LECTURTER NOTES ON ENGINEERING MATHEMATICS-III

(FOR ELECTRICAL ENGINEERING BRANCH)

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Chapter-I

COMPLEX NUMBER

A number of the form z=x+iy, where x and y are real number, is called a complex number.

Here $i = \sqrt{-1}$

and is an imaginary number.

Algebra of complex number:-

if $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are two complex number then $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$ and $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$z_1 * z_2 = (x_1 * x_2 - y_1 * y_2) + i(x_1 * y_2 + x_2 * y_1)$

Ex:- if z_1 =2+i3 and z_2 =5+i9 are two complex numbers then $z_1 + z_2 = (2 + 5) + i(3 + 9) = 7 + i12$

and $z_1 - z_2 = (2 - 5) + i(3 - 9) = -3 - i6$

$z_1 * z_2 = (x_1 * x_2 - y_1 * y_2) + i(x_1 * y_2 + x_2 * y_1) = 2 * 3 - 5 * 9 + i(5 * 3 + 2 * 9) = -39 + i33$

if a+ib=c+id then a=c and b=d

In$z = x + iy$ x is called as the real part of z and y is called as imaginary part of z.

Symbolically Re z=a and Im z=b.

ex:- z =2+i3 here Re z=2 and Im z=3

If $z = x + iy$ then $\frac{1}{z} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

z =2+i3 then $\frac{1}{z} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2} = \frac{2}{2^2+3^2} - i\frac{3}{2^2+3^2} = \frac{2}{13} - i\frac{3}{13}$

Complex number set is not well ordered as like real number.

Complex conjugate:-

If $z = x + iy$ then $\bar{z} = x - iy$

z =2+i3 then $\bar{z} = x - iy = 2 - i3$

Hence $\overline{(\bar{z})} = z$ and z. $\bar{z} = x^2 + y^2 = \bar{z}.z$ and also $\bar{z}.z = 0$ if and only if $z = 0$

If $z = x + iy$ then $\sqrt{x^2 + y^2}$ is called as the modulus of z and argument of z is $\theta = \tan^{-1}\frac{y}{x}$

z =2+i3 then modulus of $z = \sqrt{x^2 + y^2} = \sqrt{13}$ and argument of z is $\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{3}{2}$

Square root of a complex number

Find the square root of the complex number 3+i4

Solution:- Let z=x+iy be the square root of 3+i4

=>$z^2 = (x + iy)^2 = x^2 - y^2 + i2xy$

=> $z^2 = x^2 - y^2 + i2xy = 3 + i4$

=> $x^2 - y^2 = 3$ & $2xy = 4$

=> $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2 = 9 + 16 = 25$

=> $x^2 + y^2 = \pm 5$

Now $2x^2 = 5 + 3 = 8$ => $x^2 = 4$ => $x = \pm 2$

=> $2y^2 = 2$ => $y^2 = 1$ => $y = \pm 1$

Therefore $\sqrt{3 + i4} = \pm 2 \pm i$

Cube root of unity:-

$z^3 = 1$ then z=1,ω, ω^2 are the cube root of unity

Properties of cube root of unity

$1 + \omega + \omega^2 = 0$ & $\omega^3 = 1$

Example:- If $1, \omega, \omega^2$are the cube root of unity then prove that $(1 + w - w^2)^6 + (1 - w + w^2)^6 = 128$

Solution:-

L.H.S $(1 + w - w^2)^6 + (1 - w + w^2)^6$

=$(-w^2 - w^2)^6 + (-w - w)^6$

$= (-2w^2)^6 + (-2w)^6$

$= 2^6(w^{12} + w^6)$

$= 64(1 + 1) = 64 * 2 = 128$

Hence proved

De-movire's Theorem:-

If $z = cos\theta + isin\theta$ then $z^n = cosn\theta + isinn\theta$

Some questions:-

Short answer question.

1. Write in a+ib form $\frac{2-3i}{2+4i}$
2. Find the argument and modulus of the complex number 3+i5
3. Find the real part and imaginary part of a+ib
4. Find the complex conjugate of 2-8i

5. (2+3i)+(3-4i)=______
6. (5+4i)(2-5i)=______

Long answer type question

1. Find the square root of 2+i3
2. If z = (cosθ + $i$sinθ) , show that $z_n + \frac{1}{z_n} = 2\cos n\theta$ and $z_n - \frac{1}{z_n} = i2\sin n\theta$
3. If 1,w, w^2 are the cube root of unity then prove that $(1+w-w^2)^6 + (1-w+w^2)^6 = 128$
4. If 1,w, w^2 are the cube root of unity then prove that
$$(1+w)(1+w^2)(1+w^4)\ldots\ldots(1+w)^{2^{11}} = 1$$
5. If 1,w, w^2 are the cube root of unity then prove that then $(1+w)^3 - (1+w^2)^3 = 0$

Chapter-II

Rank of a matrix

A number r is said to be the rank of a non zero mxn matrix A. If

a. There is atleast one r x r sub matrix of A whose determinant is not equal to zero.
b. The determinant of every (r+1) rowed square sub-matrix in A is zero.
In other words, rank of a matrix is the greatest possible +ve integer r such that A has at least one non zero minor of order r.

Example:-

Find the rank of $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

Solution: - Let A=$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

$|A| = 1(15-16) - 2(10-12) + 3(8-9) = -1 + 4 - 3 = 0$

Here A is a singular matrix in which there is atleast one 2 x 2 sub-matrix for example $A_1 = \begin{pmatrix} 3 & 4 \\ 4 & 5 \end{pmatrix}$

|A|=$15 - 16 = -1 \neq 0$

Hence rank of the matrix is 2.

Elementary transformation:-

Elementary matrix:- The matrix obtained from a unit matrix I after one or more elementary row/column operations are called elementary matrix.

