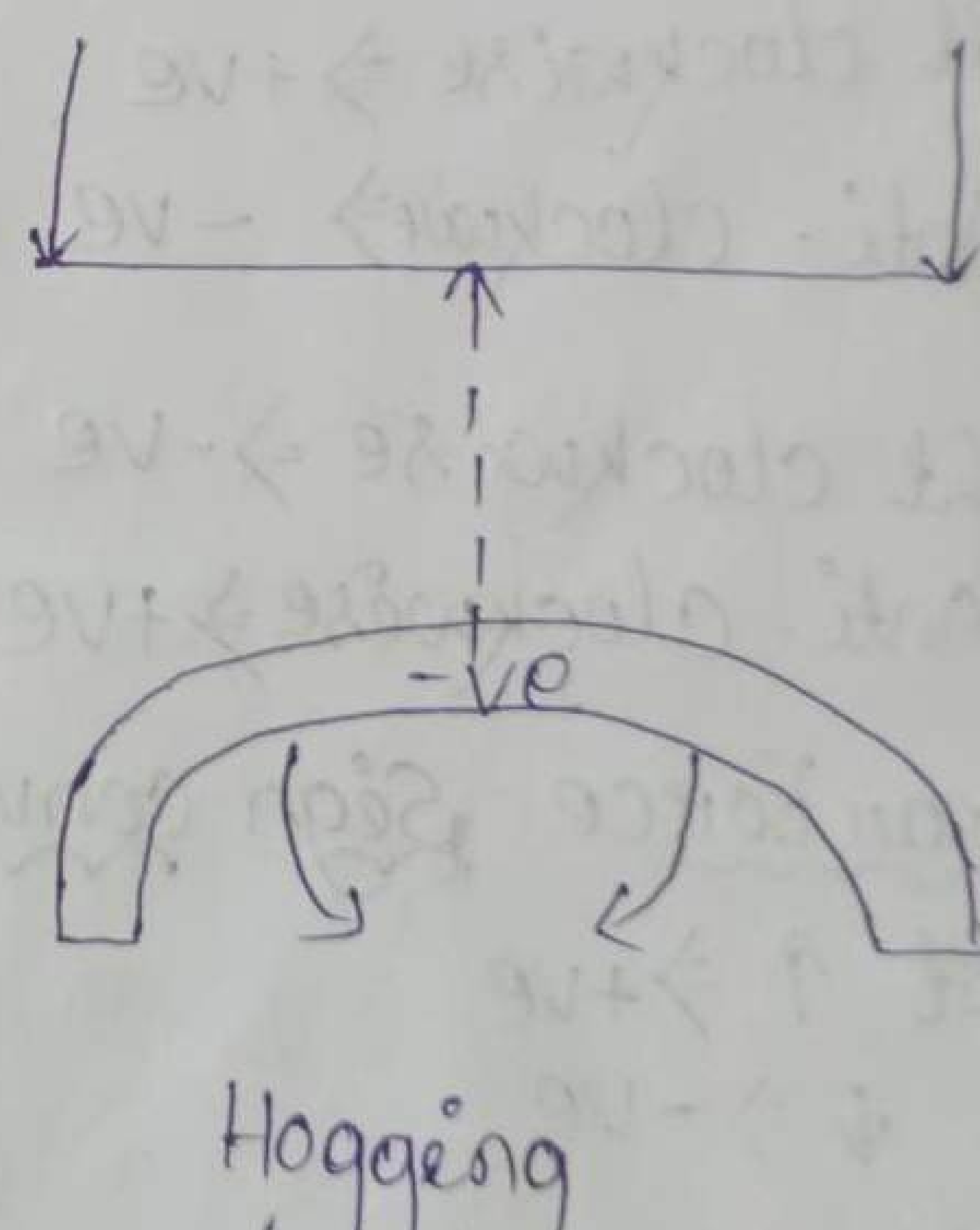
Sign convention	
* Right ↑ ⇒ -ve	* Left ↑ ⇒ +ve
↓ ⇒ +ve	↓ ⇒ -ve

ending moment:-

The bending moment at the cross-section of a beam may be defined as the algebraic sum of the moment f the vertical force to the left or right of the ection.

concept of Hogging and Sagging





Relation between the load, Shearforce and Bending moment.

) The ratio of change of S.F (Sheare Force) at any Section represents the ratio of loading at the Section he rate of change of Bending moment at any section epresents the Shear force at the section.

$$S.F = \frac{d(BM)}{dx}$$

So, $$\boxed{W = \frac{-d(S.F)}{dx} = \frac{-d^2(B.M)}{dx^2}}$$

At a point where $\frac{d(B.M)}{dx} = 0$ or shear force is zero, the Bending moment will have maximum value. That point also the position where shear force changes sign.

nding moment sign convention:

ft clockwise ⇒ +ve

Anti-clockwise ⇒ -ve

ht clockwise ⇒ -ve

Anti-clockwise ⇒ +ve

ear force sign convention:-

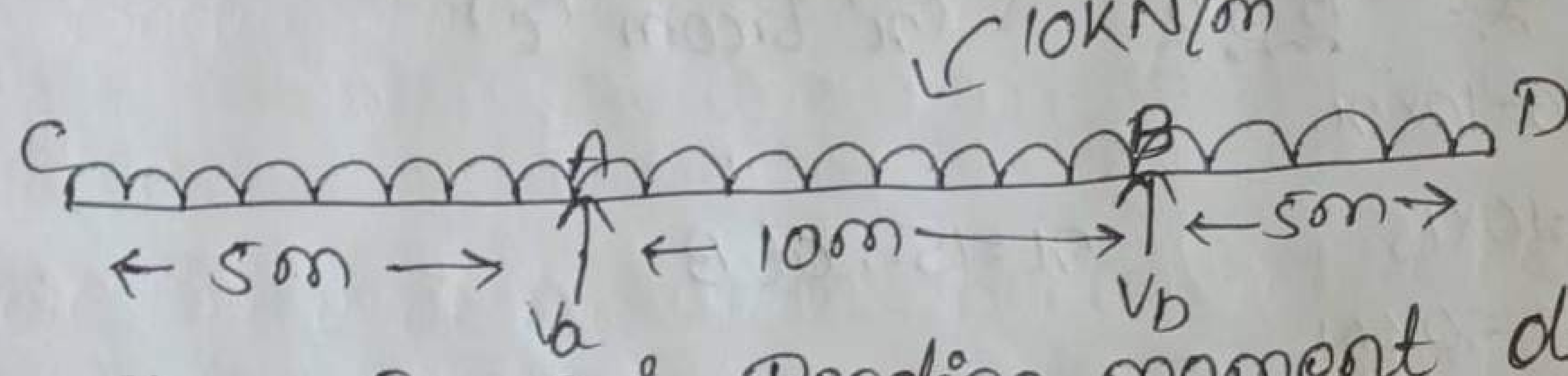
ft ↑ ⇒ +ve

↓ ⇒ -ve

ht ↑ ⇒ -ve

↓ ⇒ +ve

Q:



Draw Shear Force & Bending moment diagram?

S) $\Sigma V = 0$

$\Rightarrow V_a + V_b = 10\times10 + 10\times(5+5)$

$= 200 KN$

$\Sigma M_A = 0$

$\Rightarrow -V_b \times 10 + \cancel{10}^{5} \times 15 \times \frac{15}{\cancel{2}} - \cancel{10}^{5} \times 5 \times \frac{5}{\cancel{2}} = 0$

$\Rightarrow V_b = \frac{1000}{10}$

$\Rightarrow V_b = 100 KN$

$V_a = 200 - V_b$

$= 200 - 100 = 100 KN$

Shear Force

Take section between C and A

'x' from 'C'

$(S.F)_{xx} = -10x$

$(S.F)_C = -10\times0 = 0KN$, $x=0$ at C

$(S.F)_A = -10\times5 = -50KN$ $x=5$ at A

Take section betn A & B 'x' from 'C'

$(S.F)_{xx} = 100 - 10x$

$(S.F)_A = 100 - 10\times5$ $x=5$, at A

$= 100 - 50 = 50KN$

Shear Force betn B & D ('x' from 'c')

$S.F)_{xx} = 100+100-10\times x$

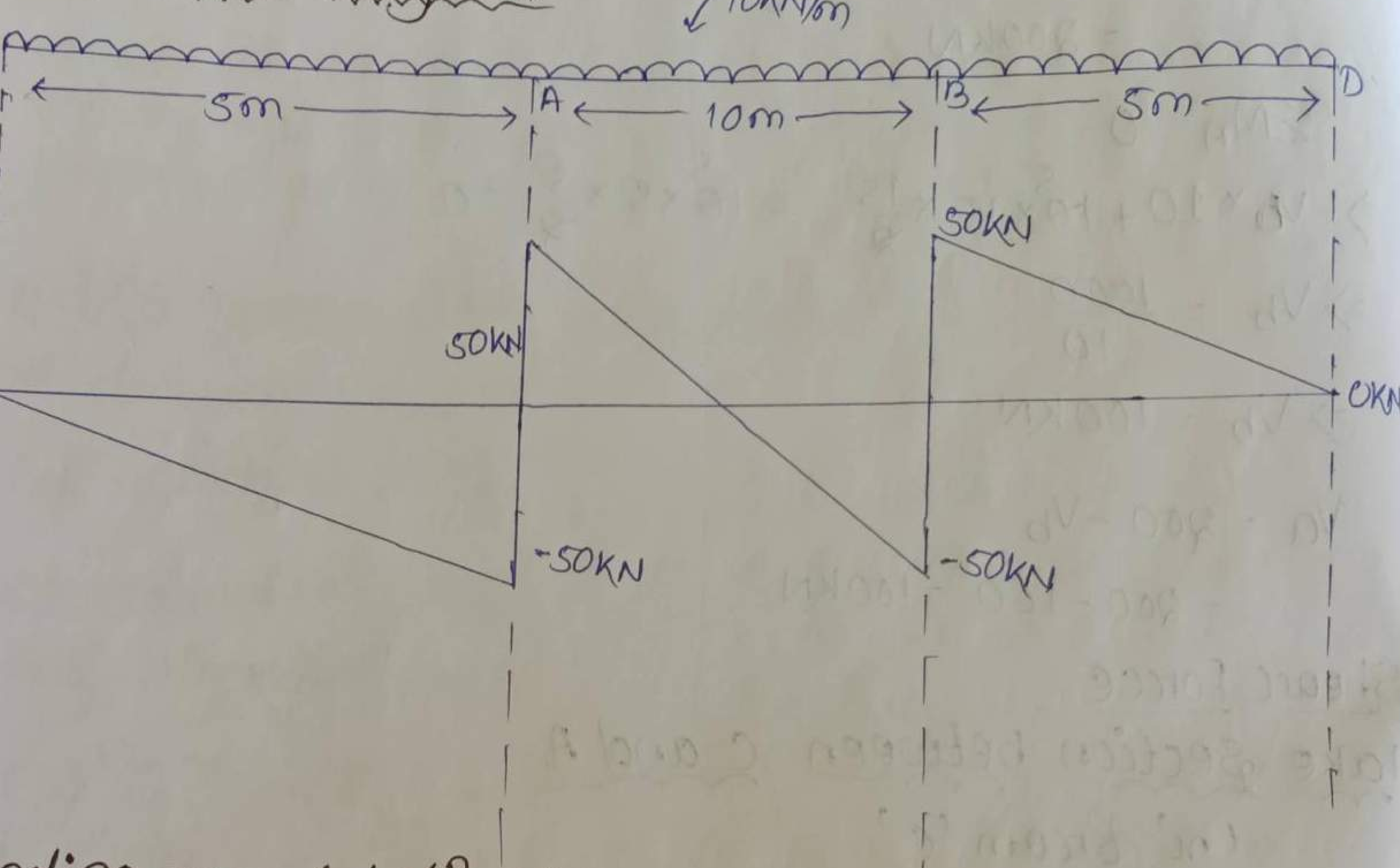
$S.F.)_B = 100+100-10\times 15$ $x=15$, at B

$= 200-150 = 50KN$

$S.F)_D = 200-10\times 20$

$= 200-200 = 0KN$ $x = 20$ at D

Shear Force diagram



Bending moment betn C & A ('x' from 'c')

$B.M)_{xx} = -10\times x\times \frac{x}{2}$

$B.M)_C = -10\times 0\times 0/2 = 0KNm$ $x=0$, at C

$B.M)_A = -\cancel{10}^{5}\times 5\times 5/\cancel{2} = -125KNm$ $x=5$, at A

Bending moment betn A & B ('x' from 'c')

$B.M)_{xx} = -10\times \frac{x^2}{2} + 100(x-5)$

$B.M)_{xx} = -5x^2+100(x-5)$

$B.M)_A = -5\times(5)^2+100(5-5)$ $x=5$, at 'A'

So, we have to calculate Bending moment at 'm'.

$(B.M)_m = -5 \times (10)^2 + 100(10-5)$

$= -500 + 500 = 0$ KN-m $x = 10$, at 'M'

$(B.M)_B = -5 \times (15)^2 + 100(15-5)$

$= -125$ KN-m $x = 15$, at B

Take section betⁿ B & D: 'x' from 'C'

$(B.M)_{xx} = -10 \times \frac{x^2}{2} + 100(x-5) + 100(x-15)$

$= -5x^2 + 100(x-5) + 100(x-15)$

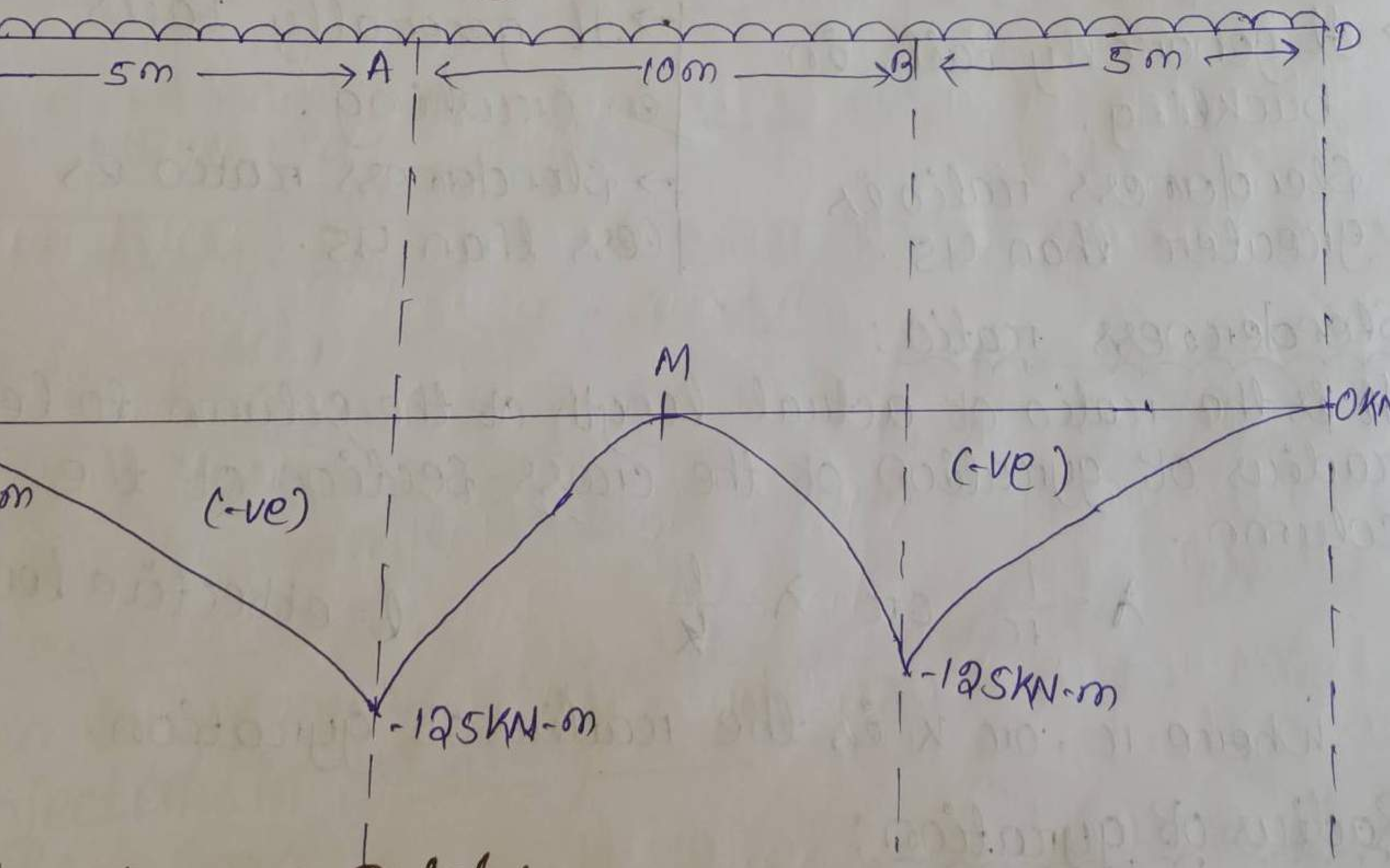
$(B.M)_B = -5 \times (15)^2 + 100(15-5) + 100(15-15)$ $x = 15$, at B

$= -125$ KN-m

$(B.M)_D = -5 \times (20)^2 + 100(20-5) + 100(20-15)$ $x = 20$, at D

$= 0$ KN-m

Bending moment diagram



Contraflexure Point:

Point at which the Bending moment is zero is called

lumn and Struts

olumn is a vertical member, it sustain compressive oad.

trut is a vertical, horizontal and inclined member, it also sustain compressive load.

Long Column	Short Column
e column whose lateral mension is small when mpared to its length.	→ The column whose lateral dimension is very large when compared to its length.
tio of effective length least lateral mension is greater an 12. $\frac{l}{D \text{ or } b} \geq 12$	→ Ratio of effective length to least lateral dimension is less than 12. $\frac{l}{D \text{ or } b} < 12$
t generally fails in uckling.	→ It generally fails in crushing.
enderness ratio is eater than 45.	→ Slenderness ratio is less than 45.

nderness ratio:

s the ratio of actual length of the column to least ius of gyration of the cross-section of the mn.

$$\lambda = \frac{l}{r} \quad \text{or} \quad \lambda = \frac{l}{k} \qquad l = \text{effective length}$$

here r, or k is the radius of gyration.

us of gyration:

s defined as the distance between the reference s to the centre of gravity. $r = \sqrt{\frac{I}{A}}$

valent length or effective length:-

the
ar
th

the same material and cross-section in which both end hinged and having the value of crippling load equal to that of the given column.

Sl. NO.	Endcondition	Relation of effective and actual length	crippling load
1	Both Hinged	$L_e = l$	$P_E = \frac{\pi^2 EI}{l_e^2}$ $P_E = \frac{\pi^2 EI}{l^2}$
2.	Both end Fixed	$L_e = l/2$	$P_E = \frac{\pi^2 EI}{(l/2)^2}$ $= \frac{4\pi^2 EI}{l^2}$
3.	one end fixed other end free	$L_e = 2l$	$P_E = \frac{\pi^2 EI}{(2l)^2}$ $P_E = \frac{\pi^2 EI}{4l^2}$
4.	one end fixed other end hinged.	$L_e = l/\sqrt{2}$	$P_E = \frac{\pi^2 EI}{(l/\sqrt{2})^2}$ $P_E = \frac{2\pi^2 EI}{l^2}$

E = Young's modulus , I = moment of Inertia

at is axially loaded column?
olumn subjected to load acting along the longitudinal xis or centroid of the column section. when short olumn is axially loaded, it will be subjected crushing load.
hen long column is axially loaded, it will be ubjected to buckling load.

ushing load:
en the load is gradually increased, the short umn will reach a stage at which it is subjecte ultimate crushing stress, beyond the stress

Buckling load:-
When a long column is subjected to a compressive stress, If the load is gradually increased the column will reach a stage when it will start buckle. The load at which the column just buckle is called buckling load.