Canonical matrix: - The matrix obtained by applying a series of elementary row/column operations such that there are some non zero rows in the top and the remaining rows consist of all zeros is called a canonical matrix

By an elementary transformation of matrix any one of the following operations are hold good.

i) Interchanging any two rows or columns

ii) Multiplication of the ith rows by a non zero number k.

iii) Adding a row or column by multiplying a non zero k to a row or column.

Example: - find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{pmatrix}$

Sol: -A=$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & -2 & -4 \end{pmatrix}$ $(R_2 \to R_2 - 3R_1 \; and \; R_3 \to R_3 - 4R_1)$

$\sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{pmatrix}$ $(R_3 \to R_3 - R_2)$

System of linear equation:-

Consider the following ‘m’ equations with ‘n’ unknowns :

$a_{11}x_1 + a_{12}x_2 + \cdots \ldots \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots \ldots \ldots + a_{2n}x_n = b_2$

………… ……….

$a_{m1}x_1 + a_{m2}x_2 + \cdots \ldots \ldots + a_{mn}x_n = b_n$

Which is in matrix form AX=B

Here A is called coefficient matrix and B is called right hand side matrix and K= [A|B] which is the augumented matrix.

If the elements in B are not all zero, then the system of linear equation is non-homogenous. Otherwise it is called homogeneous system.

Rouche’s Theorem:-

$a_{11}x_1 + a_{12}x_2 + \cdots \ldots \ldots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots \dots \dots + a_{2n}x_n = b_2$

………… ……….

$a_{m1}x_1 + a_{m2}x_2 + \cdots \dots \dots + a_{mn}x_n = b_n$

This system of equation is consistent iff the coefficient matrix A and the augumented matrix K are of same rank otherwise the system is inconsistent.

i) Rank of A=Rank of K=r (r ≤ the smaller of the numbers m and n). The system is consistent and have infinite number of solution.

ii) If rank of A= rank of K= n then the system is consistent and have unique solution

iii) If Rank of A ≠ Rank of K then the system is in consistent and have no solution.

Example: - solve $2x - 3y + z = 1, x + 2y - 3z = 4$ *and* $4x - y - 2z = 8$

The system of equation can be written as

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & -3 \\ 4 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$$

Here A= $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 2 & -3 \\ 4 & -1 & -2 \end{bmatrix}$; **B=** $\begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$ and K = $\begin{bmatrix} 2 & -3 & 1 & 1 \\ 1 & 2 & -3 & 4 \\ 4 & -1 & -2 & 8 \end{bmatrix}$

Now K = $\begin{bmatrix} 2 & -3 & 1 & 1 \\ 1 & 2 & -3 & 4 \\ 4 & -1 & -2 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 4 \\ 2 & -3 & 1 & 1 \\ 4 & -1 & -2 & 8 \end{bmatrix}$ (exchanging row one by column one)

$\sim \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 7 & -7 \\ 0 & 5 & -4 & 6 \end{bmatrix} (R_2 \to R_2 - 2R_1 \; and \; R_3 \to R_3 - 2R_2)$

$\sim \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 5 & -4 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} (R_3 \to R_3 - 5R_2)$

Hence rank of K= rank of A =3

Hence the system is consistent and have unique solution.

The system can be written as $x + 2y - 3z = 4, y - z = 1$ and $z = 1$

Hence $x = 3, y = 2$ and $z = 1$

Some problems:-

Short answer type questions

1. Define upper triangular matrix with an example.
2. Define row reduced echelon form of a matrix.
3. Define Rouche's theorm.

4. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$
5. Define rank of a matrix.

Long answer type question

1. Solve $x+2y-z=3;$
   $3x-y+2z=1;$
   $2x-2y+3z=2$
2. For what value of γ *and* μ do the system of equations $x+y+z=6$
   $x+2y+3z=10$
   $x+2y+\gamma=\mu$
   have i) no solution ii) unique solution iii) infinite solutions
3. Solve the system of linear equation.
   $x-y+z=0$
   $x+2y-z=0$
   $2x+y+3z=0$
4. Test the consistency of the linear equation $5x+3y+7z=4;$
   $3x+26y+2z=13;$
   $7x+2y+10z=5$

Differential Equation

Defⁿ:- A eqⁿ consisting of independent variable, dependent variable or variables and diff. derivatives of dependent variables w.r.t independent variable.

Order of diff. eqⁿ:-

The order of diff. eqⁿ is the order of the highest order derivative occouring in it.

Ex:- $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ order - 2

Degree of diff eqⁿ:-

The degree of a diff eqⁿ is the highest exponent of the highest order derivative after the eqⁿ has been freed from radicals and fractions as far as derivatives are concerned.

Ex:- $\frac{dy}{dx} + x = 0$ order-1 ; deg-1

$\frac{d^2y}{dx^2} = K\left[1+\left(\frac{dy}{dx}\right)^2\right]$ order-2 ; deg-1.

Linear diff. eqⁿ:-

A diff. eqⁿ is said to be linear if

a) every dependent variable & it's derivative are of first degree.

b) Dependent variables & its derivatives are not multiplied together.

Ex:- $\frac{dy}{dx} = x$; $\frac{d^2y}{dx^2} = \sin x$; $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$ are linear

But $\left(\frac{dy}{dx}\right)^2 = x+y$; $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$ are nonlinear.

Homogeneous Linear Differential Equation:-

The general linear diff. eqn is

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}}{dx^{n-1}} y + \cdots + a_{n-1} \frac{dy}{dx} + a_n y = f(x) \quad (1)$$

$a_0 \neq 0$. : coefficient a_i; $i = 0,1,2 \ldots$ are consts & $f(x)$ is func of x

if $f(x) = 0$ then eqn (1) become a Homogeneous

Soln of Linear diff eqn:-

For Homogeneous Only to find Complementary func (C.F.)

for non-Homo. Soln is Complementary func + Particular Integral (P.I)

Procedure for finding C.F.:-

$f(D)y = 0$ be a homogeneous linear diff eqn

* $f(m) = 0$ is the auxillary eqn.

find m in $f(m) = 0$

then rules for C.F is

i) If all roots are diff. i.e $m_1, m_2 \ldots m_n$ are real & diff.

then $C.F = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \cdots + c_n e^{m_n x}$.

ii) if m_1 repeated K times then

$C.F = (c_1 + c_2 x + \cdots + c_K x^{K-1}) e^{m_1 x} + c_{K+1} e^{m_{K+1} x} + \cdots + c_n e^{m_n x}$.

iii) suppose $f(m) = 0$ has two complex roots $m_1 = \alpha + i\beta$ & $m_2 = \alpha - i\beta$

then $y\,(C.F) = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

Solve ① $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

$\left[\because D = \frac{d}{dx}\right]$

Symbolically $D^2y - 5Dy + 6y = 0$

$\Rightarrow (D^2 - 5D + 6)y = 0$

Now Auxillary eqn: $m^2 - 5m + 6 = 0$

$\Rightarrow (m-2)(m-3) = 0$

$\therefore m = 2 \ \& \ 3$

$\therefore C.F = C_1 e^{2x} + C_2 e^{3x}$

② $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

Symbolically $D^2y - 4Dy + 4y = 0$

$\Rightarrow (D^2 - 4D + 4)y = 0$

Auxillary eqn $m^2 - 4m + 4 = 0$

$\Rightarrow (m-2)^2 = 0$

$\therefore m = 2, 2.$

$C.F = (C_1 + C_2 x)e^{2x}$

③ $\frac{d^3y}{dx^3} + y = 0$

Symbolically $D^3y + y = 0$

$\Rightarrow (D^3 + 1)y = 0$

A.E. $m^3 + 1 = 0$

$\Rightarrow (m+1)(m^2 - m + 1) = 0$

$\therefore m = -1 \ ; \ m = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{3}i}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$\therefore m = -1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

$\therefore C.F = C_1 e^{-x} + e^{x/2}\left[C_2 \cos\frac{\sqrt{3}}{2}x + C_3 \sin\frac{\sqrt{3}}{2}x\right]$

Particular Integral
(Non Homogeneous Linear diff eqn)

Sup: The non Homogeneous linear diff eqn is
$f(D)y = f(x)$

then $P.I = \frac{1}{f(D)} f(x)$

Case-1 Suppose $f(x) = e^{ax}$
In this case replace D by a in f(D).
P.I will be
if $f(a) = 0$ then P.I

$$P.I = \frac{x}{f'(D)} f(x)$$

again replace D by a in $f'(D)$.
If $f'(a) = 0$ (repeat as above)

Ex ① Find PI $(D^2-4)y = e^x$

$$PI = \frac{1}{D^2-4} e^x = \frac{1}{1-4} e^x = -\frac{e^x}{3}$$

② $(D^2-4)y = e^{2x}$

$$PI = \frac{1}{D^2-4} e^{2x} = \frac{1}{4-4} e^{2x} \quad [f(2) = 0]$$

$$\therefore PI = \frac{x}{2D} e^{2x} = \frac{x}{4} e^{2x}.$$

For nonHomogeneous linear diff eqn
soln is C.F + P.I.
for C.F follow the procedure as
in Homogeneous eqn (linear diff)

Case-2 Suppose $f(D)y = \cos ax / \sin ax$ be a nonHomogenous linear diff eqn

To find P.I.

$PI = \frac{1}{f(D)} \cos ax$ Replace D^2 by $-a^2$.

if $\frac{1}{f(D)} = \frac{1}{0}$ after replacement

then $PI = \frac{x}{f'(D)} \cos ax$

then repeat Same Procedure.

Ex PI for $(D^2+4)y = \cos 2x$.

$PI = \frac{1}{D^2+4} \cos 2x$ as $f(D) = 0$ after replacing D^2 by -4. i.e. $-4+4=0$

$\therefore PI = \frac{x}{2D} \cos 2x$

$= \frac{x}{2} \int \cos 2x \, dx$

$= \frac{x}{2} \cdot \frac{\sin 2x}{2} + c$

$= \frac{x \sin 2x}{4} + c$

$= \frac{xD}{+D^2} \cdot \cos 2x$

$\frac{xD}{-4} \cos 2x$

$-\frac{1}{4} \cdot D(x \cos 2x)$

$-\frac{1}{4}[\cos 2x + x \cdot 2 \sin 2x]$

$\frac{\cos 2x}{4} + \frac{x}{2} \sin 2x$

(2) find the soln of $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \cos 2x$.

As this is non Homogenous, we find C.F & P.I.

C.F

Auxillary Eqn $m^2 + m + 1 = 0$

$\therefore m = \frac{-1 \pm \sqrt{1 - 4.1.1}}{2.1} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$\therefore C.F = e^{-\frac{1}{2}x}\left[c_1 \cos \frac{\sqrt{3}}{2} + c_2 \sin \frac{\sqrt{3}}{2}\right]$

$$P.I = \frac{1}{D^2+D+1} \cdot \cos 2x.$$

Replace D^2 by $-a^2 = -4$

$$= \frac{1}{-4+D+1} \cdot \cos 2x = \frac{1}{D-3} \cdot \cos 2x$$

$$= \frac{D+3}{(D-3)(D+3)} \cdot \cos 2x$$ [∵ there is no D^2 term]

$$= \frac{D+3}{D^2-9} \cdot \cos 2x$$

$$= \frac{D+3}{-4-9} \cdot \cos 2x$$

$$= \frac{D+3}{-13} \cos 2x$$

$$= -\frac{1}{13}\left(D.\cos 2x + 3\cos 2x\right)$$

$$= -1/13\left(-2\sin 2x + 3\cos 2x\right)$$

∴ General sol^n $= C.F + P.I$

$$= e^{-1/2 x}\left(C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x\right) - 1/13\left(-2\sin 2x + 3\cos 2x\right)$$

Partial differential eqⁿ (P.D.E.)