Assumption in Euler's theory:-
Initially the column is perfectly straight and the load applied is truly axial.
The cross-section of the column is uniform throughout its length.
The column is perfectly elastic homogenous and isotropic and obey's hooke's law.
The length of the column is very large as compare to its cross-section.
The shortening of column due to direct compressive is neglected.
The failure of column occurs due to buckling alone.

$$P_E = \frac{\pi^2 EI}{(l_e)^2}$$

Isotropic - properties are equal in every direction.
Orthotropic - mutually perpendicular & diff^n direction
Anistropic - properties are different in all direction.
Homogenous - uniform mass

Moment of Inertia

$$I = \frac{\pi d^4}{64} \rightarrow \text{circule}$$

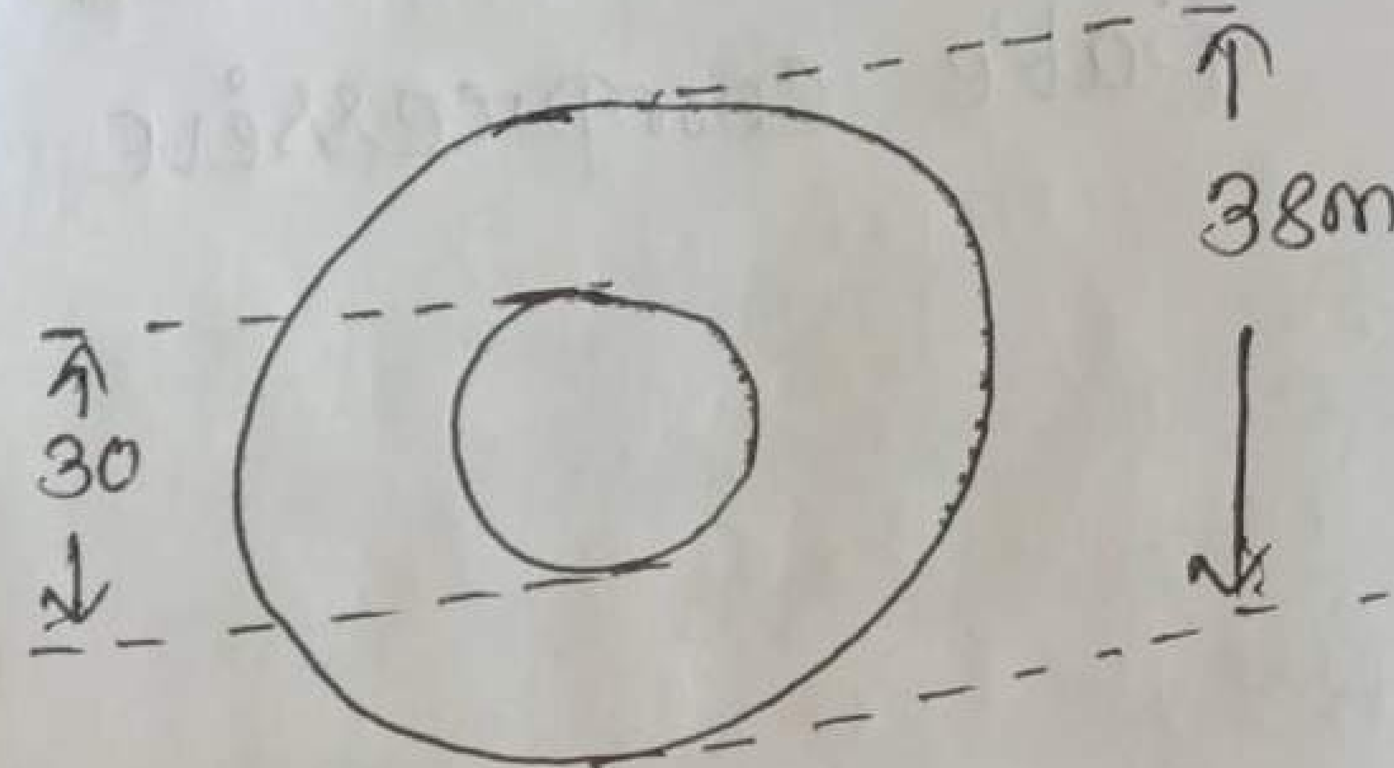
$$I = \frac{\pi (D^4 - d^4)}{64} \rightarrow \text{Holo}$$

Q.7 A
di
bo
T
Ans

Q.1 A mild steel tube 4 meters long 30mm internal diameter and 4mm thick is used as a strut with both end hinged. Find the collasping load.

Take $E = 2.1 \times 10^5 \ N/mm^2$

Ans



$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(38^4 - 30^4)$$

$$= 62561.36 \ mm^4$$

Since both ends of the column are hinged

Effective length $= L = 4m = 4000 \ mm$

$$P_E = \frac{\pi^2 EI}{l^2}$$

$$= \frac{\pi^2 \times 2.1 \times 10^5 \times 62561.36}{(4000)^2}$$

$$= 8095.89 \ N = 8.095 \ KN$$

A solid round bar 60mm in diameter and .5m long is used as a strut. one end of the trut is fixed while its other end is hinged. ind the safe compressive load for the strut ing Euler's Formula. Take $E = 200 \ GN/m^2$ and ke Factor of safety = 3.0

Diameter of solid round bar.

$D = 60mm = 0.006 \ m$

odulus of Elasticity, $E = 200 \ GN/m^2$

$F.O.S = 3$

er's crippling load is given by :-

$$= \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 200 \times 10^9 \times \frac{\pi}{64} \times (60)^4}{(1.768)^2} \times 10^{-3} k$$

$$= 401.124 kN$$

Safe compressive load $= \frac{P}{F.O.S.}$

$$= \frac{401.124}{3}$$

$$= 133.708 kN$$

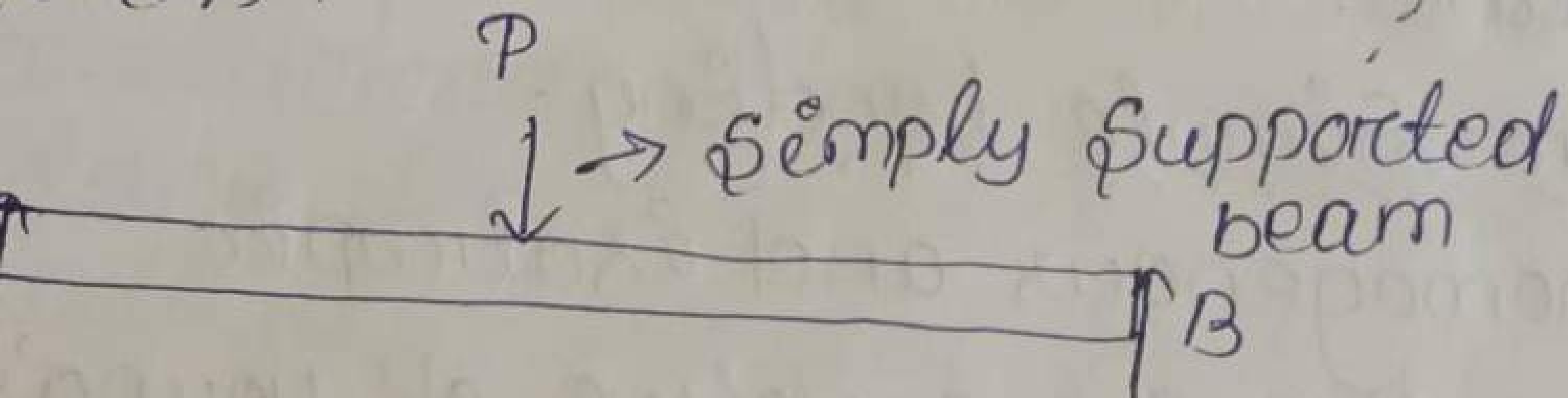
Bending Stress

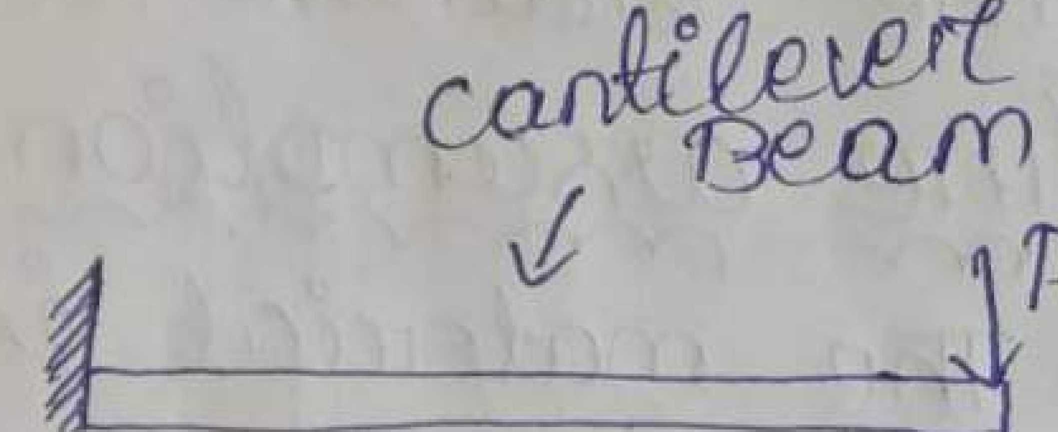
Purce bending:-

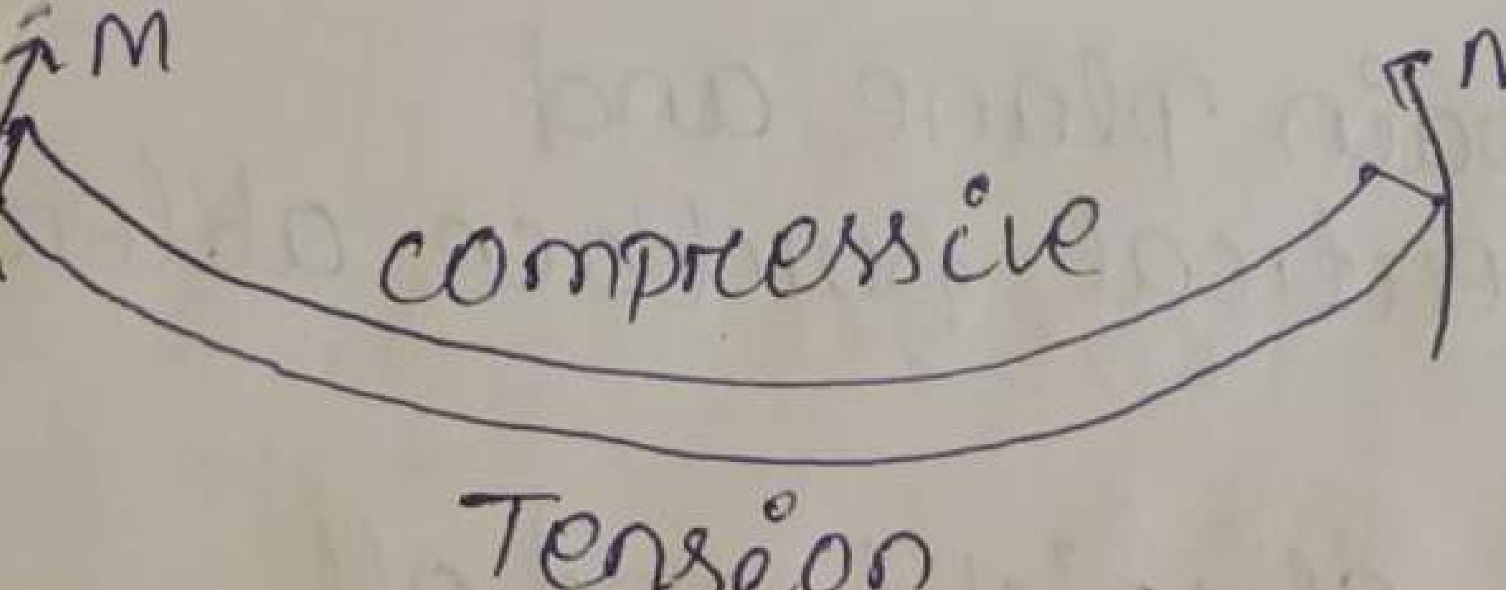
If a member is subjected to equal and opposite couple acting in the longitudinal plane, then the member is said to be purce bending.

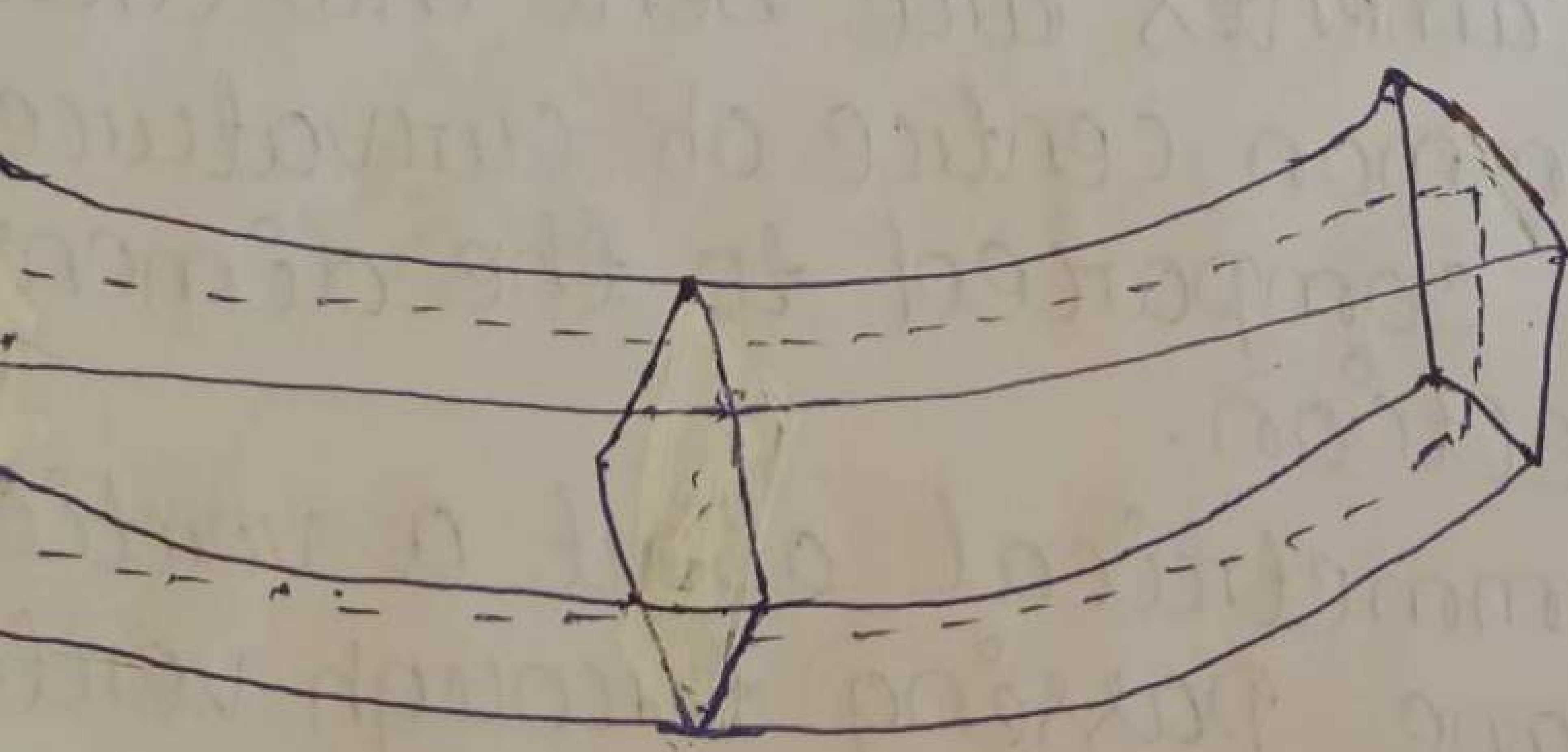
Bending Stress:-

If a constant bending moment no sheare force acts on some length of a beam, the stresses set up on any cross-section on that part of the beam constitute a purce couple, the magnitude of which is equal to the bending moment. The interenal stress developed in the beam are known as Bending stress.









Neutral layer :-
It is the layer in the beam in which longitudinal fiber do not change in length. In this layer, stress and strain is zero.
The line of intersection of neutral layer with the cross-section of the beam is known as neutral axis.

The theory of simple Bending :-
The following theory is applicable to the beam Subjected to simple or pure bending when the cross-section is not subjected to a shear force. Since that will cause a distortion of the transverse plane.

The assumption for simple bending :-
.) The material is homogenous and isotropic
For example:- It has the same value of young's modulus in tension and compression.
) Transverse planes remain plane and perpendicular to the neutral surface after bending.
Initially the beam is straight and all longitudinal filaments are bent into circular arcs with a common centre of curvature which is large compared to the dimensions of the cross-section.
The beam is symmetrical about a vertical longitudinal plane passing through vertical axis of symmetry for horizontal beams.
The stress is purely longitudinal and ...

Bending equation for a beam:-

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

M = Bending moment of any cross-section of beam

I = Moment of Inertia.

f, σ_b = Bending stress.

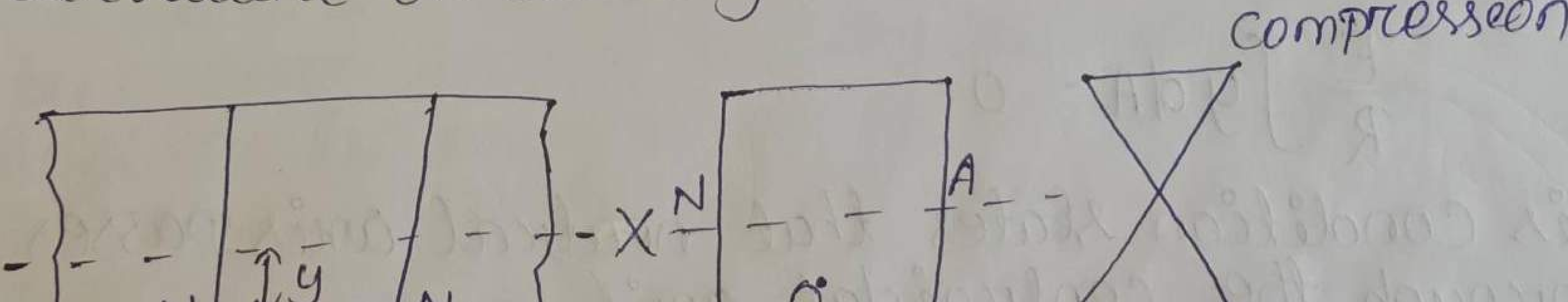
y = centroidal axis/distance

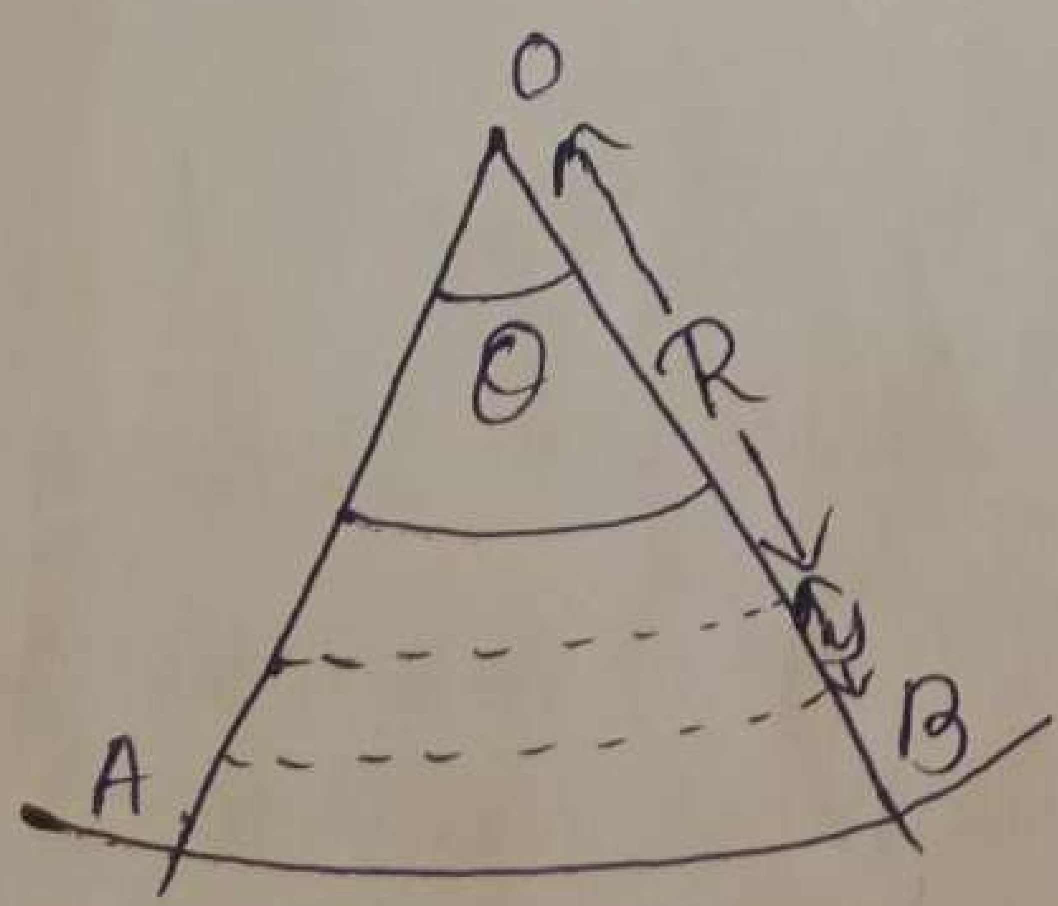
E = Modulus of Elasticity

R = Radius of curvature.

Proof:-

consider a length of beam under the action of c
bending moment 'M'. N-N is the original leng
considered of beam. The neutral surface is a
Plane through : x-x. In the side view NA
indicates neutral axis. 'O' is the   of
curvature on bending.





$l = R\theta$

R = Radius of curvature of the neutral surface

filament of original length NN at a distance
from the neutral axis will be elongated to a
ngth AB.

e Strain in AB $= \frac{AB - NN}{NN}$

$$\frac{f}{E} = \frac{(R+y)\theta - R\theta}{R\theta}$$

$$\boxed{\frac{f}{E} = \frac{y}{R}} \quad \text{—— (1)}$$

or pure bending:

Net normal forces is zero.

$$\int f \cdot dA = 0$$

$$\int \frac{E}{R} y \, dA = 0$$

$$\frac{E}{R} \int y \, dA = 0$$

condition states that neutral axis passes
ough the centroidal axis.

$$M = \int (f \cdot dA) y$$

$$= \int y \times \frac{E}{R} \times dA \times y$$

$$= \frac{E}{R} \int y^2 \, dA$$

$$M = \frac{E}{R} \times I$$

Eq

Se
Tr
Th
r

quating (i) & (ii)

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$

ction modulus :-

e ratio of I/y, where y is the furthest or the
ost distant point of the section from neutral
xis is called section modulus.