A partial diff eqⁿ is a relⁿ betⁿ the independent variables, dependent variable, & its partial derivatives.

Ex: $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 2u + 3xy$

If $z = f(x, y)$ x, y - independent variable
z - dependent variable.

Then $\frac{\partial z}{\partial x} = p$; $\frac{\partial z}{\partial y} = q$; $\frac{\partial^2 z}{\partial x^2} = r$

$$\frac{\partial^2 z}{\partial x \cdot \partial y} = \frac{\partial^2 z}{\partial y \partial x} = s; \quad \frac{\partial^2 z}{\partial y^2} = t$$

Formation of P.D.E :-

① Form a P.D.E. $z = ax + by + a^2 + b^2$ —— ①

Diff both sides partially w.r.t. x, we get-

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left[ax + by + a^2 + b^2\right]$$

$$= a \cdot \frac{\partial x}{\partial x} + b \frac{\partial y}{\partial x} + 0 + 0 \quad (\because y \to \text{constant})$$

$$= a \cdot 1$$

Again diff ① w.r.t. y partially, we get-

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left[ax + by + a^2 + b^2\right]$$

$$= a \cdot \frac{\partial x}{\partial y} + b \cdot \frac{\partial y}{\partial y} + 0 + 0$$

$$= b.$$

putting value of a & b in ①, we get-

$$z = x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

$$= x \cdot p + y \cdot q + p^2 + q^2$$

(2) $z = f(x^2 - y^2)$ —— (1)

Diff both side w.r.t. x partially, we get.

$$\frac{\partial z}{\partial x} = f'(x^2-y^2) \cdot \frac{\partial}{\partial x}(x^2-y^2)$$

$$= f'(x^2-y^2) \cdot 2x \quad (\because y \text{ is const.}) \quad \text{—— (2)}$$

Diff both side partially w.r.t y, we get.

$$\frac{\partial z}{\partial y} = f'(x^2-y^2) \cdot \frac{\partial}{\partial y}(x^2-y^2)$$

$$= f'(x^2-y^2) \cdot (-2y) \quad (\because x \text{ is const}) \quad \text{—— (3)}$$

Divide eqn (2) by eqn (3)

$$\frac{\partial z/\partial x}{\partial z/\partial y} = \frac{f'(x^2-y^2) \cdot 2x}{f'(x^2-y^2)(-2y)}$$

$$\Rightarrow \frac{p}{q} = -x/y \Rightarrow py = -qx$$

$$\Rightarrow \boxed{py + qx = 0}$$

<u>Exercise</u> $z = f(xy/z)$

<u>Linear Diff. eqn of 1st order:-</u>

To find the soln to linear diff. eqn of the form $Pp + Qq = R$.

<u>Procedure:-</u>

(1) write the subsidiary eqn as

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

then solve ~~for~~ ~~x,y,z~~ by taking any pair

Ex-1
Solve $p\sqrt{x}+q\sqrt{y}=\sqrt{z}$ — ①

$\equiv pP+q.Q=R$

$P=\sqrt{x};\ Q=\sqrt{y};\ R=\sqrt{z}$.

The subsidiary eqn to eqn ① is

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$$

$$\Rightarrow \frac{dx}{\sqrt{x}}=\frac{dy}{\sqrt{y}}=\frac{dz}{\sqrt{z}} \quad —①$$

Considering first two ratio, we get.

$$\frac{dx}{\sqrt{x}}=\frac{dy}{\sqrt{y}}$$

Integrate bothside. $\int\frac{dx}{\sqrt{x}}=\int\frac{dy}{\sqrt{y}}$

$$\Rightarrow \frac{x^{-1/2+1}}{-1/2+1}=\frac{y^{-1/2+1}}{-1/2+1}+C_1$$

$$\Rightarrow \frac{\sqrt{x}}{1/2}=\frac{\sqrt{y}}{1/2}+C_1$$

$$\Rightarrow \sqrt{x}=\sqrt{y}+\sqrt{2}C_1$$

$$\Rightarrow \sqrt{x}-\sqrt{y}=C_2. \quad — ②$$

Considering 2nd & 3rd ratio, we get.

$$\frac{dy}{\sqrt{y}}=\frac{dz}{\sqrt{z}}$$

On integration $\int\frac{dy}{\sqrt{y}}=\int\frac{dz}{\sqrt{z}}$

$$\Rightarrow \sqrt{y}=\sqrt{z}+C_3$$

$$\Rightarrow \sqrt{y}-\sqrt{z}=C_3 \quad — ③$$

Required soln $\phi(\sqrt{x}-\sqrt{y},\sqrt{y}-\sqrt{z})=0$

$$\Rightarrow \sqrt{x}-\sqrt{y}=f(\sqrt{y}-\sqrt{z})$$

(Long) (2) $x(y-z).p + y(z-x).q = z(x-y)$

Subsidiary eq^n $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$

$P = x(y-z)$; $Q = y(z-x)$; $R = z(x-y)$

Now we'll find P', Q', R' s.t. $PP' + QQ' + RR' = 0$

$\therefore$ $P' = 1$; $Q' = 1$; $R' = 1$.

as $x(y-z) + y(z-x) + z(x-y)$

$= xy - xz + yz - yx + zx - zy = 0$.

Hence $P'dx + Q'dy + R'dz = 0$ is integrable.

$\Rightarrow dx + dy + dz = 0$ is integrable

On integration $x + y + z = c_1$ —— (1)

Next, we'll find P'', Q'', R'' s.t. $PP'' + QQ'' + RR'' = 0$.

$\therefore$ $P'' = \frac{1}{x}$; $Q'' = \frac{1}{y}$; $R'' = \frac{1}{z}$.

as $\frac{1}{x}.x(y-z) + \frac{1}{y}.y(z-x) + \frac{1}{z}z(x-y)$

$= y - z + z - x + x - y = 0$

Hence $P''dx + Q''dy + R''dz = 0$ is integrable

$\Rightarrow \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$ is integrable

On integration $\int \frac{1}{x}.dx + \int \frac{1}{y}.dy + \int \frac{1}{z}.dz = C_2$

$\Rightarrow \ln x + \ln y + \ln z = C_2$

$\Rightarrow \ln(xyz) = C_2$

$\Rightarrow xyz = e^{C_2} = C_3$. —— (2)

Sol^n is $\phi(x+y+z, xyz) = 0$

$\Rightarrow x+y+z = f(xyz)$

Laplace's Transform:-

Gamma function:-

i) Γ(n+1)=nΓ(n)
ii) Γ(n+1)=n! For a +ve integer n.
iii) $\Gamma(\frac{1}{2})=\sqrt{\pi}$

Ex:-Γ(5)=Γ(4+1)=4!=24

Γ(6)=5!=120

$\Gamma(\frac{5}{2})=\Gamma(\frac{3}{2}+1)=\frac{3}{2}\Gamma(\frac{3}{2})=\frac{3}{2}\,\Gamma(\frac{1}{2}+1)=\frac{3}{2}.\,\frac{1}{2}\,\Gamma(\frac{1}{2})=\frac{3}{2}.\,\frac{1}{2}.\,\sqrt{\pi}=\frac{3}{4}\sqrt{\pi}$

$\Gamma(\frac{9}{2})=\Gamma(\frac{7}{2}+1)=\frac{7}{2}\Gamma\left(\frac{7}{2}\right)=\frac{7}{2}\Gamma\left(\frac{5}{2}+1\right)=\frac{7}{2}\frac{5}{2}\,\Gamma(\frac{5}{2})=\frac{7}{2}\frac{5}{2}\frac{3}{4}\sqrt{\pi}=\frac{105}{16}\sqrt{\pi}$

Laplace's Transform:-

Let f(t) is a function of real variable t>0 then the Laplace's Transform of f(t) is

$L[f(t)]=\int_0^{\infty} e^{-st} f(t)dt$ =F(s) and s is a parameter which may be real or complex.

$L[\sin t]=\int_0^{\infty} e^{-st}\sin t\,dt=\frac{1}{s^2+1}$

Laplace's Transform of some standard function:-

i) $L[c]=\frac{c}{s}$; where c is a constant
ii) $L[t]=\frac{1}{s^2}$
iii) $L[t^n]=\frac{n!}{s^{n+1}}$ s>0, n= 0,1,2,......
iv) $L[e^{at}]=\frac{1}{s-a}$ s>a
v) $L[e^{-at}]=\frac{1}{s+a}$
vi) $L[\sin at]=\frac{a}{s^2+a^2}$
vii) $L[\cos at]=\frac{s}{s^2+a^2}$

Ex:- $L[1]=\frac{1}{s}$; $L[t^2]=\frac{2!}{s^{2+1}}=\frac{2}{s^3}$

L[sin 2t]= L[cos 4t] $L[e^{-3t}]$=

<u>Linearity Property of Laplace's Transform</u>:-

If f(t) and g(t) are two function of t, then

L[af(t)∓bg(t)]=aL[f(t)]∓bL[g(t)] where a and b are scalars (constants)

L[sin2t+cos3t]=L[sin 2t]+L[cos 3t]=$\frac{2}{s^2+4}+\frac{s}{s^2+9}=\frac{2(s^2+9)+s(s^2+4)}{(s^2+4)(s^2+9)}=\frac{2s^2+18+s^3+4s}{(s^2+4)(s^2+9)}$

L[$4e^{3t}$]= 4L[e^{3t}]=4.$\frac{1}{s-3}$

<u>Shifting Property:-</u>

If L[f(t)]= F(s) then

L[e^{at}f(t)]=F(s-a), s-a>0

EX:- L[$e^{3t}\sin 2t$]

Here f(t)= sin 2t

L[f(t)]= L[sin 2t]= $\frac{2}{s^2+4}$ =F(s) (2 marks)

By using shifting Property,

L[$e^{3t}\sin 2t$]= F(s-3)= $\frac{2}{(s-3)^2+4}=\frac{2}{s^2+9-6s+4}=\frac{2}{s^2-6s+13}$ (3 marks)

L[$sin^2 t$]=L[$\frac{\cos 2t-1}{2}$]=L[$\frac{1}{2}\cos 2t-\frac{1}{2}$]=$\frac{1}{2}$ L[cos 2t]- $\frac{1}{2}$ $L[1]=\frac{1}{2}\frac{s}{s^2+4}-\frac{1}{2}\frac{1}{s}=\frac{s^2-(s^2+4)}{2s(s^2+4)}=\frac{-4}{2s(s^2+4)}$

{Cos 2t = $cos^2 t-sin^2 t=1-sin^2 t-sin^2 t=1-2sin^2 t$=> $sin^2 t=\frac{\cos 2t-1}{2}$ }

{Sin 3t= 3sin t-4$sin^3 t$ => $-4sin^3 t=\sin 3t-3sint$ => $sin^3 t=\frac{3\sin t-\sin 3t}{4}$}

Laplace's Transform of $t^n f(t)$:

if L[f(t)]=F(s)

L[$t^n f(t)$]= $(-1)^n\frac{d^n}{ds^n}[F(s)]$

Ex:- find the laplace's transform of t.sint

Here n=1 & f(t)= sint

L[sin t]= $\frac{1}{s^2+1}$

Now L[t sin t]=$(-1)^1\frac{d}{ds}\left[\frac{1}{s^2+1}\right]$ =-1.$\frac{0.(s^2+1)-1(2s)}{(s^2+1)^2}$ = - $\frac{-2s}{(s^2+1)^2}=\frac{2s}{(s^2+1)^2}$

Ex:- 2 L[t. e^{2t}]

L[e^{2t}] = $\frac{1}{s-2}$

L[t. e^{2t}] = $(-1)^1\frac{d}{ds}\frac{1}{s-2}$ = -1$\frac{0.(s-2)-1(1)}{(s-2)^2}=\frac{1}{(s-2)^2}$