It is denoted by 'Z'

$$Z = \frac{\text{moment of inertia about } \cancel{\text{about}} \text{ neutral axis}}{\text{Distance of furthest point from neutral axis.}}$$

$$\boxed{Z = \frac{I}{y}}$$

oment of resistance :-

e maximum bending moment which can be carrie
y a given section for a given maximum value
stress is known as moment of resistance.

gnificance of section modulus :-

e strength of the beam depends on the sectio
odulus.

$$Z = \frac{I}{y}$$

lexural rigidity (F) :-

The product of modulus of elasticity and mom
of inertia is called flexural rigidity.

$$F = E \times I$$

aximum Bending Stress Formula (M_{max}):

ntilever

D.L (M_{max}) = $\frac{wl^2}{2}$

int load (M_{max}) = $P \times l$

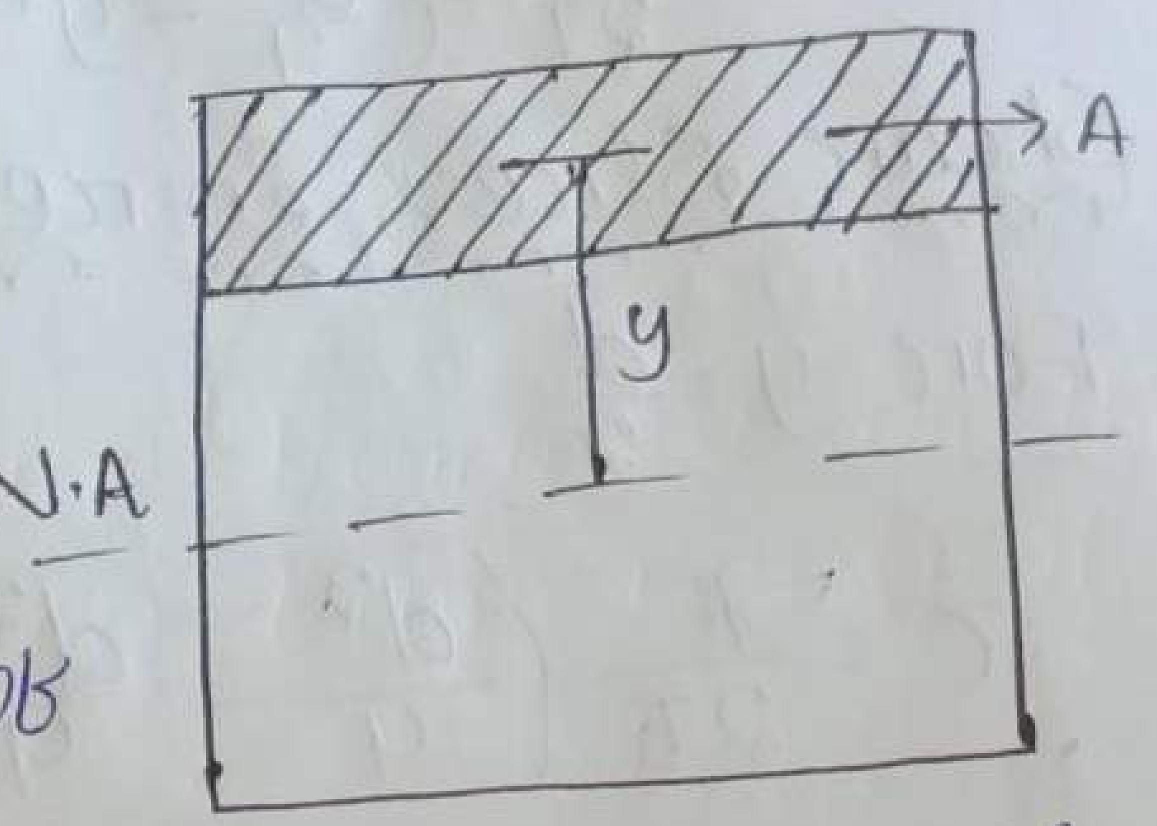
mply Supported:

D.L. (M_{max}) = $\frac{wl^2}{8}$

int load (M_{max}) = $\frac{Pl}{4}$

Sheare Stress

$$\tau = \frac{F A \bar{y}}{I b}$$



τ = Sheare Stress

F = Sheare Force

A = Area above the neutral axis of the shaded Portion.

$\bar{y}$ = distance betn centre of gravity of the section to the shaded area.

I = moment of inertia of the whole section above its neutral axis.

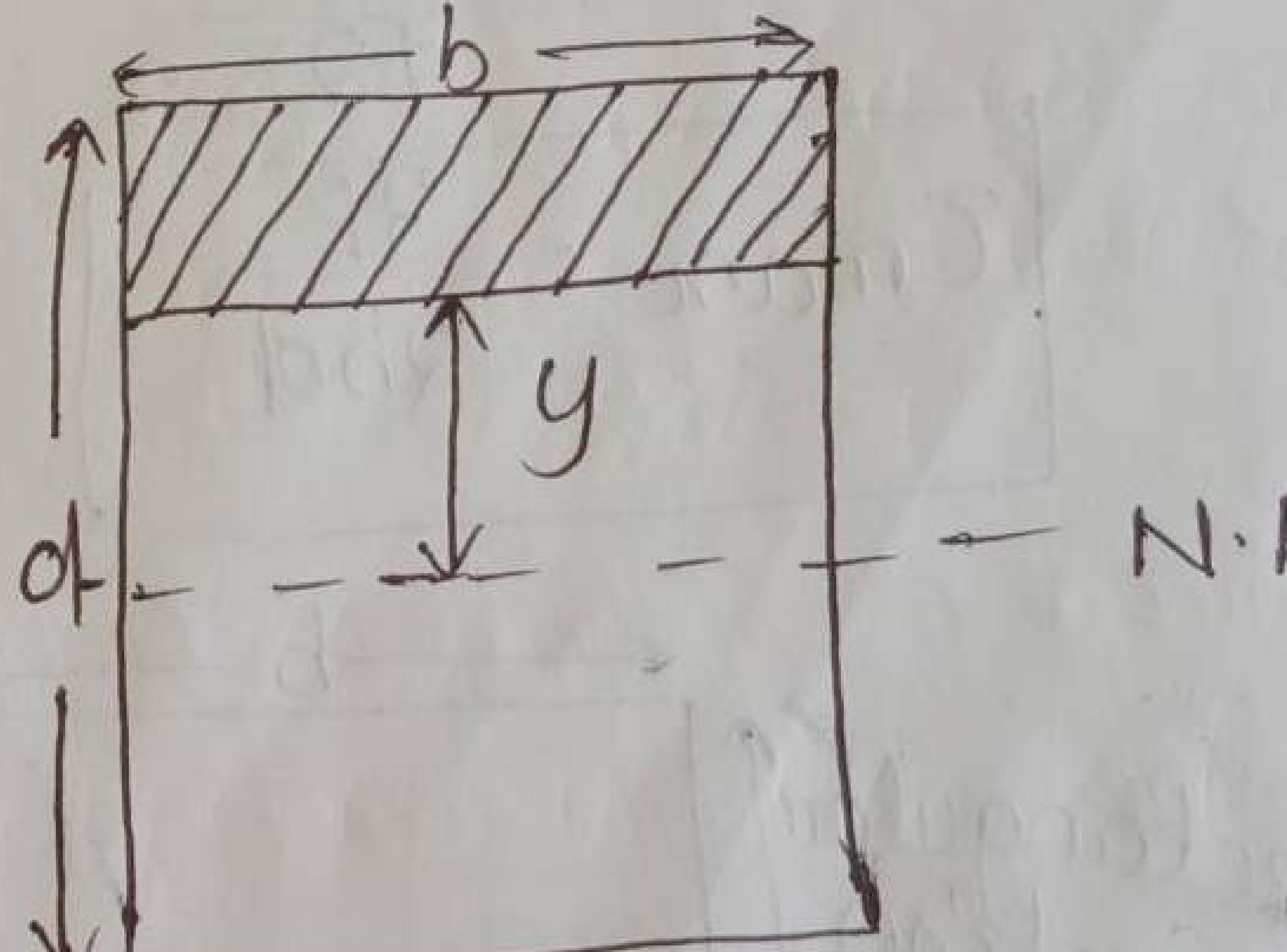
b = width at the section.

Rectangular Section:

b = width of the beam

d = depth of the beam

y = distance between neutral axis to the section of the beam.



$$\tau = \frac{F A \bar{Y}}{I b}$$

$$= \frac{F \times \left(\frac{d}{2} - y\right) \times \cancel{b} \times \frac{1}{2}\left(\frac{d}{2} + y\right)}{I \cancel{b}}$$

$$= \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

$$\bar{Y} = \frac{1}{2}\left(\frac{d}{2} - y\right) + y$$

$$= \frac{d}{4} - \frac{y}{2} + y$$

$$= \frac{d}{4} + \frac{y}{2}$$

$$= \frac{1}{2}\left(\frac{d}{2} + y\right.$$

$A = (d$

For Rectangular section:

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

Shear stress in rectangular section:

For $y = \frac{d}{2}$

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - \frac{d^2}{4}\right)$$

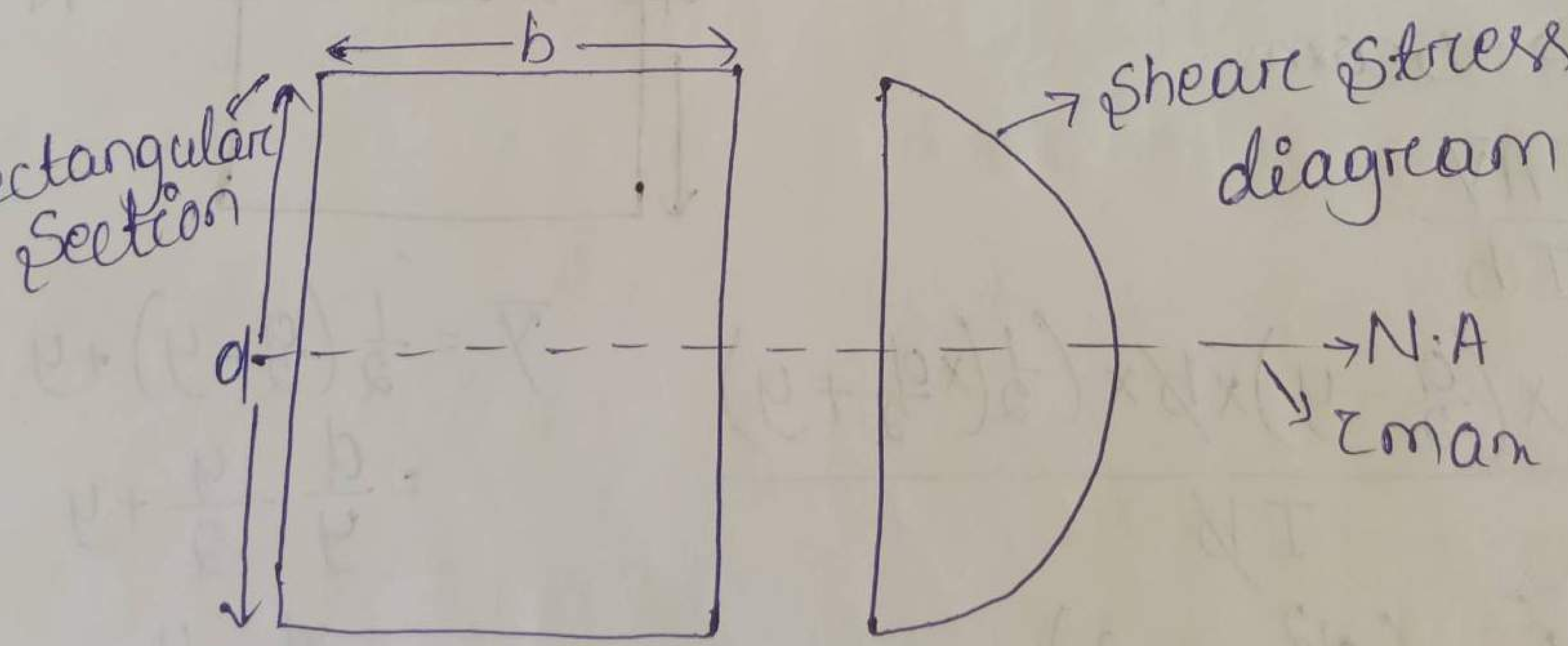
$$\tau = 0$$

For $y = 0$

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4}\right)$$

$$= \frac{F}{2 \times \frac{bd^3}{12}} \times \frac{d^2}{4}$$

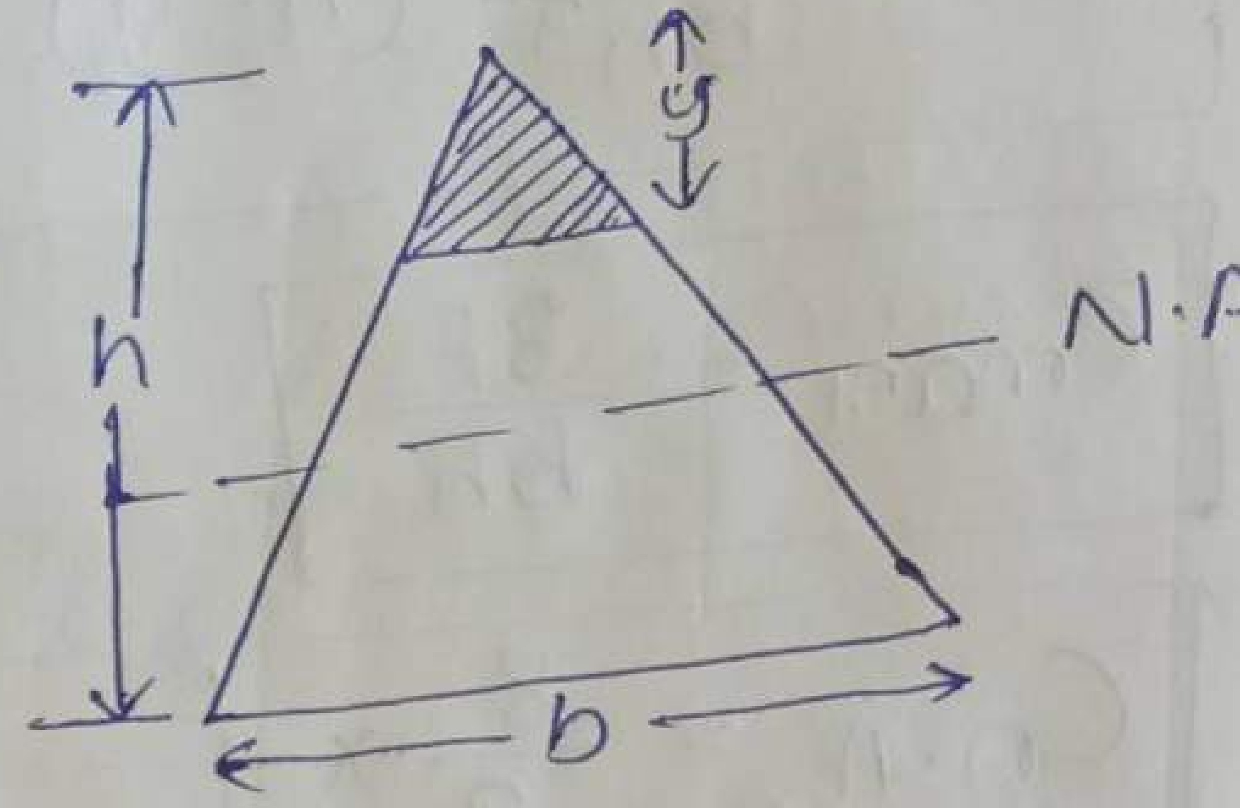
$$\boxed{\tau_{max} = \frac{3F}{2bd}}$$



Triangulare Section:-

$$\tau = \frac{FA\bar{y}}{Ib}$$

$$= \frac{F \times \frac{1}{2} \times \frac{by}{h} \times y \times \left(\frac{2h}{3} - \frac{2y}{3}\right)}{\frac{bh^3}{36} \times \frac{by}{h}}$$



$$\boxed{\tau = \frac{12F}{bh^3} y (h-y)}$$

Area of Shaded portion =

$$\frac{1}{2} \times \frac{by}{h} \times y$$

$$\frac{b'}{b} = \frac{y}{h}$$

$$\boxed{b' = \frac{y}{h} \times b}$$

$$\frac{d\tau}{dy} = \frac{12F}{bh^3}(h-2y)$$

$$0 = \frac{12F}{bh^3}(h-2y)$$

$$\boxed{y = \frac{h}{2}}$$

$$\tau_{max} = \frac{12F \times \frac{h}{2} \times (h - h/2)}{bh^3}$$

$$= \frac{3}{2} \times \frac{F}{\frac{1}{2} \times bh}$$

$$\boxed{\tau_{max} = 1.5\, \tau_{mean}}$$

Traingle centroid = h/

neutral axis $= \frac{12F}{bh^3} y(h-y)$

$$= \frac{12F}{bh^3} \times \frac{2h}{3}\left(h - \frac{2h}{3}\right)$$

$$y = \frac{2h}{3}$$

$$\boxed{\tau_{N.A} = \frac{8}{3} \times \frac{F}{bh}}$$

$$\boxed{\tau = \frac{12F \times y}{bh^3}(h-y)}$$

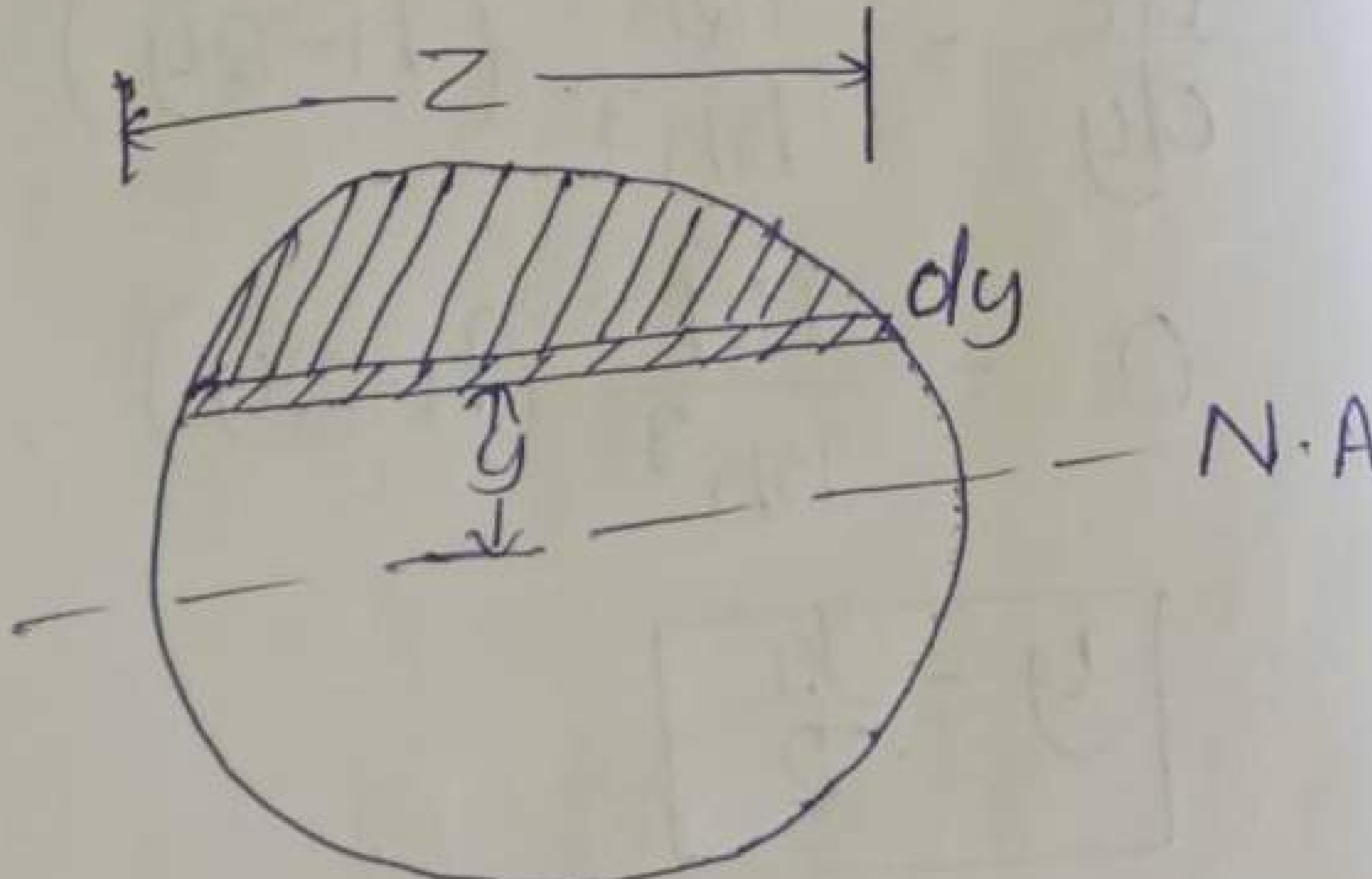
$$\boxed{\tau_{max} = \frac{3F}{bh}}$$

$$\boxed{\tau_{n.A} = \frac{8}{3} \times \frac{F}{bh}}$$

→ Triangular section

rcular Section:

$A\bar{y}$ = moment of shaded area about neutral axis



$$\tau = \frac{F \times A\bar{y}}{Ib}$$

$$\left(\frac{z}{2}\right)^2 = r^2 - y^2$$

$$z^2 = 4(r^2 - y^2)$$

ea of small strip $= z \times dy$

oment of small strip about N.A

$$= A\bar{y} = z\,dy \cdot y$$

$\bar{y}$ for shaded portion

$$= \int_y^r z \cdot dy \cdot y$$

$$z^2 = 4(r^2 - y^2)$$

$$2z \cdot dz = -8y\,dy$$

$$y\,dy = \frac{-z\,dz}{4}$$

$$= -\frac{1}{4}\int_z^0 z \cdot z \cdot dz$$

$$\tau = F \cdot \frac{1}{ZI} \cdot \frac{Z^3}{12} = F \times \frac{1}{I} \times \frac{Z^2}{12} = \frac{F}{3I}(r^2 - y^2)$$

Shear stress is parabolic nature :

$$\tau_{max}\,(y=0) = \frac{F}{3I}(r^2) = \frac{F}{3 \times \left(\frac{\pi d^4}{64}\right)} \times \left(\frac{d^2}{4}\right)$$

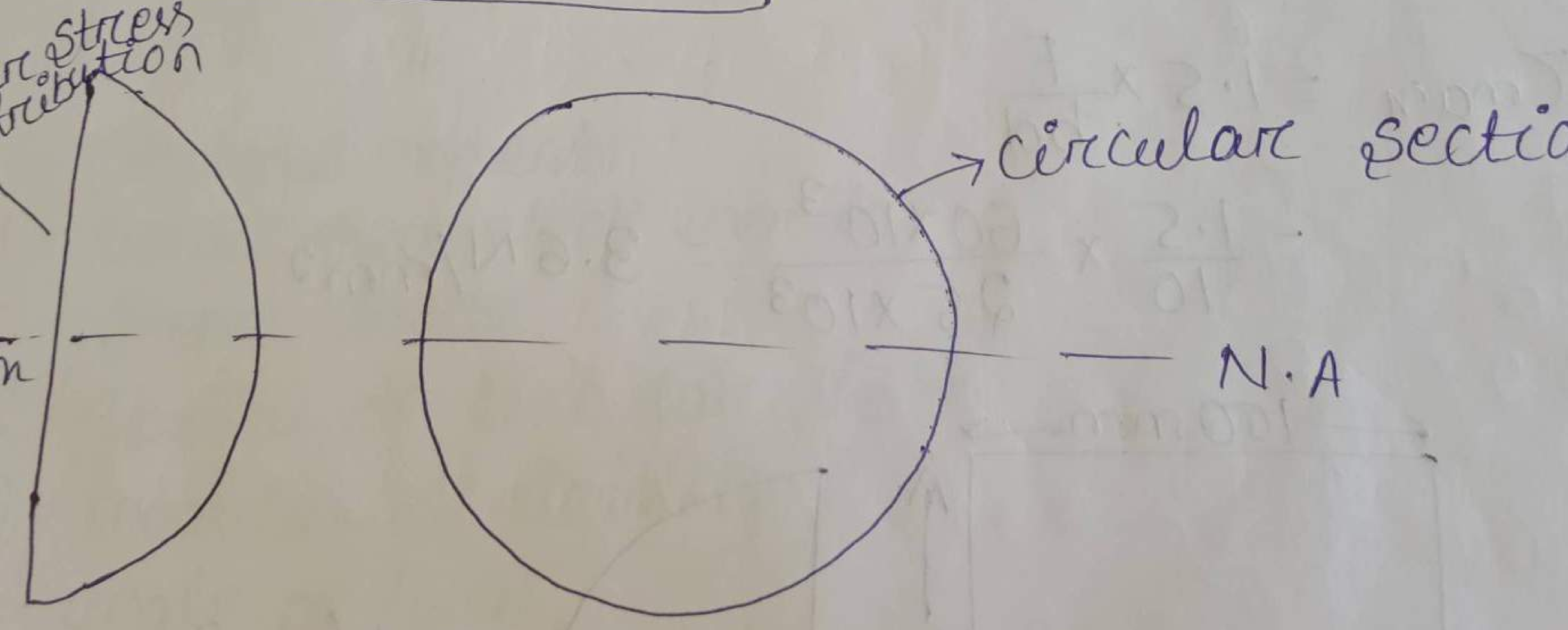
$$\boxed{\tau_{max} = \frac{4}{3}\tau_{av}}$$

Sheare Stress for circular section:

$$\boxed{\tau = \frac{F}{3I}(r^2 - y^2)}$$

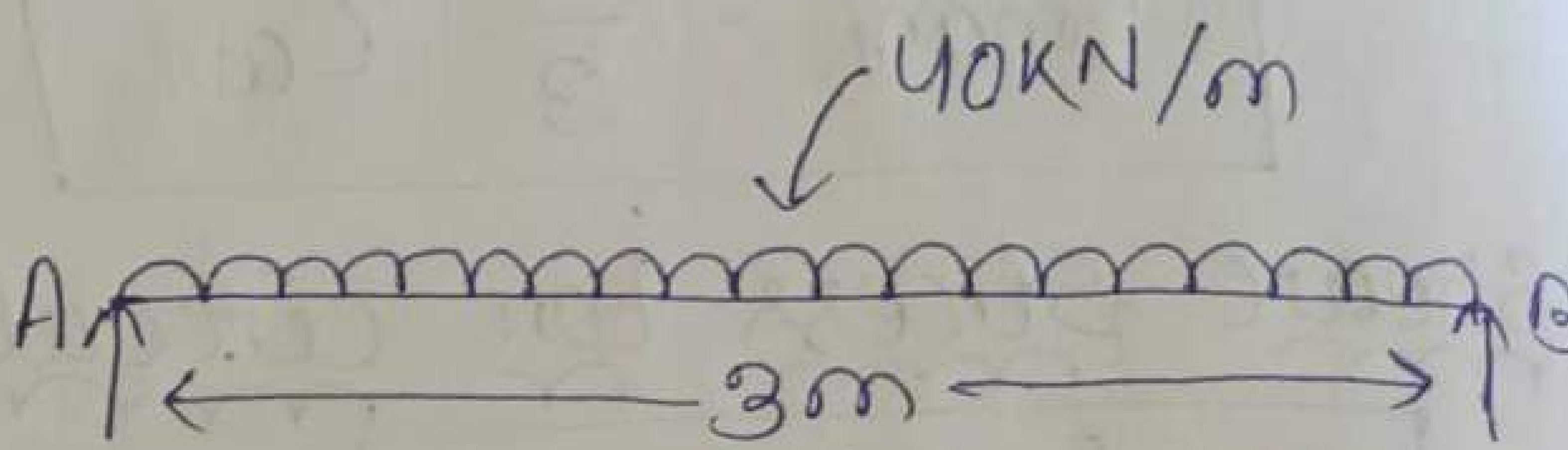
$$\boxed{\tau_{max} = \frac{16F}{3\pi d^2} = \frac{4}{3}\frac{F}{\left(\frac{\pi d^2}{4}\right)}}$$

$$\boxed{\tau_{max} = \frac{4}{3}\tau_{av}}$$



: A wooden beam 100mm wide, 250mm depth and 3m
ong carrying a U.D.L 40KN/mt. Determine the
aximum shear stress and draw the diagram
shear stress along the depth of the beam
simply supported beam).

b = 100 mm
d = 250 mm
L = 3 m
w = 40 KN/m



ax^m shear force $= \frac{w \times L}{2}$

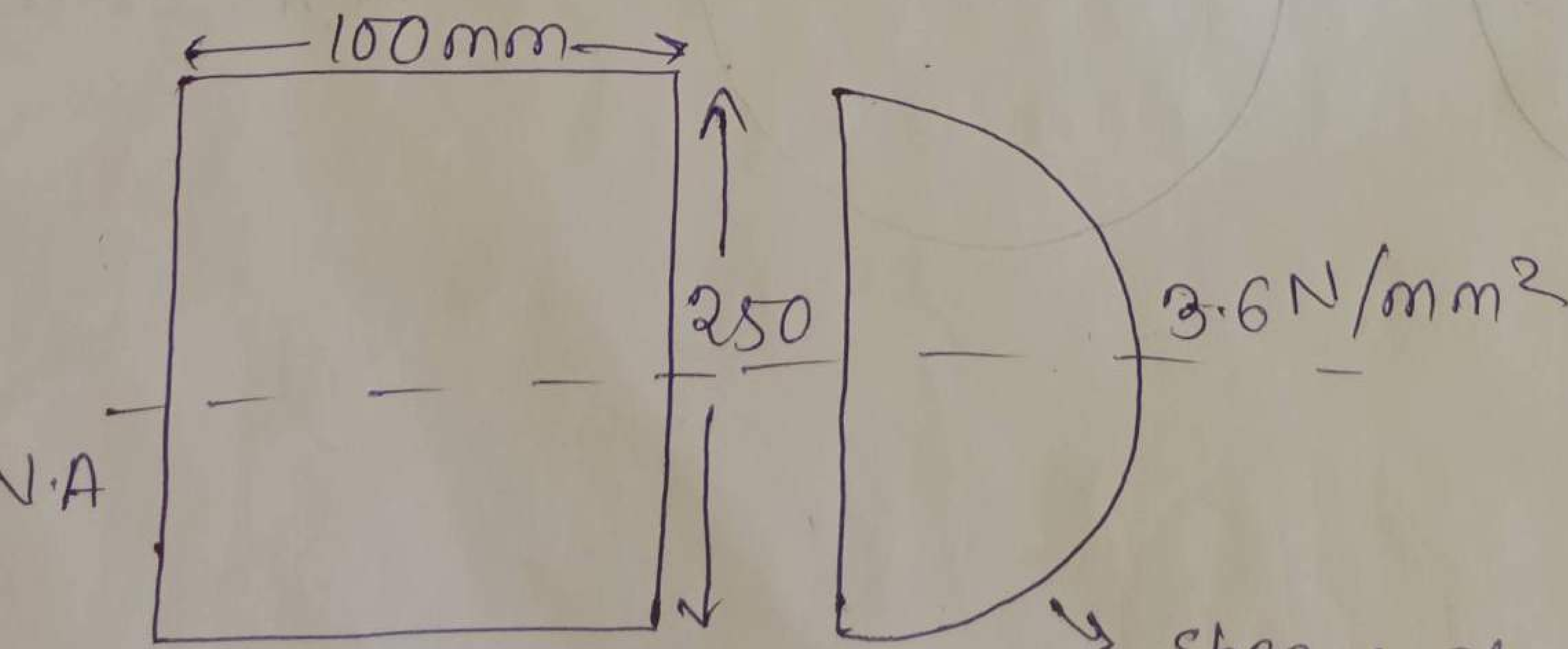
$= \frac{40 \times 3}{2}$

$= 60 kN$

$A = b \times d = 100 \times 250 = 25 \times 10^3 \, mm^2$

$\tau_{max} = 1.5 \times \frac{F}{bd}$

$= \frac{1.5}{10} \times \frac{60 \times 10^3}{25 \times 10^3} = 3.6 \, N/mm^2$



Torque

Torsional moment:
It is a moment acting about the axis which is perpendicular to the plane of cross-section.

Assumption:
The material of the shaft is homogenous isotrop
and perfectly elastic.
The material obey's Hooke's law and the stress remains within limit of proportionality.
The twisting couples acts in the transverse Pla
nly.
ll radii remain straight after torsion.
arallel planes normal to the axis do not warp or
istort after torsion.

rsional equation:-

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$

Torsional moment
Polar moment of inertia.
Shear stress
Radius of circular shaft
modulus of rigidity
angle of twist
length of shaft

r moment of inertia:-

$$J = I_x + I_y$$

r Sectional modulus:

Torsional rigidity :-

The product of modulus of rigidity and polar moment of inertia is known as Torsional rigidity (GJ).

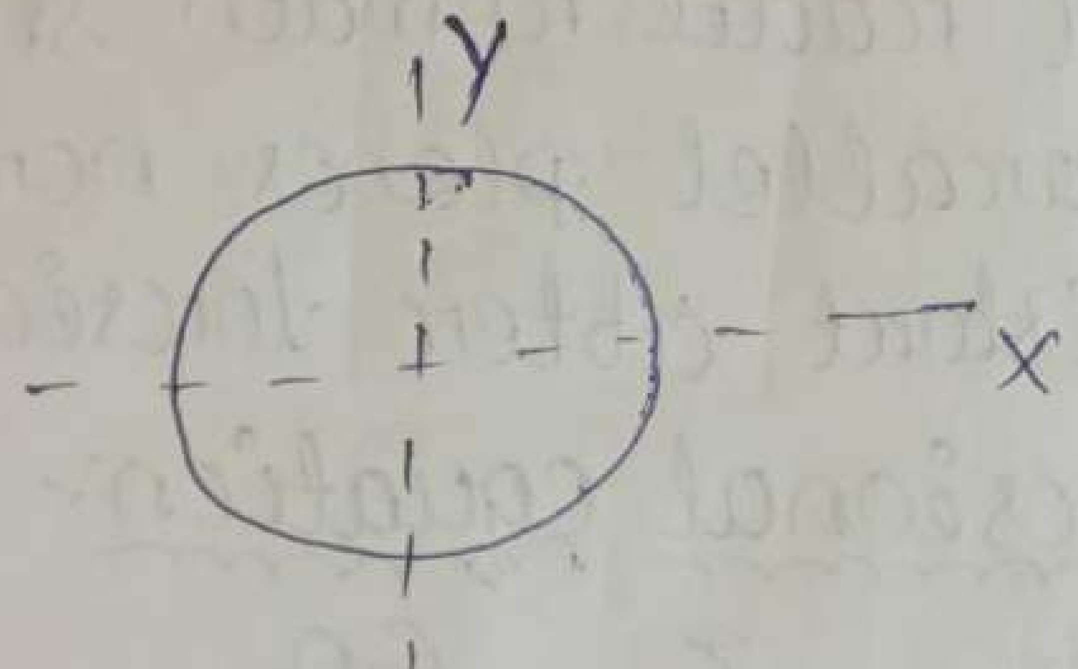
Torsional moment of resistance :

The torque which can be carried by a given section of shaft for a given maximum value of shear stress is known as Torsional moment of resistance.

1. Solid Shaft :

$$J = I_x + I_y = 2I$$

$$I_x = I_y = I$$



$$= 2 \times \frac{\pi d^4}{64}$$

$$= \frac{\pi d^4}{32}$$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}$$

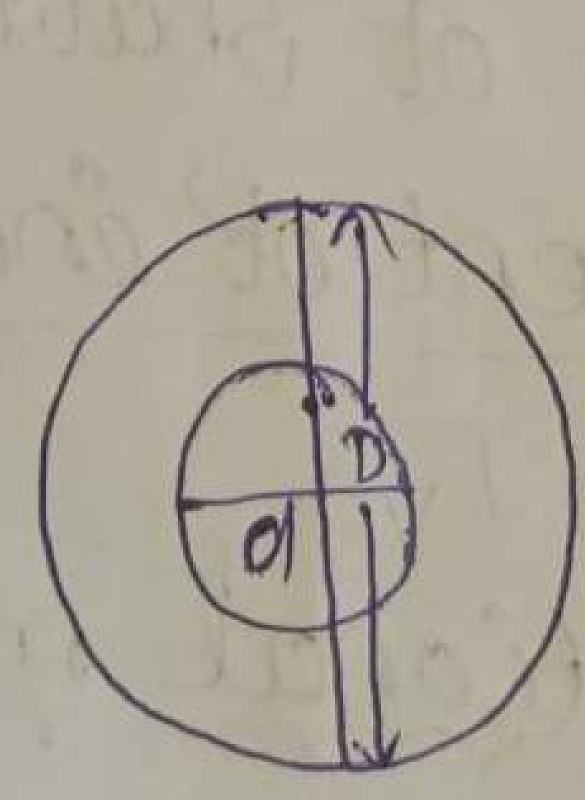
$$\frac{T}{\frac{\pi d^4}{32}} = \frac{\tau}{\frac{d}{2}}$$

$$\tau = \frac{16T}{\pi d^3} \longrightarrow \text{For solid shaft}$$

Hollow Shaft :

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\Rightarrow \frac{T}{\pi(D^4 - d^4)} = \frac{\tau}{D/2}$$



$$\text{Solid Shaft} = \tau = \frac{16T}{\pi d^3}$$

$$\text{Hollow Shaft} = \tau = \frac{16TD}{\pi(D^4 - d^4)}$$

ower transmitted by torque :-

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

$$\text{Work done} = \text{Force} \times \text{distance}$$

$$P = \frac{2\pi N T}{60}\ \text{KW}$$

N = no. of revolution

Combined bending and direct strees

Symmetrical columns with eccentric loading about one axis

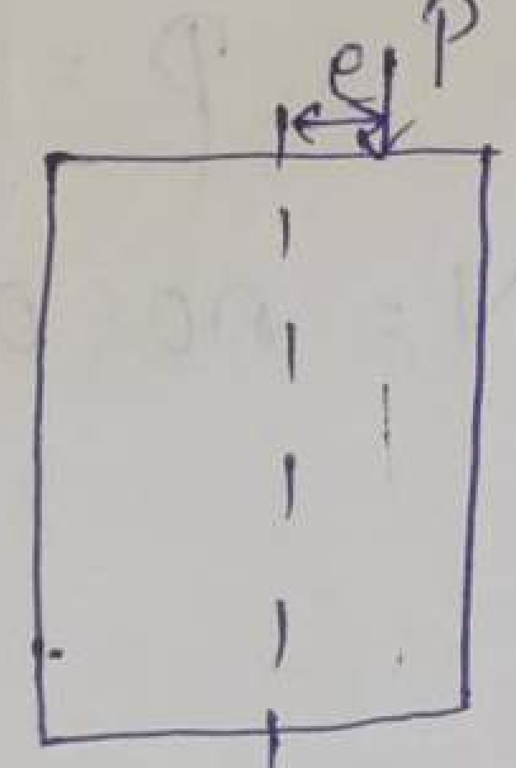
Considere a column ABCD subjected to an eccentric load about one axis (i.e. about Y-Y axis)

P = load acting on the column

e = Eccentricity of the load

b = width of the column section

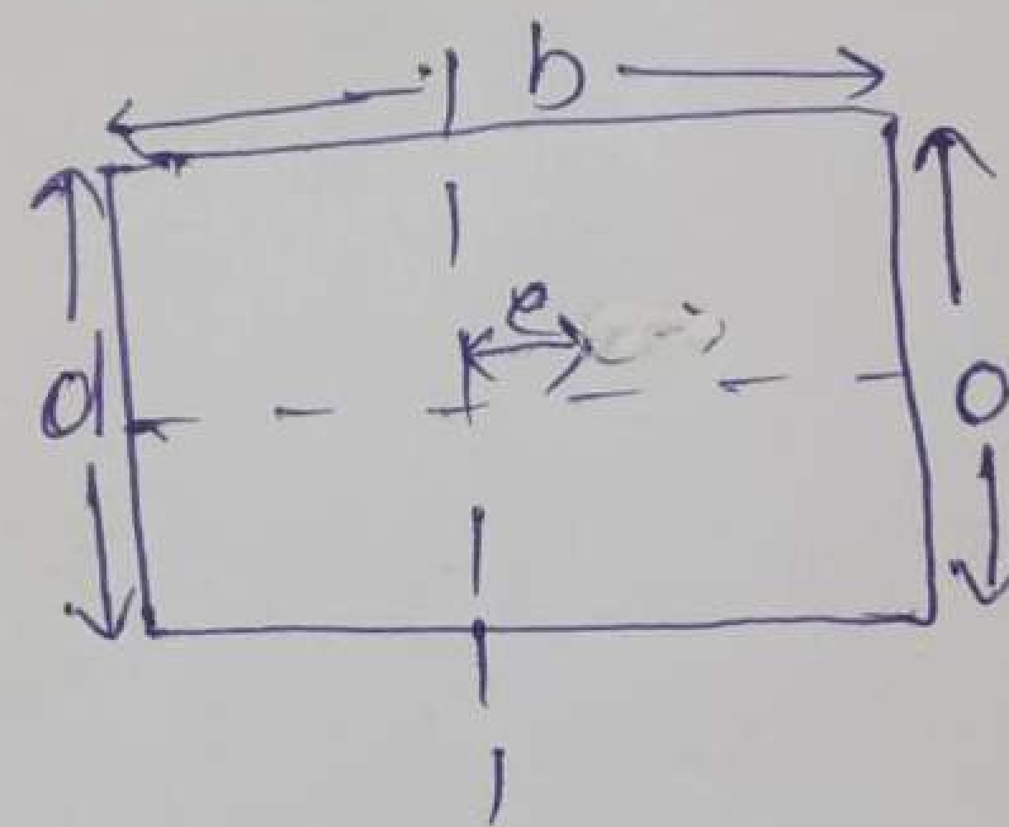
d = Thickness of column



Area of column section

$$A = bd$$

Moment of inertia of the column section about an axis



Plan

through its centre of gravity and parallel to the axis about which the load is eccentric

$$I = \frac{db^3}{12}$$

Direct stress of column due to the load:

$$\boxed{\sigma_0 = P/A}$$

Bending stress at any point of the column section at a distance y from y-y axis.

$$\sigma_b = \frac{My}{I} = \frac{M}{Z}$$

$$\sigma_b = \frac{M \times b/2}{\frac{db^3}{12}} = \frac{6M \times \cancel{b}}{d\cancel{b}^3 b^2}$$

$$\boxed{\sigma_b = \frac{6 \times Pe}{db^2}}$$

Total stress at the extreme fibre $= \sigma_0 \pm \sigma_b$

$$= \frac{P}{A} \pm \frac{M}{Z} \rightarrow \text{General equation}$$

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{6Pe}{db^2}}$$

$$\boxed{\sigma_{min} = \frac{P}{A} - \frac{6Pe}{db^2}}$$

Rectangular section for plane bisecting thickness.

$$\boxed{\sigma_{max} = \frac{P}{A} + \frac{Pe}{\left(\frac{\pi d^3}{32}\right)}}$$

$$\boxed{\sigma_{min} = \frac{P}{A} - \frac{Pe}{\frac{\pi d^3}{32}}}$$

Circular Section.