Laplace's Transform of $\frac{1}{t}f(t)$:-

If L[f(t)]= F(s) then $L[\frac{1}{t}f(t)] = \int_s^{\infty} F(s)ds$

Ex:- 1 find $L[\frac{\sin t}{t}] = L[\frac{1}{t}\sin t]$

Here f(t)= sin t

L[f(t)]= L[sin t]= $\frac{1}{s^2+1}$

$L[\frac{1}{t}\sin t] = \int_s^{\infty} \frac{1}{s^2+1}$ ds $= tan^{-1}s]_s^{\infty} = tan^{-1}\infty - tan^{-1}s = \pi/2 - tan^{-1}s$

<u>Laplace Transform of the nth derivative</u>:-

$L[f^{(n)}(t)] = s^n\ L[f(t)] - s^{n-1}\ f(0) - s^{n-2}\ f'(0) - \ldots\ldots\ldots f^{(n-1)}(0)$

$L[f^{(1)}(t) = sL[f(t)] - f(0)$

$L[f^{(2)}(t)] = s^2\ L[f(t)] - s^1\ f(0) - f'(0)$

Ex:- L[cos t]= L[(sin t)']=sL[sin t]-sin 0 = $s.\frac{1}{s^2+1} - 0 = \frac{s}{s^2+1}$

<u>Laplace's Transform of integration</u>:-

If L[f(t)]= F(s) then $L\{\int_0^t f(t)dt\} = \frac{1}{s}L[f(t)] = \frac{1}{s}\ F(s)$

Ex:- L[sin t]= $L[\int_0^t \cos t\ dt\}] = \frac{1}{s}L[\cos t] = \frac{1}{s}\frac{s}{s^2+1} = \frac{1}{s^2+1}$

<u>Inverse Laplace Transform:-</u>

i) $L[c] = \frac{c}{s}$; where c is a constant $\quad L^{-1}[\frac{c}{s}] = c$

ii) $L[t] = \frac{1}{s^2}$ $\quad L^{-1}[\frac{1}{s^2}] = t$

iii) $L[t^n] = \frac{n!}{s^{n+1}}$ s>0, n= 0,1,2,...... $\quad L^{-1}[\frac{n!}{s^{n+1}}] = t^n$

iv) $L[e^{at}] = \frac{1}{s-a}$ s>a $\quad L^{-1}[\frac{1}{s-a}] = e^{at}$

v) $L[e^{-at}] = \frac{1}{s+a}$ $\quad L^{-1}[\frac{1}{s+a}] = e^{-at}$

vi) $L[\sin at] = \frac{a}{s^2+a^2}$ $\quad L^{-1}[\frac{a}{s^2+a^2}] = \sin at$

vii) $L[\cos at] = \frac{s}{s^2+a^2}$ $\quad L^{-1}[\frac{s}{s^2+a^2}] = \cos at$

viii) $L[\sinh at] = \frac{a}{s^2-a^2}$ $\quad L^{-1}[\frac{a}{s^2-a^2}] = \sinh at$

ix) $L[\cosh at] = \frac{s}{s^2-a^2}$ $\quad L^{-1}[\frac{s}{s^2-a^2}] = \cosh at$

Inverse Laplace Transform also satisfies linearity property:-

L^{-1} [aF(s)+bG(s)]= $aL^{-1}[F(s)]$+b$L^{-1}[G(s)]$

L^{-1} [aF(s)-bG(s)]= $aL^{-1}[F(s)]$-b$L^{-1}[G(s)]$

<u>Shifting Property:-</u>

If $L^{-1}[F(s)] = f(t)\ then\ L^{-1}[F(s-a)] = e^{at}f(t)$

Ex:- $L^{-1}[\frac{2s}{s^2+9}] = 2.\ L^{-1}[\frac{s}{s^2+3^2}]$=2. Cos 3t

2. $L^{-1}[\frac{3s}{s^2+2s-8}] = L^{-1}[\frac{3s}{s^2+2s.1+1^2-9}] = L^{-1}[\frac{3s}{(s+1)^2-3^2}]= L^{-1}[\frac{\{3(s+1)-3\}}{(s+1)^2-3^2}]$

$= L^{-1}\left[\frac{3(s+1)}{(s+1)^2-3^2}\right] - L^{-1}\left[\frac{3}{(s+1)^2-3^2}\right]$ $[\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}]$

$= 3\ L^{-1}\left[\frac{(s+1)}{(s+1)^2-3^2}\right]$ - $L^{-1}\left[\frac{3}{(s+1)^2-3^2}\right]$

We know that $L^{-1}\left[\frac{s}{s^2-3^2}\right]$ = cosh 3t and $L^{-1}\left[\frac{3}{s^2-3^2}\right]$ = sinh 3t

By using shifting property,

=3e^{-t}cosh 3t-e^{-t}sinh 3t

Imp:-

Ex:- y''+5y'+6y=e^{2x},$y(0) = 2, y'(0) = -1$

Taking the Laplace's Transform on both side, we get

L[y''+5y'+6y]=L[e^{2x}]

=>L[y'']+5L[y']+6L[y]= L[e^{2x}] {Using Linearity Property}

=>{s^2 $L[y]$- s^1 y(0)- y'(0)}+5{ sL[y]-y(0)}+6L[y]=$\frac{1}{s-2}$ {Laplace's Transform of derivative}

=>s^2 $L[y]$- s^1 2+1+5sL[y]-10+6L[y]= $\frac{1}{s-2}$

=>L[y]{ s^2 $+ 5s + 6\} - 2s - 9$=$\frac{1}{s-2}$

=>L[y] {s^2 $+ 5s + 6$}=$\frac{1}{s-2}$+2s+9

=> L[y] {s^2 $+ 5s + 6$}= $\frac{1+(2s+9)(s-2)}{s-2} = \frac{1+2s^2+5s-18}{s-2} = \frac{2s^2+5s-17}{s-2}$

=>L[y]= $\frac{2s^2+5s-17}{(s-2)(s^2+5s+6)}$=$\frac{2s^2+5s-17}{(s-2)(s+2)(s+3)}$

Let's consider

$$\frac{2s^2+5s-17}{(s-2)(s+2)(s+3)} = \frac{A}{(s-2)} + \frac{B}{s+2} + \frac{C}{s+3} = \frac{A(s+2)(s+3)+B(s-2)(s+3)+C(s-2)(s+2)}{(s-2)(s+2)(s+3)}$$

$=> 2s^2 + 5s - 17 = A(s+2)(s+3) + B(s-2)(s+3) + C(s-2)(s+2)$

Put s=-2=>2.4+5(-2)-17=0+B*-4*1+0=>8-10-17=-4B=>B=19/4

Put s=-3=> 18-15-17=5C=>5C=-14=>C=14/5

Put s=2=>1=20A=>A=1/20

Therefore $\frac{2s^2+5s-17}{(s-2)(s+2)(s+3)} = \frac{\frac{1}{20}}{s-2} + \frac{\frac{19}{4}}{s+2} + \frac{\frac{14}{5}}{s+3}$

Therefore L[y]= $\frac{\frac{1}{20}}{s-2} + \frac{\frac{19}{4}}{s+2} + \frac{\frac{14}{5}}{s+3}$

=>y= $L^{-1}[\frac{\frac{1}{20}}{s-2} + \frac{\frac{19}{4}}{s+2} + \frac{\frac{14}{5}}{s+3}]$

=>y= $\frac{1}{20} L^{-1}\left[\frac{1}{s-2}\right] + \frac{19}{4} L^{-1}\left[\frac{1}{s+2}\right] + \frac{14}{5} L^{-1}[\frac{1}{s+3}]$

=>y= $\frac{1}{20} e^{2x} + \frac{19}{4} e^{-2x} + \frac{14}{5} e^{-3x}$

NUMERICAL METHODS

Bisection Method

Example :- Find a root of the following equation using bisection method correct to two decimal places (i) $x^3 - 5x + 1 = 0$ which lies between 2 & 3.

Solution:- Let f(x)=$x^3 - 5x + 1$

Since f(2) is –ve and f(3) is +ve, a root lies between 2 & 3.

First approximation to the root is $x_1 = \frac{2+3}{2} = 2.5$

$$f(x_1) = 2.5^3 - 5 * 2.5 + 1 = 4.125 > 0$$

Hence the root lies between 2.5 and 2

Second approximation to the root is $x_2 = \frac{2+2.5}{2} = 2.25$

$$f(x_2) = 2.25^3 - 5 * 2.25 + 1 = 1.140625 > 0$$

Hence the root lies between 2.25 and 2

Third approximation to the root is $x_3 = \frac{2+2.25}{2} = 2.125$

$$f(x_3) = 2.125^3 - 5 * 2.125 + 1 = -0.0293 > 0$$

This implies that the root lies between 2.125 and 2.25

Fourth approximation to the root is $x_4 = \frac{2.125+2.25}{2} = 2.1875$

So the desired root is 2.19 approximately.

EX:- Find a root of an equation $\boldsymbol{f(x) = x^3 - x - 1}$ using Bisection method

Solution:Here x3-x-1=0Let f(x)=$\boldsymbol{x^3 - x - 1}$

Since f(1) is –ve and f(2) is +ve, a root lies between 1 & 2.

First approximation to the root is $x_1 = \frac{1+2}{2} = 1.5$

$$f(x_1) = 1.5^3 - 1.5 - 1 = 0.875 > 0$$

Hence the root lies between 1 and 1.5

Second approximation to the root is $x_2 = \frac{1+1.5}{2} = 1.25$

$$f(x_2) = 1.25^3 - 1.25 - 1 = \boldsymbol{-0.29688} < 0$$

Hence the root lies between 1.25 and 1.5

Third approximation to the root is $x_3 = \frac{1.25+1.5}{2} = 1.375$

$$f(x_3) = 1.375^3 - 1.375 - 1 = \boldsymbol{0.22461} > 0$$

This implies that the root lies between 1.375 and 1.25

Fourth approximation to the root is $x_4 = \frac{1.25+1.375}{2} = 1.3125$

So the desired root is 1.31 approximately.

Newton Rapson's Method:-

Newton Raphson method Steps (Rule)	
Step-1:	Find points a and b such that $a<b$ and $f(a)\cdot f(b)<0$.
Step-2:	Take the interval $[a,b]$ and find next value $x_0=\frac{a+b}{2}$
Step-3:	Find $f(x_0)$ and $f'(x_0)$ $$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$
Step-4:	If $f(x_1)=0$ then x_1 is an exact root, else $x_0=x_1$
Step-5:	Repeat steps 2 to 4 until $f(x_i)=0$ or $\lvert f(x_i)\rvert \le$ Accuracy

Example: - Find by Newton's method, a root of the equation $x^3-3x+1=0$ correct to 3 decimal places.

solution:- let f(x)=$x^3-5x+1 f'(x)=3x^2-3$

$$f(1)-1<0 \ and \ f(2)=3>0$$

The root lies between 1 & 2.

Let $x_0=1.5$

By using Newton's formula

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=1.53333$$

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}=1.532$$

Hence the approximate root is 1.532

Exercise: -Find a root of an equation $f(x)=x^3-x+1$ using Newton Raphson method

Evaluate $\sqrt{28}$ using Newton's iteration method.