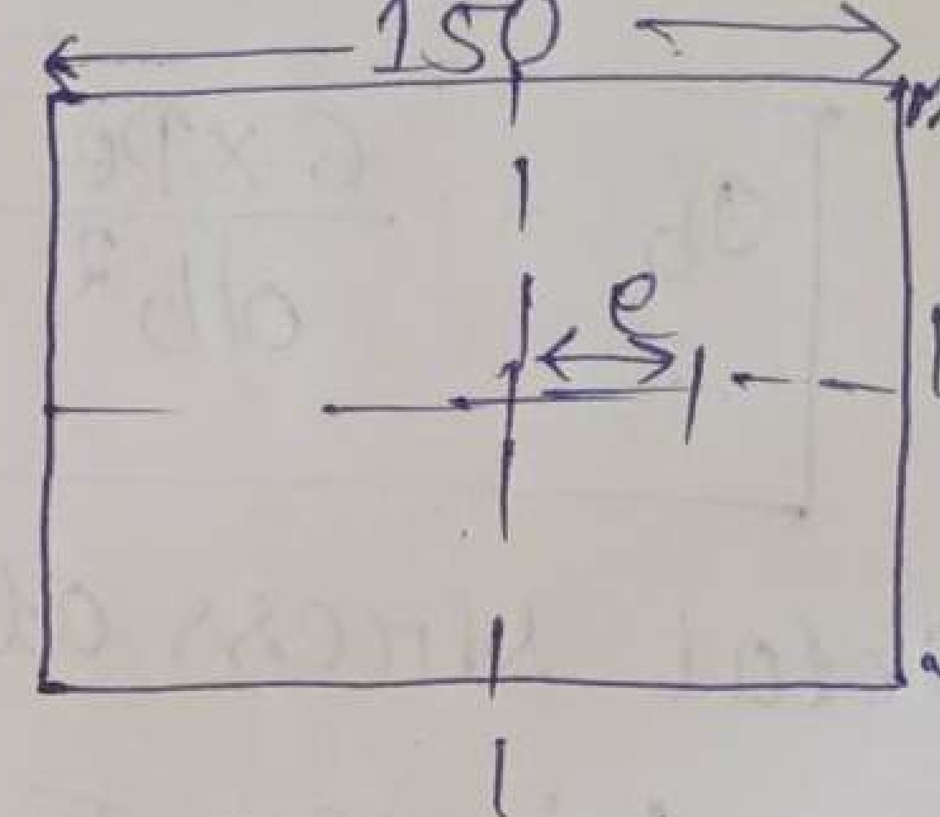
Rectangular Section { circular Section

$= \frac{P}{} + \frac{Pey}{}$

A Rectangulare strut is 150mm and 120mm thick. carreies a load of 180KN at an eccentricity of 10m a plane bisecting the thickness. Find the maximum d minimum stress in the section.

$$A = b \times d = 150 \times 120$$

$$= 18{,}000\ mm^2$$



$P = 180KN$

$b = 150\ mm$

$d = 120\ mm$

$e = 10\ mm$

$$\sigma_{max} = \frac{P}{A} + \frac{6Pe}{b^2 d}$$

$$= \frac{180 \times 10^3}{150 \times 120} + \frac{6 \times 180 \times 10^3 \times 10}{150^2 \times 120}$$

$$= 14\ N/mm^2 \text{ or } 14\ MPa$$

$$\sigma_{min} = \frac{P}{A} - \frac{6Pe}{b^2 d}$$

$$= \frac{180 \times 10^3}{150 \times 120} - \frac{6 \times 180 \times 10^3 \times 10}{150^2 \times 120}$$

$$= 6\ N/mm^2 = 6\ MPa$$

Symmetrical columns with eccentric loading about two axes

P = Load acting on the column

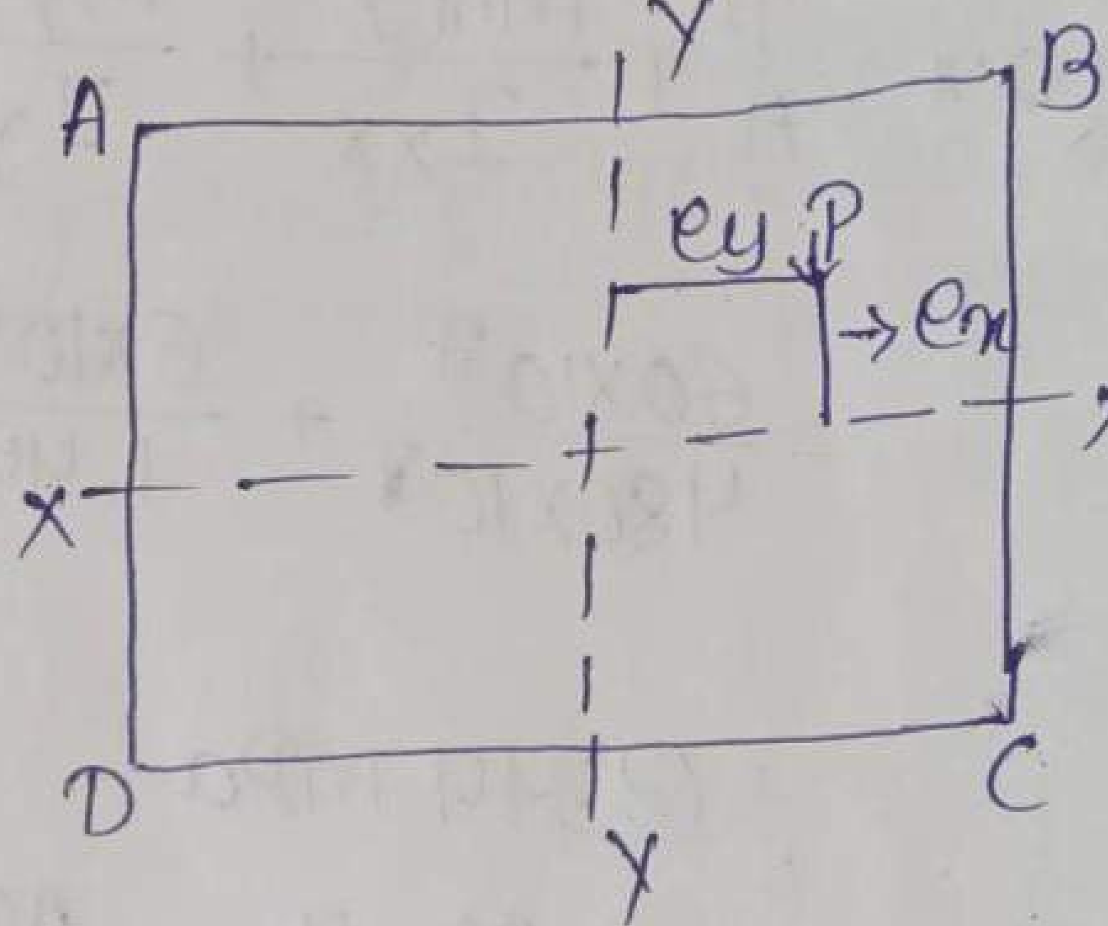
A = cross-sectional area of the column

e_x = Eccentricity of the load about x-x axis

e_y = Eccentricity of the load about y-y axis.

I_{xx} = moment of inertia about 'x' axis

I_{yy} = moment of inertia about 'y' axis



$$\sigma = \frac{P}{A} \pm \frac{M_x y}{I_{xx}} \pm \frac{M_y x}{I_{yy}}$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_x y}{I_{xx}} + \frac{M_y x}{I_{yy}}$$

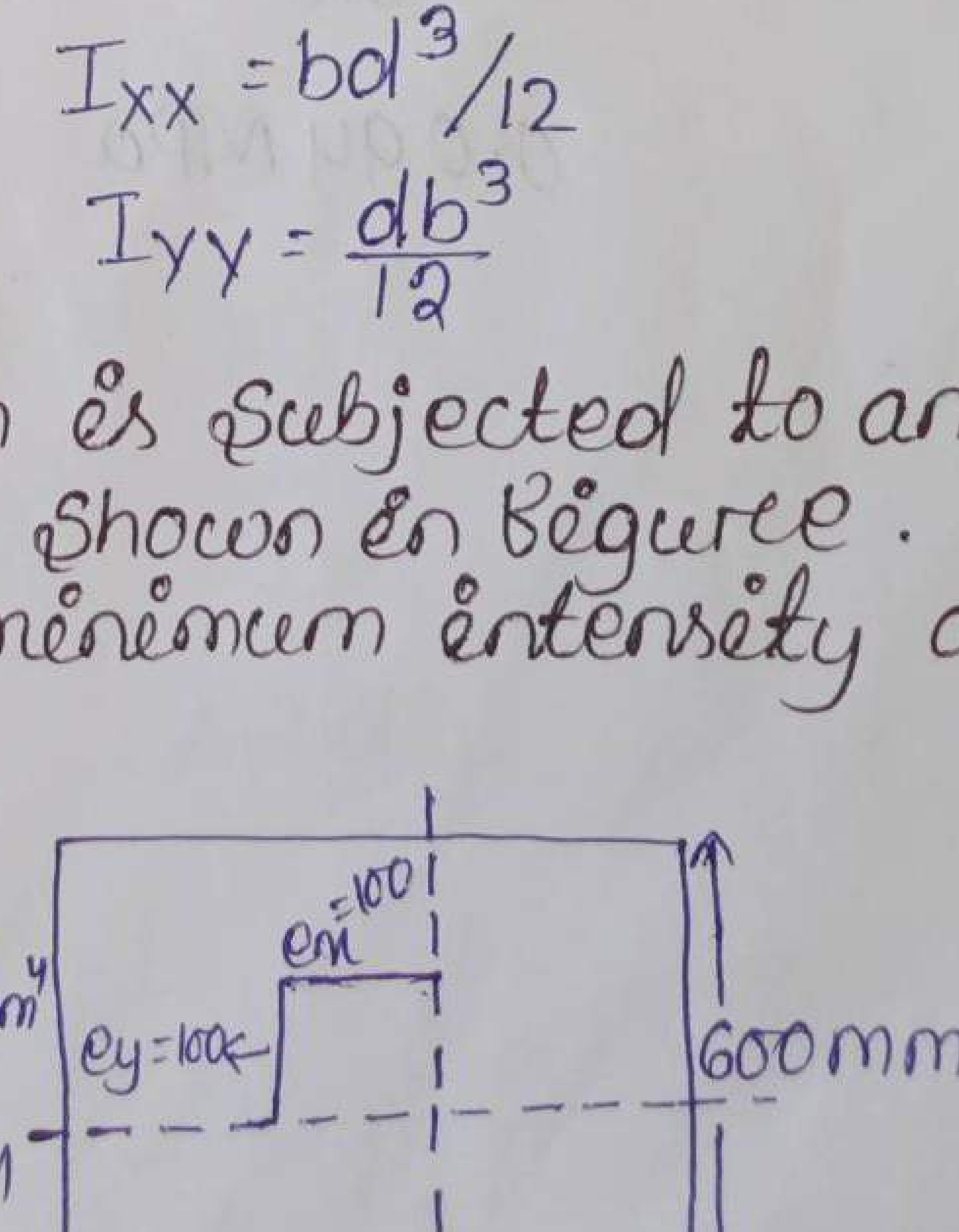
$$\sigma_{min} = \frac{P}{A} - \frac{M_x y}{I_{xx}} - \frac{M_y x}{I_{yy}}$$

$M_x = P \times e_x$ $\quad y = d/2$ $\quad I_{xx} = bd^3/12$

$M_y = P \times e_y$ $\quad x = b/2$ $\quad I_{yy} = \frac{db^3}{12}$

Q.1 A column 800 mm × 600 mm is subjected to an eccentric load of 60 kN as shown in figure. What is the maximum and minimum intensity of stress in the column.

Ans



$A = 600 \times 800 = 480 \times 10^3 \text{ mm}^2$

$I_{xx} = \frac{bd^3}{12} = \frac{800 \times 600^3}{12} = 1.44 \times 10^{10} \text{ mm}^4$

$I_{yy} = \frac{db^3}{12} = \frac{600 \times 800^3}{12} = 2.56 \times 10^{10} \text{ mm}^4$

$M_x = P \times e_x = 60 \times 10^3 \times 100 = 6 \times 10^6$ N-mm

$M_y = P \times e_y = 60 \times 10^3 \times 100 = 6 \times 10^6$ N-mm

$x = b/2 = \frac{800}{2} = 400$ mm

$y = d/2 = \frac{600}{2} = 300$ mm

$$\sigma_{max} = \frac{P}{A} + \frac{M_x Y}{I_{xx}} + \frac{M_y x}{I_{yy}}$$

$$= \frac{60 \times 10^3}{480 \times 10^3} + \frac{6 \times 10^6 \times 300}{1.44 \times 10^{10}} + \frac{6 \times 10^6 \times 400}{2.56 \times 10^{10}}$$

$$= 0.344 \text{ MPa}$$

$$\sigma_{min} = \frac{P}{A} - \frac{M_x y}{I_{xx}} - \frac{M_y x}{I_{yy}}$$

$$= \frac{60 \times 10^3}{480 \times 10^3} - \frac{6 \times 10^6 \times 300}{1.44 \times 10^{10}} - \frac{6 \times 10^6 \times 400}{2.56 \times 10^{10}}$$

$$= 0.094 \text{ N/mm}^2$$

or

0.094 MPa

Limits of eccentricity

When an eccentric load is acting on a column, it produces direct as well as bending stress. If the direct stress exceeds bending stress then there will be compressive stress will occur and if the bending stress exceeds direct stress then there will be tensile stress will occur.

As concrete is weak in tension and strong in compression, it is advisable to have less or zero tension. The condition for zero tension is:

$$\sigma_{Direct} \geq \sigma_{bending}$$

$$P/A \geq \frac{PeI}{y}$$

$$\boxed{e \leq Z/A}$$

$Z = \frac{I}{y}$ → section modulus

Limit of eccentricity for a rectangular section

$$e \leq \frac{Z}{A}$$

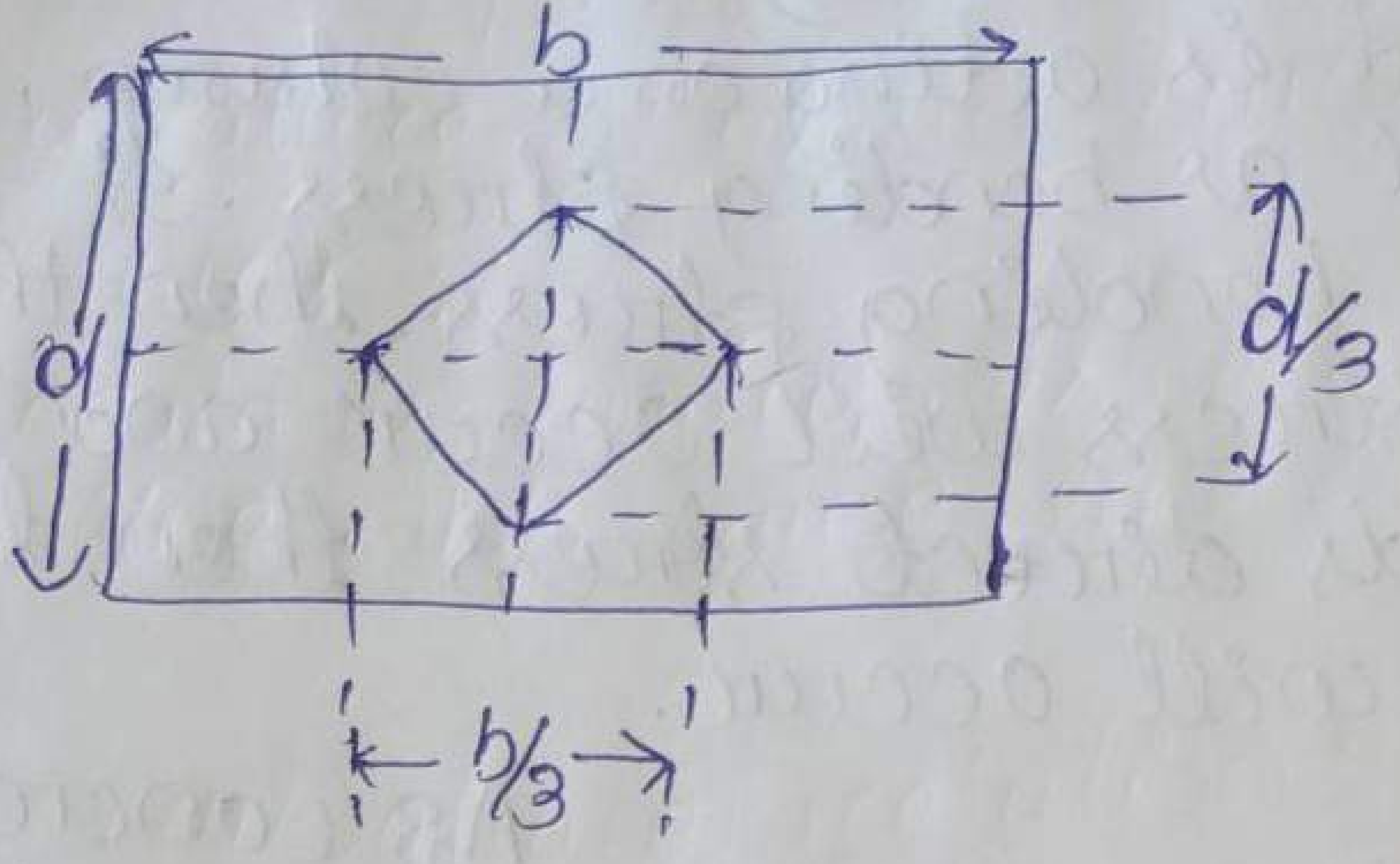
$$Z = \frac{I}{y}$$

$$I = \frac{bd^3}{12}, \quad y = d/2$$

$$Z = \frac{\frac{bd^3}{12}}{d/2} = \frac{bd^2}{6}$$

$$e \leq \frac{bd^2/6}{bd}$$

$A = bd$



the line of action of the load is within middle third as shown in figure, then the stress is compressive.

Limit of eccentricity of column section:

$$e \leq \frac{Z}{A} \qquad Z = I/y$$

$$Z = \frac{I}{y}$$

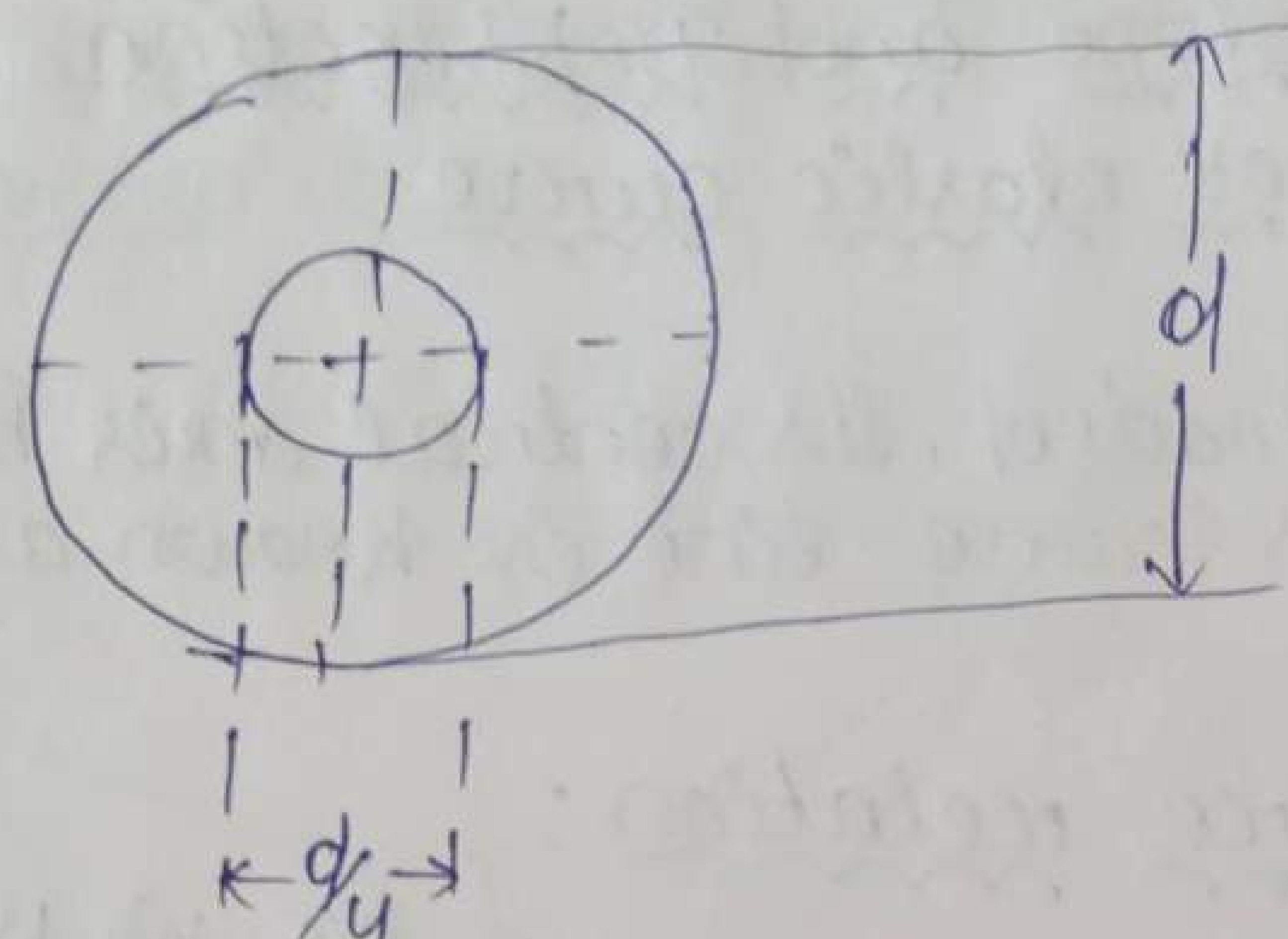
$$I = \frac{\pi d^4}{64} \quad y = d/2$$

$$Z = \frac{\pi d^4}{64} \Big/ d/2$$

$$= \frac{\pi d^3}{32}$$

$$e \leq \frac{\pi d^3}{32} \Big/ \frac{\pi}{4} d^2$$

$$\boxed{e \leq d/8}$$



hus if the line of action of the load is within a
ircle of diameter equal to one - fourth of main
ircle, then the stress will be compressive.