Solution:- Let N=28. Then by using Newton's iteration formula

$$x_{n+1}=\frac{1}{2}(x_n+\frac{28}{x_n})$$

Sincean approximate value of $\sqrt{28}=5$,

$$\text{We take } x_0=5$$

$$x_1=\frac{1}{2}\left(x_0+\frac{28}{x_0}\right)=\frac{1}{2}\left(5+\frac{28}{5}\right)=5.3$$

$$x_2=\frac{1}{2}\left(x_1+\frac{28}{x_1}\right)=\frac{1}{2}\left(5.3+\frac{28}{5.3}\right)=5.29150$$

$$x_3=\frac{1}{2}\left(x_2+\frac{28}{x_2}\right)=\frac{1}{2}\left(5.29150+\frac{28}{5.29150}\right)=5.29151$$

Hence $\sqrt{28} = 5.2915$

Operators: -

i) Shift Operators:-Ef(x)=f(x+h)

ii) Forward difference operator Δf(x)=f(x+h)-f(x)

iii) Backward difference operator ∇f(x)=f(x)-f(x-h)

Relation between operators

Δf(x)=f(x+h)-f(x)=f(x+h)-f(x)=Ef(x)-f(x)=(E-1)f(x)

⇨ Δ=E-1

Evaluate $\Delta^n(a^x)$

Solution: -

$$\Delta^n(a^x) = \Delta^{n-1}\{\Delta(a^x)\}$$
$$= \Delta^{n-1}[a^{x+1} - a^x]$$
$$= \Delta^{n-1}.a^x(a-1)$$
$$= (a-1)\Delta^{n-1}.a^x$$
$$= (a-1)\Delta^{n-2}\{\Delta(a^x)\}$$
$$= (a-1)\Delta^{n-2}a^x(a-1)$$
$$= (a-1)^2\Delta^{n-2}a^x$$

Proceeding in this way we get; $\Delta^n(a^x) = (a-1)^n a^x$

Example:-By forming a difference table find the missing values in the following table assuming that the fourth differences are equal to zero.

X	0	5	10	15	20	25
y	6	10	?	17	?	31

Solution:- Let's assume the missing value be y_1 & y_2.

The difference table is as follows.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	6				
		4			
5	10		y_1-14		
		y_1-10		41-3y_1	
10	y_1		27-2y_1		y_2+6y_1-102
		17-y_1		y_2+3y_1-61	
15	17		y_2+y_1-34		$-4y_2-4y_1+143$
		y_2-17		$-3y_2-y_1$+82	
20	y_2		48-2y_2		
		31-y_2			
25	31				

As it is given that the fourth approximation is zero i.e$\Delta^4 y$=0

Therefore y_2+$6y_1$-102=0 and $-4y_2 - 4y_1 + 143$=0

y_2+$6y_1$-102=0 ------(1)

$-4y_2 - 4y_1 + 143$=0------(2)

Solve for x & y.

Multiplying equation 1 by 4, we get $4y_2$+$24y_1$-408=0 --------------(3)

Add equation 2 & 3

$20y_1$-265=0 =>y_1 = 265/20=13.25

=> y_2 = 102-6*13.25=102-79.50=22.5

2)find the missing value using difference table.

X	0	1	2	3	4
y	1	3	9	?	81

solution:-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		2			
1	3		4		
		6		c-19	
2	9		c-15		124-4c
		c-9		90-2c-(c-15)	
3	C		90-2c		
		81-c			
4	81				

Therefore the polynomial so formed is of degree 4. Hence 4^{th} difference must be zero.

$\Delta^4 y$ =0 =>124-4c=0=>c=124/4=31

Exercise:-Find the sixth term of the sequence 2,6,12,20,30....

X	0	1	2	3	4	5
y	2	6	12	20	30	?

LAGRANGE INTERPOLATING POLYNOMIAL

Example: -Find the interpolating polynomial by using the given data and find the value of f(3)

X	0	1	2
y	1	3	9

Solution:-

x		Y=f(x)	
x_0	0	y_0=f(x_0)	1
x_1	1	y_1	3
x_2	2	y_2	9

Lagrange interpolating polynomial

P(x)=$\frac{(x-x_1)(x-x_2)\ldots\ldots\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots\ldots\ldots(x_0-x_n)}$ f(x_0)+$\frac{(x-x_0)(x-x_2)\ldots\ldots\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots\ldots\ldots(x_1-x_n)}$ f(x_1)+

.....................+$\frac{(x-x_0)(x-x_1)\ldots\ldots\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots\ldots\ldots\ldots(x_n-x_{n-1})}$ f(x_1)

P(x)=$\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$f($x_0$)+$\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$f($x_1$)+$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$f($x_2$)

=$\frac{(x-1)(x-2)}{(0-1)(0-2)}1+\frac{(x-0)(x-2)}{(1-0)(1-2)}3+\frac{(x-0)(x-1)}{(2-0)(2-1)}9$

=$\frac{x^2-3x+2}{2}+\frac{x^2-2x}{-1}3+\frac{x^2-x}{2}9$

$$=\frac{-(x^2-3x+2)+6(x^2-2x)-9(x^2-x)}{-2}$$

=$\frac{-x^2+3x-2)+(6x^2-12x)-9x^2+9x)}{-2}$

=$\frac{-4x^2-2}{-2}$

=$2x^2+1$

f(5)= $2*3^2+1=2*9+1=19$

Find the interpolating polynomial by using newton's forward method

X	0	1	2	3	4
y	1	3	9	31	81

Solution:-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		2			
1	3		4		
		6		12	
2	9		16		0
		22		12	
3	31		28		
		50			
4	81				

Netwon's forward interpolation formula is

P(x)= $y_0+\frac{x-x_0}{h}\ \Delta y_0$ + $\frac{(x-x_0)(x-x_1)}{2!h^2}\Delta^2 y_0$ +.........+ $\frac{(x-x_0)(x-x_1)........(x-x_{n-1})}{n!h^n}\Delta^n y_0$

=1+$\frac{x-0}{1}$2+$\frac{(x-0)(x-1)}{2.1}$ 4+$\frac{(x-0)(x-1)(x-2)}{3.1}$12

=1+2x+2x^2-2x+$\frac{x(x^2-3x+2)}{}$.4

=1+2x^2+4x^3-12x^2+8x

= 4x^3-10x^2+8x+1