) limit of eccentricity of hollow section

$$e \le \frac{Z}{A} \qquad Z = I/y$$

$$I = \frac{\pi(D^4-d^4)}{64}$$

$$y = D/2$$

$$Z = \frac{\frac{\pi(D^4-d^4)}{\cancel{64}\,32}}{D/\cancel{2}}$$

$$Z = \frac{\pi}{32D}(D^4-d^4)$$

$$e \le \frac{\frac{\cancel{\pi}}{8\,\cancel{32}D}(D^4-d^4)}{\frac{\cancel{\pi}}{\cancel{4}}(D^2-D^4)}$$

$$(D^4-d^4) = (D^2+d^2)(D^2-$$

$$\boxed{e \le \frac{D^2+d^2}{8D}}$$

Slope and Deflection

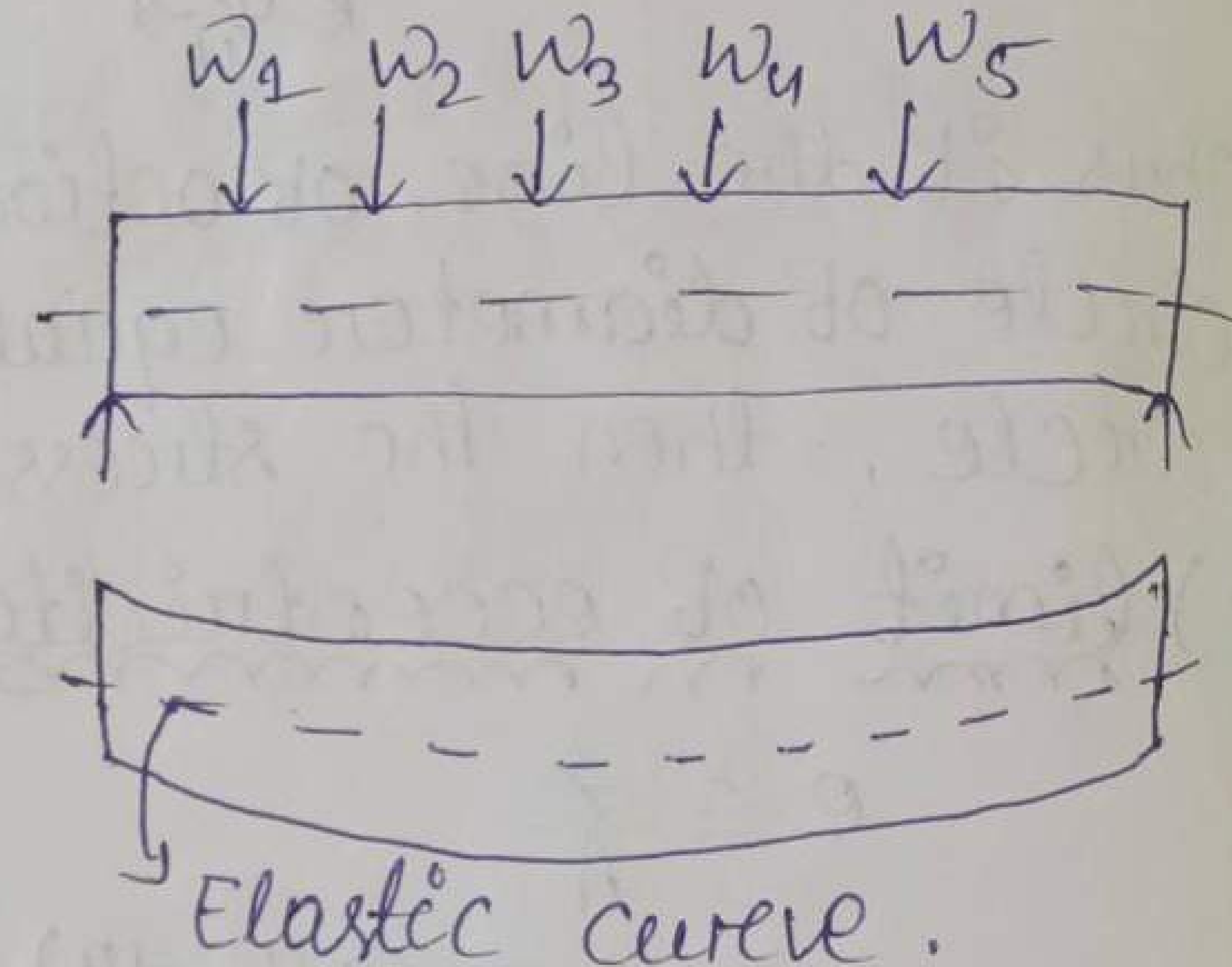
Shape and nature of elastic curve

Elastic curve:

When a beam is loaded, its central axis becomes a curved line. This curve line is known as Elastic curve.

Moment - curvature relation:

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$



Slope:

Slope is the angle formed by the tangent of the curve the horizontal axis.

Deflection:

is the translational movement of the beam from original position is called deflection.

Radius of curvature:

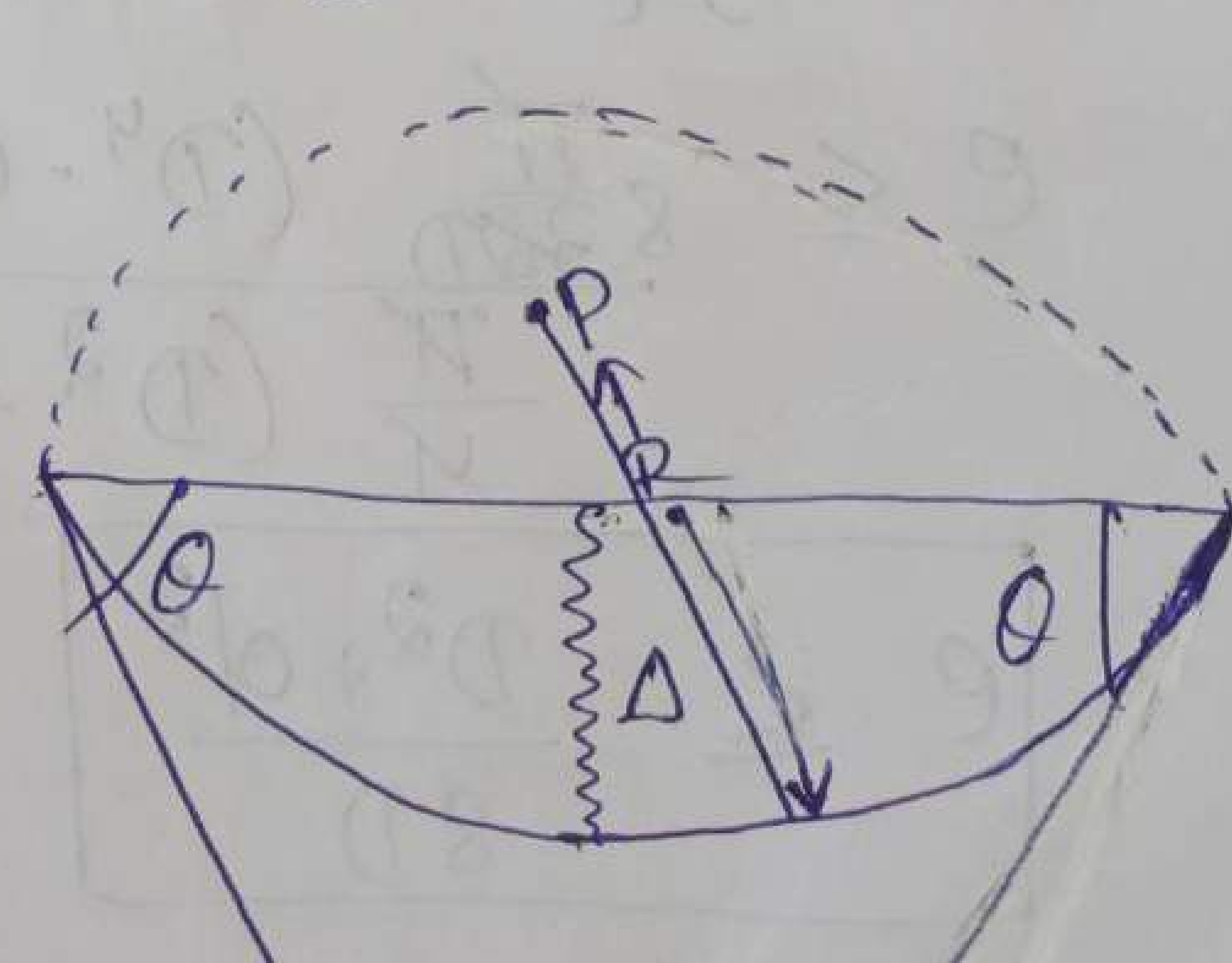
Radius of curvature of a curve at a point is a measure the radius of the circular arc.

θ = Slope

Δ = deflection

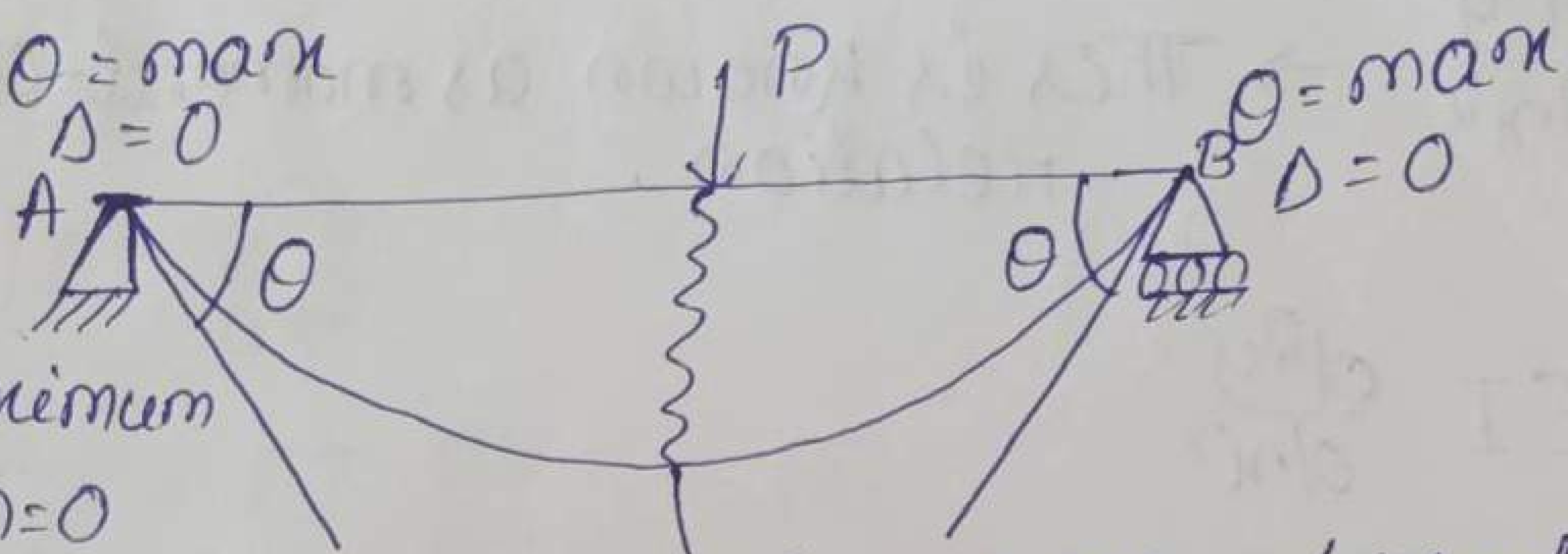
R = Radius of curvature.

(Horizontal direction
Slope in maximum



Shape of elastic curve for different beam :

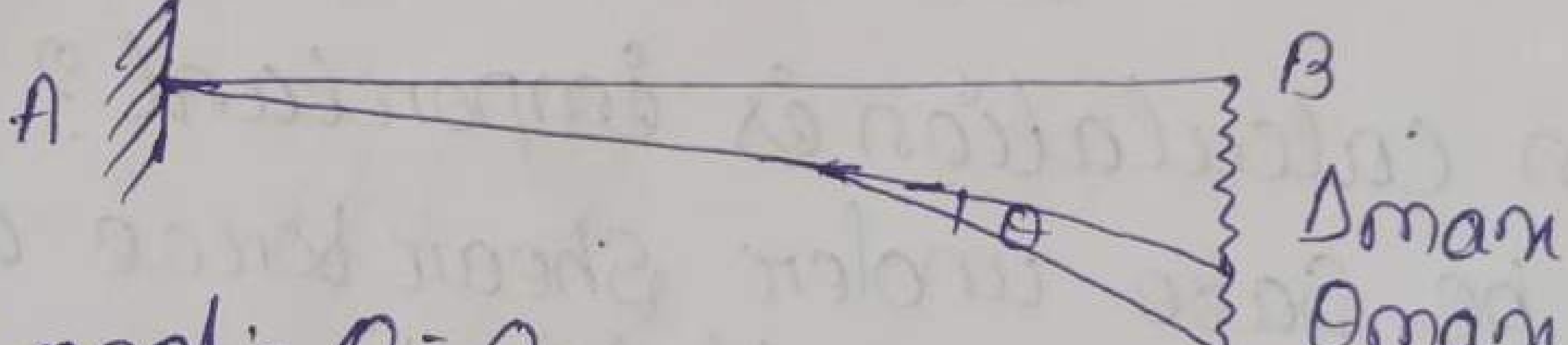
1. Simply supported beam



At ends
slope = maximum
deflection = 0

In middle deflection is maximum slope is zero.

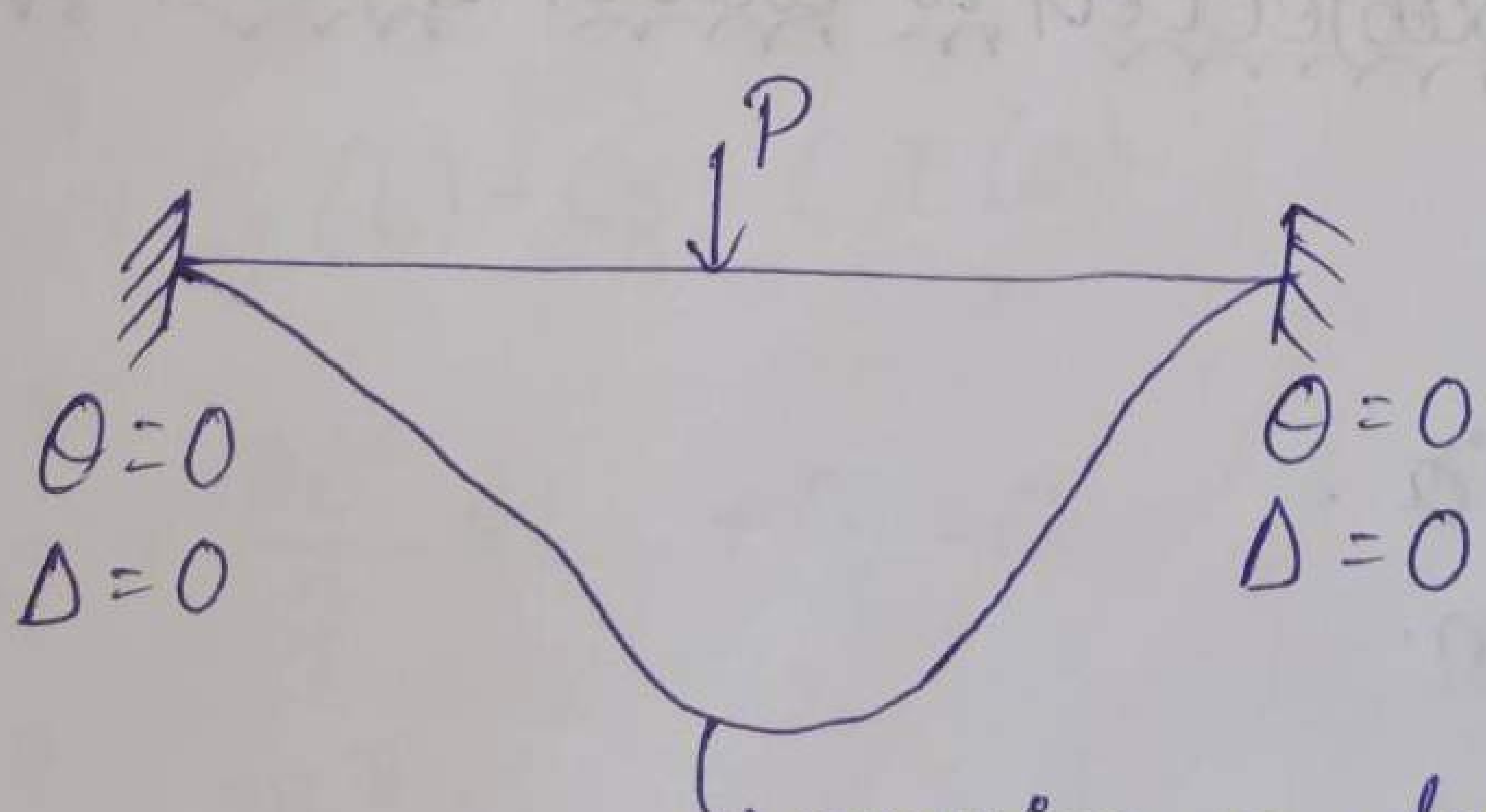
cantilever beam:



Fixed end :- $\theta = 0$
$\Delta = 0$

Free end :- slope is maximum
deflection is maximum.

Fixed beam:



maximum deflection

Relation between slope, deflection and Radius of curvature:

np

$$M = EI\frac{d^2y}{dx^2}$$ → This is known as moment-curvature relation.

$$\int M = \int EI\frac{d^2y}{dx^2}$$

$$\boxed{\int \frac{M}{EI}\,dx + C_1 = \frac{dy}{dx}}$$ → slope

$$\boxed{\int\left[\int \frac{M}{EI}\,dx\right]dx + C_1x + C_2 = y}$$

↳ Deflection

Why deflection calculation is important?

: A beam may be safe under shear force and
ding moment but it is unsuitable because its
ection under the calculated safe load is excessi
to control it, the deflection check is important.

Deflection is caused by loads, temperature,
truction error, settlement.

ection of beam subjected to loads depend upon:

oading pattern.

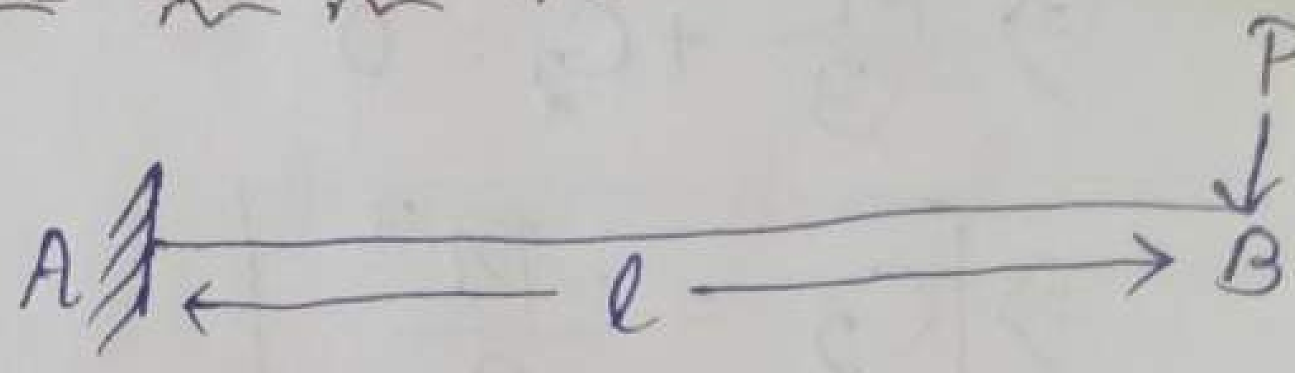
lastic modulus

oment of inertia.

upport condition.

ength of beam.

Slope and deflection of cantilever due to point load:



$M = -P.x$

$M = EI \frac{d^2y}{dx^2}$

$\int -P.x = \int EI \frac{d^2y}{dx^2}$

$-\frac{Px^2}{2} + C_1 = EI \frac{dy}{dx}$ — (i)

$\int -\frac{Px^2}{2} + \int C_1 = \int EI \frac{dy}{dx}$

$-\frac{Px^3}{6} + C_1 x + C_2 = EI (y)$ — (ii)

At, $x = L$, $\Delta = 0$

At, $x = L$, $\Delta = 0$

$-\frac{Px^2}{2} + C_1 = EI \frac{dy}{dx}$

$\Rightarrow -\frac{PL^2}{2} + C_1 = EI(0)$

$\Rightarrow C_1 = \frac{PL^2}{2}$

$-\frac{Px^3}{6} + C_1 x + C_2 = EI (y)$

$-\frac{PL^3}{6} + C_1 (L) + C_2 = EI (0)$

$-\frac{PL^3}{6} + \frac{Pl^2}{2} \times L + C_2 = 0$

$-\frac{PL^3}{6} + \frac{PL^3}{2} + C_2 = 0$

$$\Rightarrow \frac{PL^3}{3} + C_2 = 0$$

$$\Rightarrow \boxed{C_2 = \frac{-PL^3}{3}}$$

aximum

$$-\frac{Px^2}{2} + C_1 = EI\left(\frac{dy}{dx}\right)$$

$(x = 0)$

$$\Rightarrow \frac{-P\times 0^2}{2} + C_1 = EI\left(\frac{dy}{dx}\right)$$

$$\Rightarrow C_1 = EI\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \boxed{\left(\frac{dy}{dx}\right)_{max} = \frac{PL^2}{2EI}}$$

$$-\frac{Px^3}{6} + C_1 x + C_2 = EI(y)$$

$(x = 0)$

$$\Rightarrow -\frac{P\times 0^3}{6} + C_1\times 0 + C_2 = EI(y)$$

$$\Rightarrow C_2 = EI(y)$$

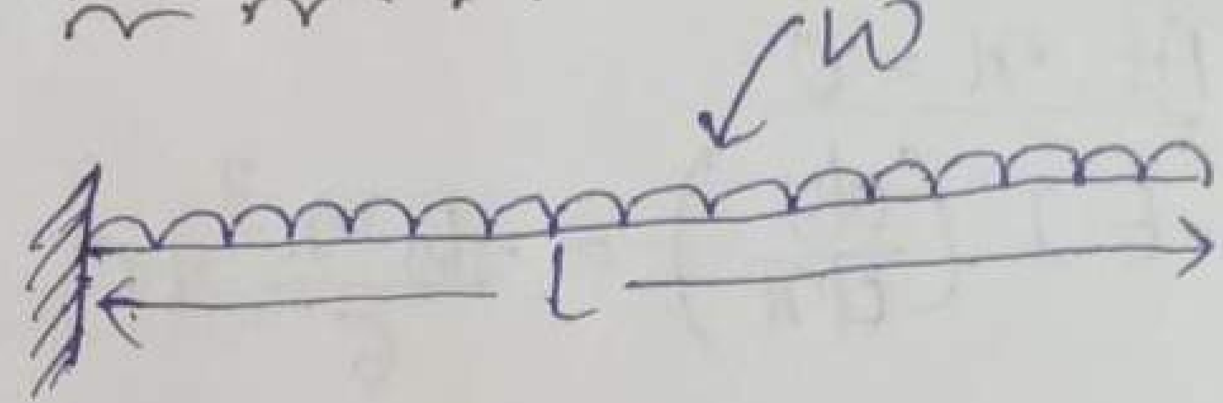
$$\Rightarrow EI(y) = C_2$$

$$\Rightarrow EI(y) = \frac{-PL^3}{3}$$

$$\Rightarrow \boxed{(y)_{max} = \frac{-PL^3}{3EI}}$$

Slope and deflection of cantilever due to UDL :

$M = \frac{-wx^2}{2}$



$EI \frac{d^2y}{dx^2} = \int -\frac{wx^2}{2}$

$EI \frac{dy}{dx} = \frac{-wx^3}{6} + C_1 \quad -(i)$

$EI \frac{dy}{dx} = \int -\frac{wx^3}{6} + \int C_1$

$EI(y) = \frac{-wx^4}{24} + C_1 x + C_2 \quad -(ii)$

At $x = L$, $\theta = 0$
At $x = L$, $\Delta = 0$

$EI\left(\frac{dy}{dx}\right) = -\frac{wx^3}{6} + C_1$

$\Rightarrow EI(0) = \frac{-wL^3}{6} + C_1$

$\Rightarrow 0 = \frac{-wL^3}{6} + C_1$

$\Rightarrow \boxed{C_1 = \frac{wL^3}{6}}$

$EI(y) = \frac{-wx^4}{24} + C_1 x + C_2$

$\Rightarrow 0 = \frac{-wL^4}{24} + \frac{wL^3}{6} \times L + C_2$

$\Rightarrow 0 = \frac{-wL^4 + 4wL^4}{24} + C_2$

Maximum

At, $x = 0$

$$EI\left(\frac{dy}{dx}\right) = -\frac{wx^3}{6} + C_1$$

$$\Rightarrow EI\left(\frac{dy}{dx}\right) = \frac{wL^3}{6}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{wL^3}{6EI}}$$

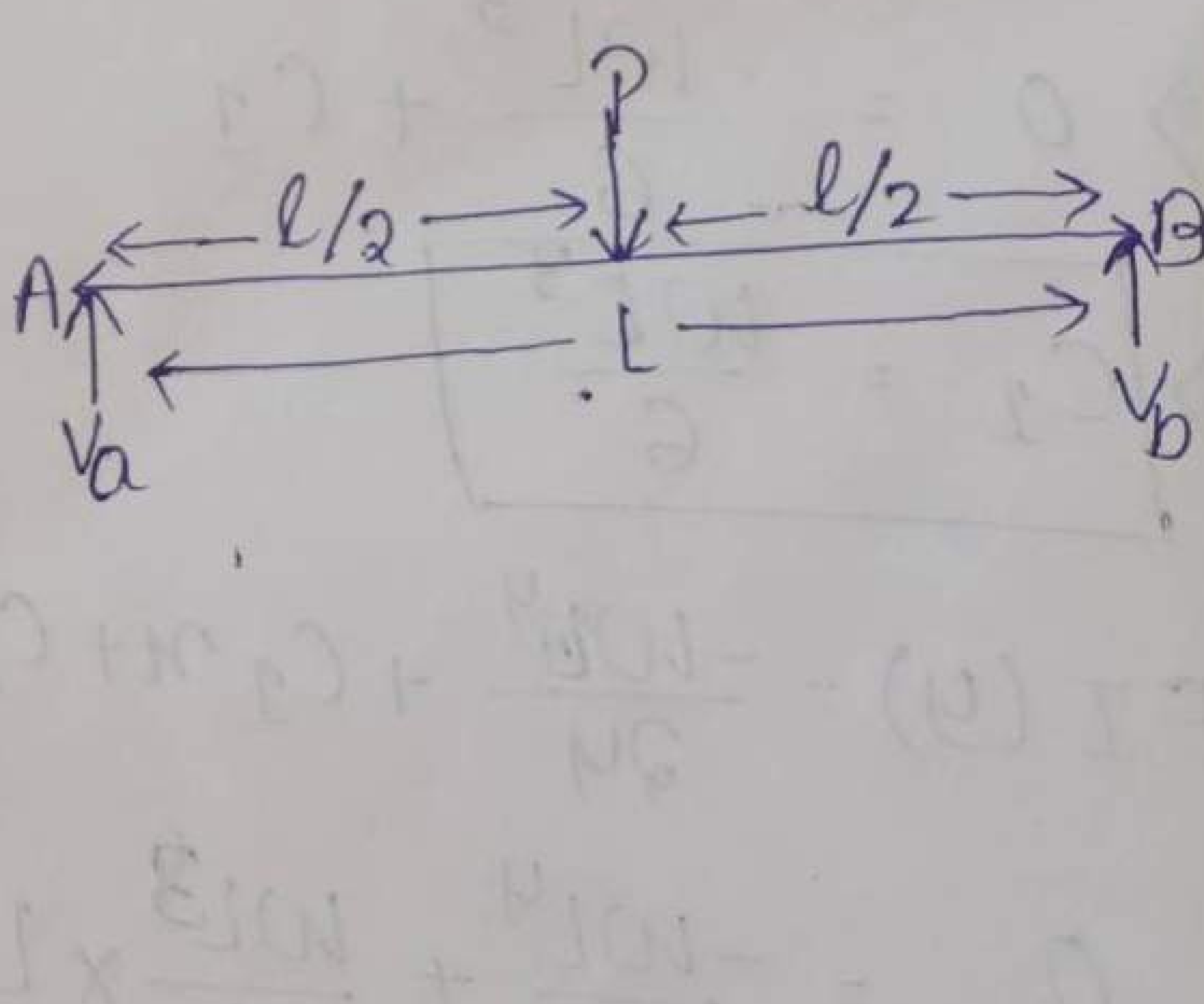
At, $x = 0$

$$EI(y) = -\frac{w\cancel{x^4}}{24} + \cancel{C_1 x} + C_2$$

$$\Rightarrow EI(y) = -\frac{wL^4}{8}$$

$$\boxed{(y)_{max} = \frac{-wL^4}{8EI}}$$

Slope and deflection of simply supported for point load:



$\Sigma V = 0$

$V_a + V_b = P$

$\Sigma M_B = 0$

$V_a \times L - P \times L/2 = 0$

$\boxed{V_a = P/2}$ $\boxed{V_b = P/2}$

$M = P/2\, x$

$$\frac{P}{2}x = \int EI\frac{d^2y}{dx^2}$$

$$\int \frac{Px^2}{4} + \int C_1 = \int EI\left(\frac{dy}{dx}\right)$$

$$\frac{Px^3}{12} + C_1 x + C_2 = EI(y) \quad -(ii) \rightarrow \text{Deflection}$$

At, $x = 0$, $y = 0$

At, $x = L/2$, $\frac{dy}{dx} = 0$

$$EI\left(\frac{dy}{dx}\right) = \frac{Px^2}{4} + C_1$$

$$0 = \frac{P(L/2)^2}{4} + C_1$$

$$\boxed{C_1 = \frac{-PL^2}{16}}$$

$$\frac{Px^3}{12} + C_1 x + C_2 = EI(y)$$

$$0 + 0 + C_2 = EI(0)$$

$$\boxed{C_2 = 0}$$

ximum

$$EI\frac{dy}{dx} = \frac{Px^2}{4} + C_1$$

, $x = 0$

$$EI\left(\frac{dy}{dx}\right) = \frac{Px^2}{4} + C_1$$

$$EI\left(\frac{dy}{dx}\right) = 0 + C_1$$

$$\left(\frac{dy}{dx}\right) \quad = \frac{-PL^2}{}$$

At, $x = L/2$

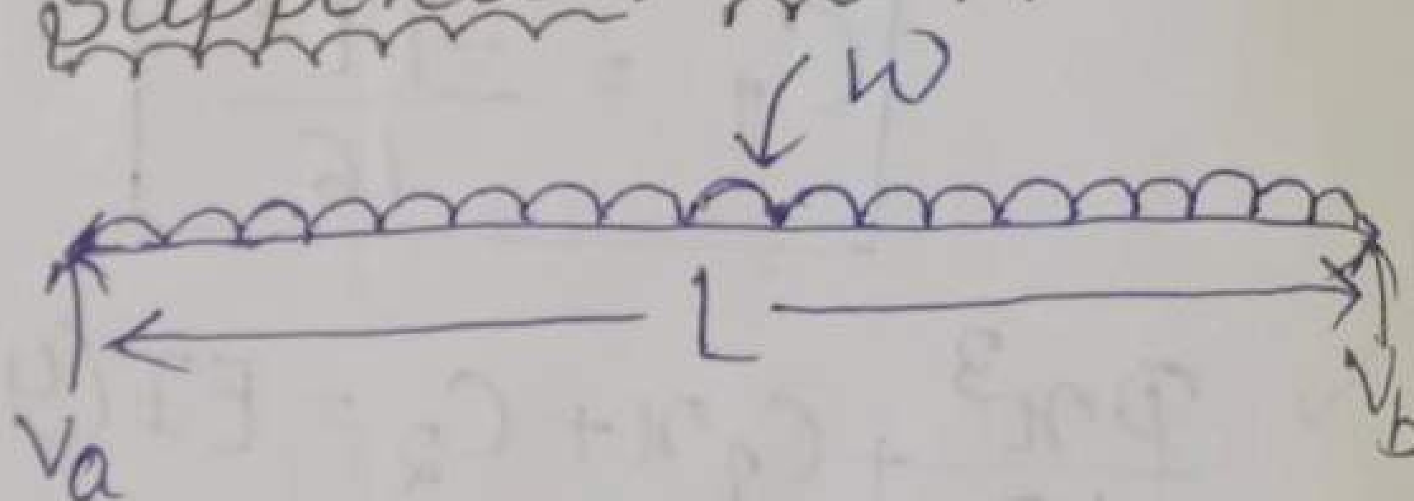
$$\frac{Px^3}{12} + C_1 x + C_2 = EI(y)$$

$$\Rightarrow \frac{P(L/2)^3}{12} + \frac{-PL^2}{16} \times \frac{L}{2} + 0 = EI(y)$$

$$\Rightarrow \frac{PL^3}{96} - \frac{PL^3}{32} = EI(y)$$

$$\boxed{(y)_{max} = \frac{-PL^3}{48EI}}$$

lope and deflection of simply supported for UDL:



$\Sigma V = 0$

$V_a + V_b = WL$

$\Sigma M_A = 0$

$V_a \times L - W \times L \times \frac{L}{2} = 0$

$$\boxed{V_a = \frac{WL}{2}, \quad V_b = \frac{WL}{2}}$$

$$M = \frac{WL}{2} x - \frac{Wx^2}{2}$$

$$\frac{WL}{2} x - \int \frac{Wx^2}{2} = \int EI\left(\frac{d^2y}{dx^2}\right)$$

$$\frac{WL}{2} \times \frac{x^2}{2} - \frac{Wx^3}{6} + C_1 = EI\left(\frac{dy}{dx}\right)$$

$$\frac{WLx^2}{4} - \int \frac{Wx^3}{6} + \int C_1 = \int EI\left(\frac{dy}{dx}\right)$$

At, $x = 0, y = 0$

$x = L/2, \frac{dy}{dx} = 0$

$$\frac{wL \times (L/2)^2}{4} - \frac{w(L/2)^3}{6} + C_1 = EI\left(\frac{dy}{dx}\right)$$

$$\frac{wL^3}{16} - \frac{wL^3}{48} + C_1 = EI(0)$$

$$\frac{2wL^3}{48} + C_1 = 0$$

$$\boxed{C_1 = \frac{-wL^3}{24}}$$

$$\frac{wLx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2 = EI(y)$$

$$\boxed{C_2 = 0}$$

<u>maximum</u>

At, $x = 0$

$$EI\left(\frac{dy}{dx}\right) = \frac{wLx^2}{4} - \frac{wLx^3}{6} + C_1$$

$$EI\left(\frac{dy}{dx}\right) = \frac{-wL^3}{24}$$

$$\boxed{\left(\frac{dy}{dx}\right)_{max} = \frac{-wL^3}{24EI}}$$

At, $x = L/2$

$$EI(y) = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2$$

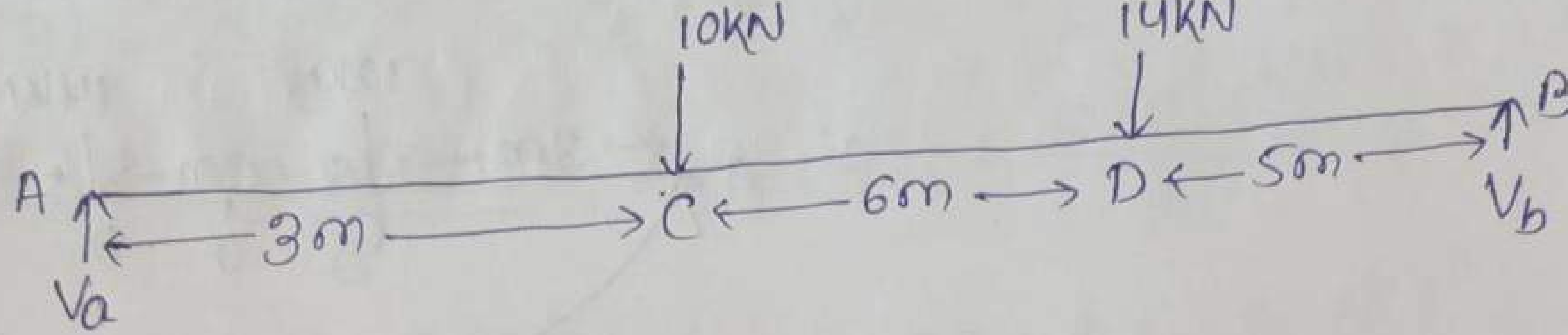
$$EI(y) = \frac{WL^4}{96} - \frac{WL^4}{384} - \frac{WL^4}{48}$$

$$EI(y) = \frac{-5WL^4}{384}$$

$$\boxed{y = \frac{-5WL^4}{384EI}}$$

S. NO	Slope	deflection
Cantilever point load	$\frac{PL^2}{2EI}$	$\frac{PL^3}{3EI}$
Cantilever U.D.L	$\frac{WL^3}{6EI}$	$\frac{WL^4}{8EI}$
Simply supported point load	$\frac{PL^2}{16EI}$	$\frac{PL^3}{48EI}$
Simply supported (UDL)	$\frac{WL^3}{24EI}$	$\frac{5WL^4}{384EI}$

Macaulay's method



Ans $\Sigma V = 0$

$\Rightarrow V_a + V_b = 24KN$

$\Sigma M_B = 0$

$\Rightarrow V_a \times 14 - 10 \times 11 - 14 \times 5 = 0$

$\Rightarrow V_a \times 14 = 180$

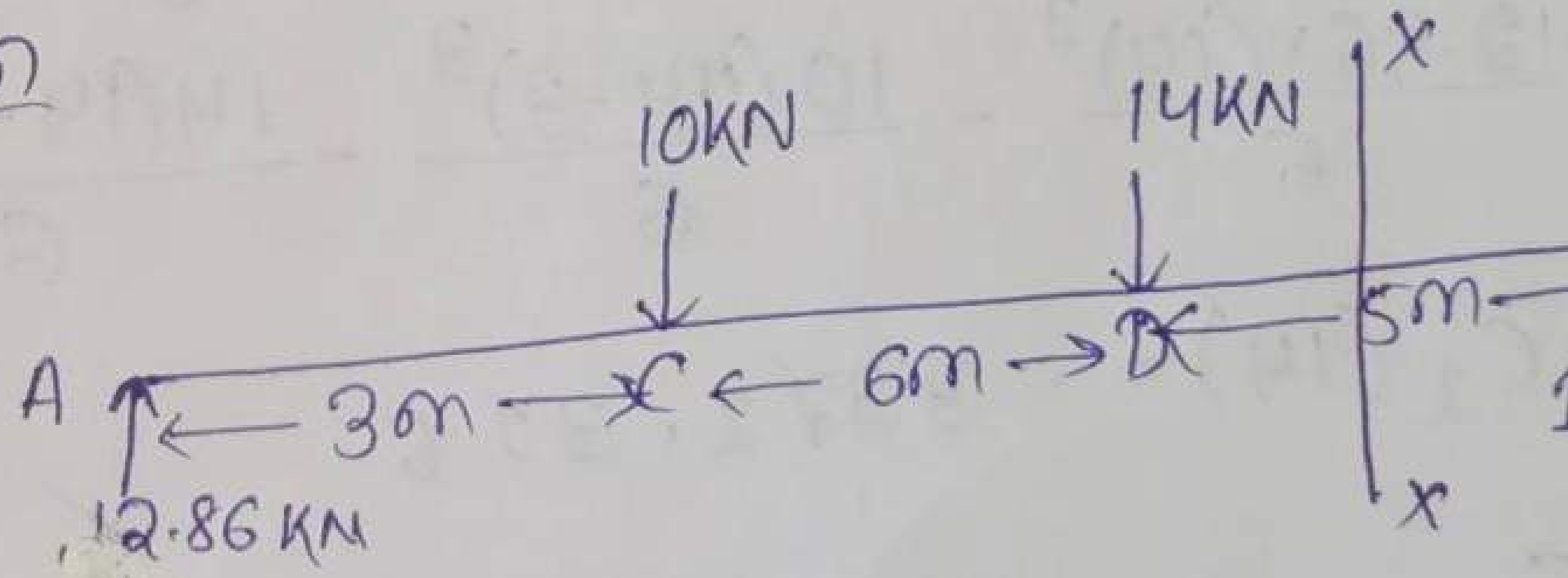
$\Rightarrow V_a = \frac{180}{14}$

$\Rightarrow V_a = 12.86 KN.$

$V_b = 24 - 12.86 = 11.14 KN$

Moment curvature relation

$M = EI \frac{d^2y}{dx^2}$



$M = 12.86x - 10(x-3) - 14(x-9)$

$\Rightarrow 12.86x - 10(x-3) - 14(x-9) = EI \frac{d^2y}{dx^2}$

$\Rightarrow \int 12.86x - \int 10(x-3) - \int 14(x-9) = \int EI \frac{d^2y}{dx^2}$

$\Rightarrow \frac{12.86x^2}{2} - \frac{10(x-3)^2}{2} - \frac{14(x-9)^2}{2} + C_1 = EI \frac{dy}{dx} \quad -(1)$

$\int \frac{12.86x^2}{2} - \int \frac{10(x-3)^2}{2} - \int \frac{14(x-9)^2}{2} + \int C_1 = \int EI \frac{dy}{dx}$

$\frac{12.86x^3}{} - \frac{10(x-3)^3}{} - \frac{14(x-9)^3}{} + C_1 x + C_2 = EI(y) -$

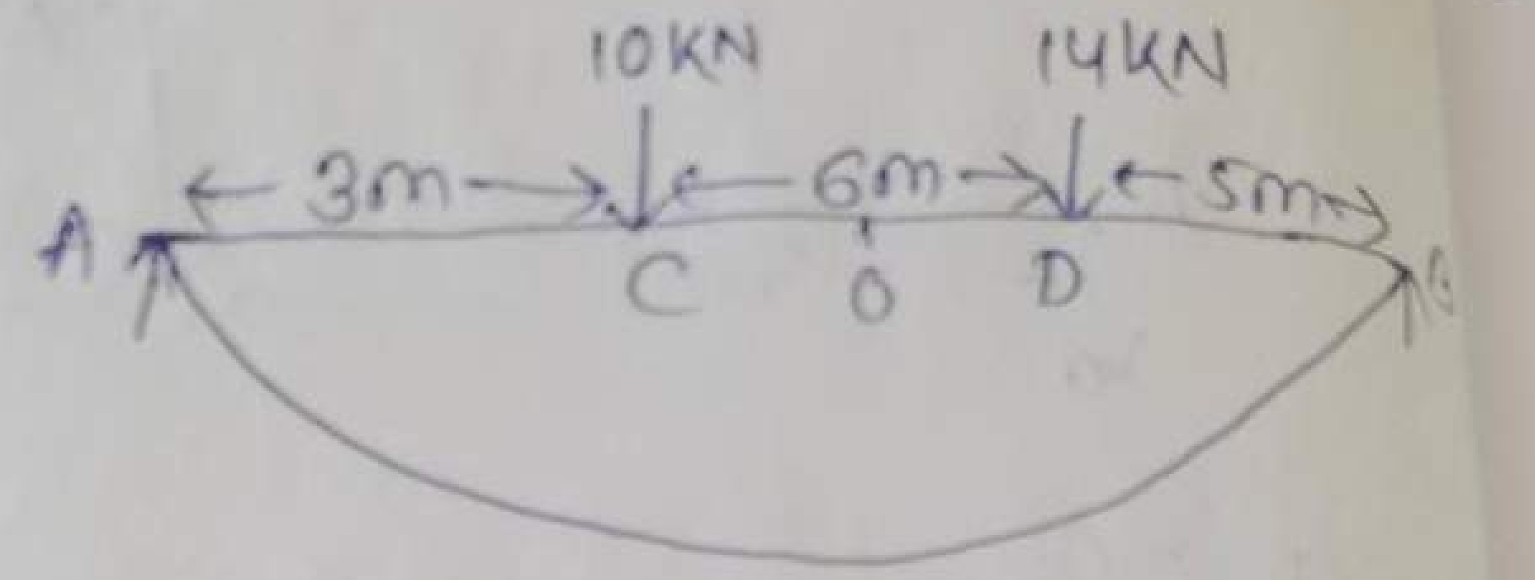
low, the boundary condition

$x = 0$

$y = 0$

$x = L$

$y = 0$



$x = 0, \quad y = 0$

$$\frac{2.86x^3}{6} + C_1 x + C_2 = EI(y)$$

$$\Rightarrow 0 + 0 + C_2 = EI(0)$$

$$\Rightarrow C_2 = 0$$

$x = 14, \ y = 0$

$$\frac{2.86x^3}{6} - \frac{10(x-3)^3}{6} - \frac{14(x-9)^3}{6} + C_1 x + C_2 = EI(y)$$

$$\frac{12.86 \times (14)^3}{6} - \frac{10(14-3)^3}{6} - \frac{14(14-9)^3}{6} + C_1 \times 14 + 0 = EI(0)$$

$$C_1(14) = -3371.316$$

$$C_1 = -240.81$$

eflection at C, $x = 3m$

$$\frac{2.86x^3}{6} + C_1 x + \cancel{C_2}^{0} = EI(y)$$

$$\frac{12.86(3)^3}{6} + (-240.81) \times 3 = EI(y)$$

$$-664.56 = EI(y)$$

ection at D $x = 9m$

$$EI(y) = \frac{12.86x^3}{6} + C_1x + C_2 - \frac{10(x-3)^3}{6}$$

$$EI(y) = \frac{12.86(9)^3}{6} + C_1(9) - \frac{10(9-3)^3}{6}$$

$$= \frac{12.86(9)^3}{6} - 240.81 \times 9 - \frac{10(6)^3}{6}$$

$$= -964.8$$

$$\boxed{y_D = \frac{-964.8}{EI}}$$

ection at mid $x = 7m$

$$(y) = \frac{12.86x^3}{6} + C_1x + C_2 - \frac{10(x-3)^3}{6}$$

$$(y) = \frac{12.86 \times (7)^3}{6} - 240.81(7) + 0 - \frac{10(7-3)^3}{6}$$

$$(y) = -1057.17$$

$$\boxed{= \frac{-1057.17}{EI}}$$

Indeterminate beam

Statically determinate structure:

The structure having unknown force is less than or equal to available equilibrium equation is called statically determinate structure.

Statically indeterminate structure:

The structure having unknown force more than available equilibrium equation is called statically indeterminate structure.

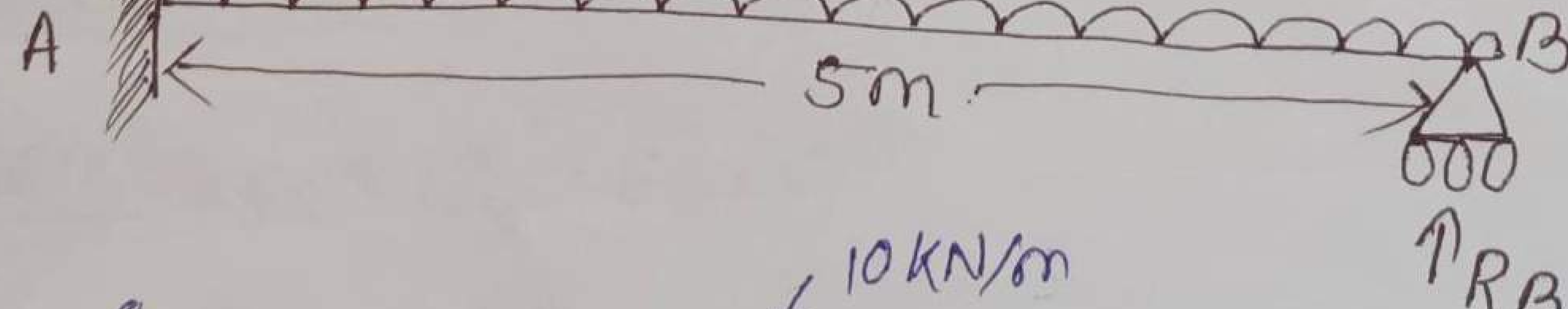
Principle of superposition:

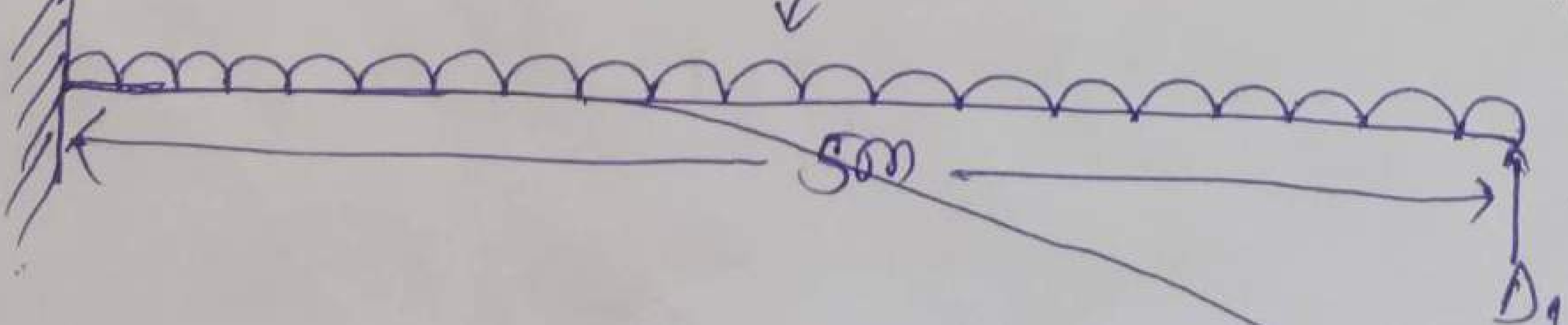
It simply states that on a linear elastic structure the combined effect of several load action simultaneously equal to algebraic sum of the effect of each load acting individually.

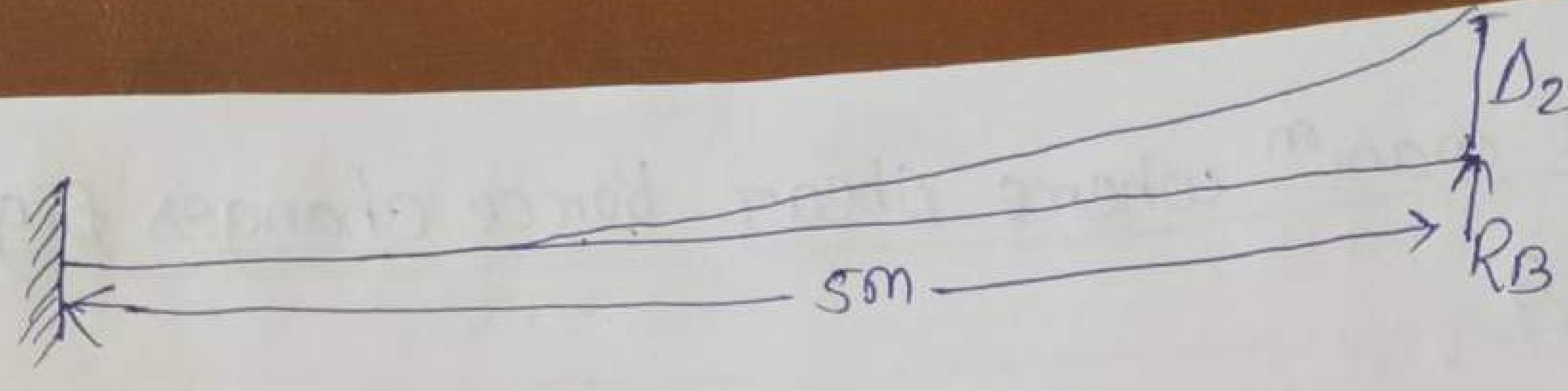
Method of consistent deformation:

It is a force method which is used to analyse indeterminate beam with degree of indeterminancy 1 or 2.

Propped cantilever beam:







$\Delta_1 - \Delta_2 = 0$

$\Delta_1 = \Delta_2$

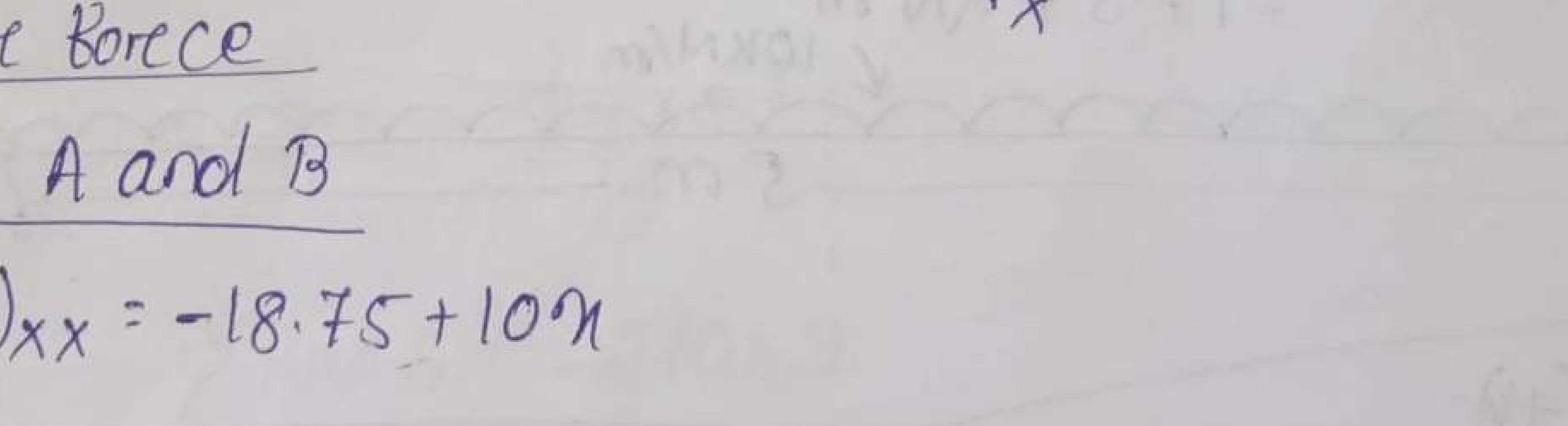
$\Rightarrow \dfrac{w l^4}{8EI} = \dfrac{R_B \times l^3}{3EI}$

$\Rightarrow R_B = \dfrac{3wl}{8}$

$\Rightarrow R_B = \dfrac{3}{8} \times 10 \times 5$

$\Rightarrow R_B = \dfrac{75}{4}$

$\Rightarrow R_B = 18.75 \, KN$



Shear Force

Betn A and B

$(S.F)_{xx} = -18.75 + 10x$

At B

$(S.F)_B = -18.75 + 10 \times 0 \qquad x = 0$

$= -18.75 \, KN$

At A

$(S.F)_A = -18.75 + 10 \times 5$

B.M will max^m where shear force changes sign.

2

Bet^n A & B

$(B.M)_{XX} = 18.75x - 10 \times x \times \frac{x}{2}$

At B $x = 0$

$(B.M)_B = 0$

At A $x = 5$

$(B.M)_A = 18.75 \times 5 - 10 \times 5 \times \frac{5}{2}$

$= -31.25$ KN-m

$\frac{d}{dx}(B.M_{XX}) = 18.75 - 10x = 0$

$18.75 - 10x = 0$

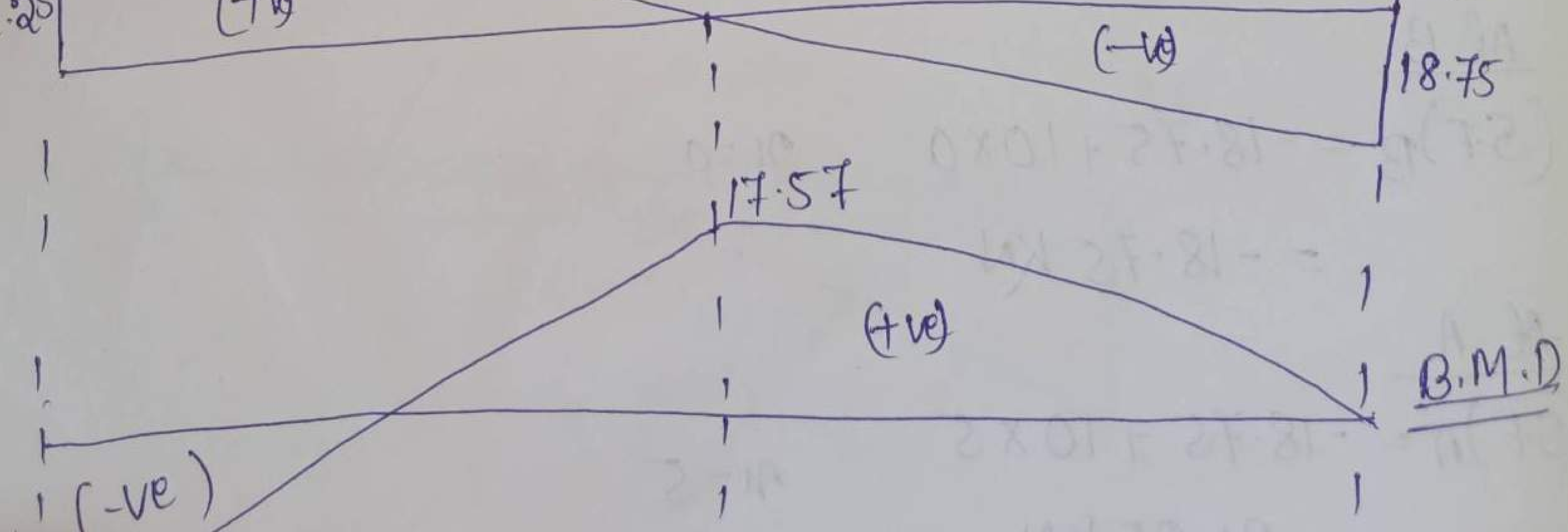
$x = \frac{18.75}{10}$

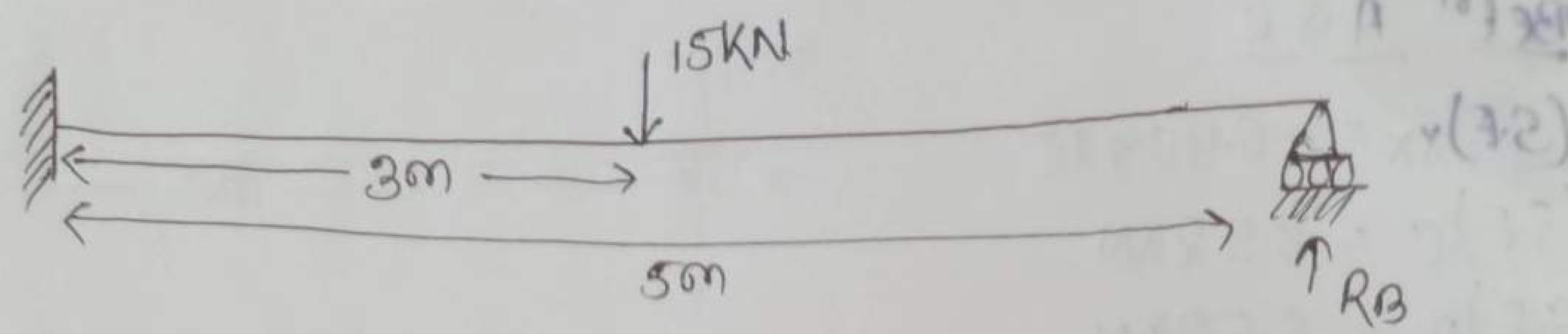
$x = 1.875$ m

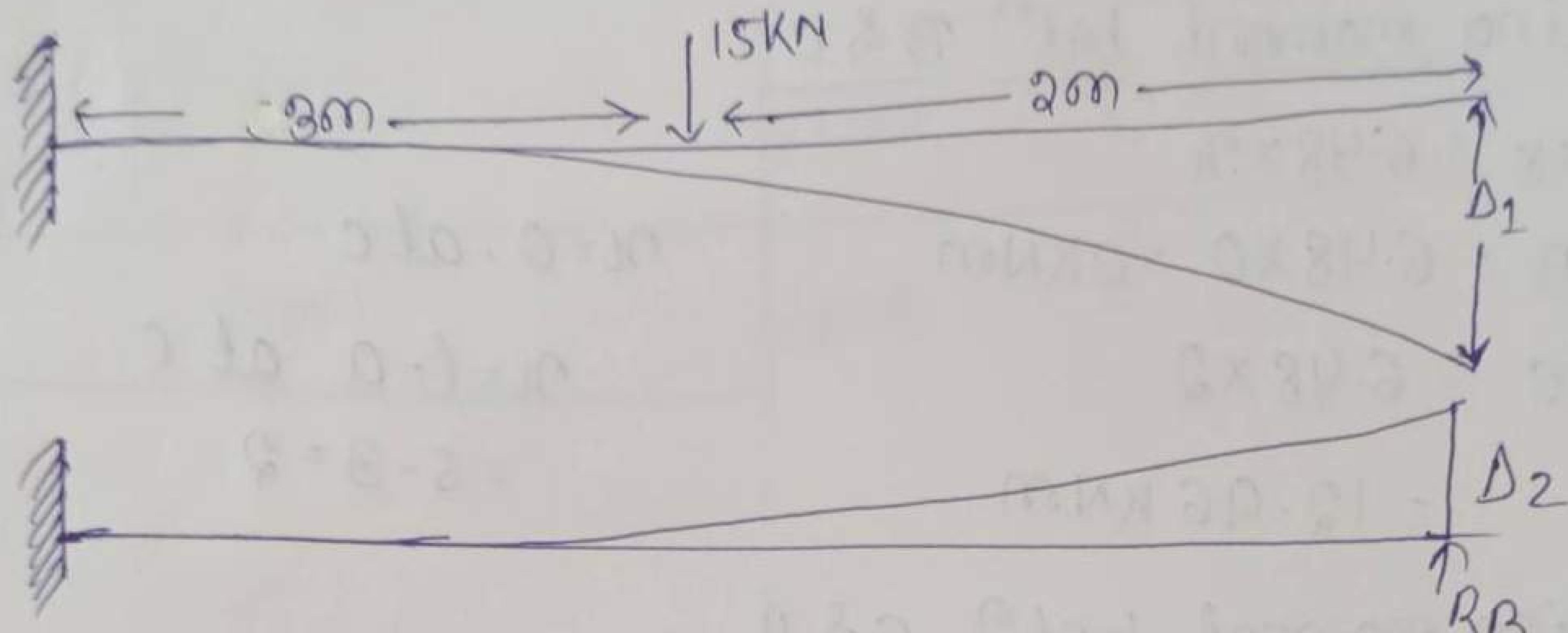
$(B.M)_{max} = 18.75 \times 1.875 - 10 \times 1.875 \times \frac{1.875}{2}$

$= 17.57$ KN m

Diagram :







$$\Delta_1 - \Delta_2 = 0$$

$$\Delta_1 = \Delta_2$$

$$\cancel{150\times(3)}\ \frac{Pa^3}{3EI} + \frac{Pa^2}{2EI}(l-a) = \frac{R_B\times(5)^3}{3EI}$$

$$\Rightarrow \frac{15\times(3)^3}{3\cancel{EI}} + \frac{15\times 3^2}{2\cancel{EI}}(5-3) = \frac{R_B\times 5^3}{3\cancel{EI}}$$

$$\Rightarrow 270 = \frac{RB\times 5^3}{3}$$

$$\Rightarrow R_B\times 5^3 = 270\times 3$$

$$\Rightarrow R_B = \frac{270\times 3}{5^3} = 6.48\ KN$$

KN

Shearforce diagram Betn C & B

et^n A & C

$S.F)_{xx} = -6.48 + 15$

$.F)_C = 8.52 KN$

$F)_A = 8.52 KN$

ending moment bet^n B & C

$.M)_{xx} = 6.48 \times x$

$.M)_B = 6.48 \times 0 = 0 KNm$

$x = 0$, at C

$.M)_C = 6.48 \times 2$

$x = l - a$ at C

$= 5 - 3 = 2$

$= 12.96 KNm$

ending moment bet^n C & A

$B.M)_{xx} = 6.48 \times x - 15(x-2)$

$B.M)_C = 6.48 \times 2 - 15(2-2)$

$= 12.96 KNm$

$B.M)_A = 6.48 \times 5 - 15(5-2)$

$= -12.6 KNm$.

Diagram :