QUESTION BANK ON ENGINEERING MATHEMATICS-I

(COMMON FOR ALL ENGINEERING BRANCH)

PREPARED BY

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DETERMINANT AND MATRIX

1. 02 Marks Questions

I. Evaluate $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$.

II. Solve $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$.

III. Find the minor and cofactor of the elements 4 and 0 in the determinant $\begin{vmatrix} 1 & 2 & -3 \\ 4 & 5 & 0 \\ 2 & -1 & 1 \end{vmatrix}$.

IV. Evaluate $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

V. What is the maximum value of $\begin{vmatrix} \sin x & \cos x \\ -\cos x & 1+sinx \end{vmatrix}$.

VI. Without expanding evaluate $\begin{vmatrix} sin^2\theta & cos^2\theta & 1 \\ cos^2\theta & sin^2\theta & 1 \\ -10 & 12 & 2 \end{vmatrix}$,

VII. Without expanding, find the value of $\begin{vmatrix} 1/a & 1 & bc \\ 1/b & 1 & ca \\ 1/c & 1 & ab \end{vmatrix}$.

VIII. If X + $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, Then find X.

IX. Find x and y, if $\begin{bmatrix} 2 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$.

2. 05 Marks Questions

I. Solve $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix} = 0$

II. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

III. Prove that $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$.

IV. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

V. Prove that $\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$

VI. Prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$

VII. Prove that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

VIII. Prove that $(AB)^T = B^T A^T$, where $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ -1 & -2 \end{bmatrix}$.

IX. Find the adjoint of the matrix $\begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}$

X. If $= \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 1 \\ 3 & 1 & -2 \end{bmatrix}$, find adjoint of A.

XI. Find the inverse of the matrices $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$

XII. Solve by Cramer's rule: $2x - 3y = 8\,,\quad 3x + y = 1$

XIII. Solve by matrix method : $5x - 3y = 1,\quad 3x + 2y = 12$

3. 10 Marks Questions

I. Prove that $\begin{vmatrix} sin^2A & \cot A & 1 \\ sin^2B & \cot B & 1 \\ sin^2C & \cot C & 1 \end{vmatrix} = 0$, where A, B and C are the angles of a triangle.

II. Prove that $\begin{vmatrix} -1 & \cos C & \cos B \\ \cos C & -1 & \cos A \\ \cos B & \cos A & -1 \end{vmatrix} = 0$, where $A + B + C = \pi$.

TRIGNOMETRY AND INVERSE TRIGNOMETRY

1. 02 Marks Question

i. State $\cot 375°$ and $cosec\ 271°$ are positive or negative.

ii. Find the value of $\cos 1° \cos 2° \ldots\ldots\ldots\ldots \cos 100°$.

iii. Find the value of $cos30°\ sin45° + sin90°\ tan60°$.

iv. Find the value of $\frac{\tan 30°}{\sqrt{1-cos^2 45°}}$

v. Calculate $\frac{cos45° + sin45°}{cos45° - sin45°}$

vi. Evaluate $\cos(270° - \theta)\sec(-\theta)\tan(180° - \theta) + \sec(360° + \theta)\sin + cot(90° - \theta)$

vii. Evaluate $\sin 150° + \cos 300° - \tan 315° + \sec 3660°$

viii. Show that $sec^2 135°\ sec^2 30°\ sin^3 90°\ \cos 60° = \frac{4}{3}$

ix. Evaluate $\sin 15°$.

x. Find the value of $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

xi. If $\sin^{-1}\frac{x}{5} + \text{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$. Find the value of x.

xii. Find the value of $\sin(2\sin^{-1} 0.6)$

xiii. Find the value of $\tan\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right)$

xiv. Find the value of $x^{-1}\sin\left(\text{cosec}^{-1}\frac{1}{x}\right)$

xv. Find the value of $\sin\cos^{-1}\tan\sec^{-1}\sqrt{2}$.

2. 05 Marks Question

i. Verify that $\tan 225° \cot 405° + \tan 765° \cot 675° = 0$

ii. Prove that $tan\left(\frac{\pi}{4} - \frac{A}{2}\right) = \sqrt{\frac{1-\sin A}{1+\sin A}} = \sec A - \tan A$

iii. If A, B and C are the angles of a triangle,

Then show that $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

iv. If $A + B + C = \pi$, Prove that $\frac{\sin 2A - \sin 2B + \sin 2C}{sin\ 2A + sin\ 2B - sin\ 2C} = \frac{\tan B}{\tan C}$

v. Prove that $\sin 20°\ sin\ 40°\ sin\ 60°\ sin\ 80° = \frac{3}{16}$

vi. Prove that $\sin 4\theta = 4\sin\theta\ cos^3\theta - 4\cos\theta\ sin^3\theta$

vii. Prove $\sqrt{\frac{1-cos\theta}{1+cos\theta}} = cosec\theta - cot\theta$

viii. Prove $\frac{1}{cosec\theta - cot\theta} - \frac{1}{sin\theta} = \frac{1}{sin\theta} - \frac{1}{cosec\theta + cot\theta}$

ix. Prove $sin^6\theta + cos^6\theta = 1 - 3sin^2\theta . cos^2\theta$

x. Find the value of $tan75°$ and hence prove that $tan75° + cot75° = 4$

xi. Prove that $\tan 70^{\circ} = 2\tan 50^{\circ} + \tan 20^{\circ}$

xii. If $A + B = 45^{\circ}$, show that $(1 + \tan A)(1 + \tan B) = 2$

xiii. Prove that $\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

xiv. Prove that $\cot^{-1}\frac{pq+1}{p-q} + \cot^{-1}\frac{qr+1}{q-r} + \cot^{-1}\frac{rp+1}{r-p} = 0$

3. 10 Marks Question

i. Prove that $\sin 3A + sin\ 2A - \sin A = 4\sin A\ \cos\frac{3A}{2}\cos\frac{A}{2}$

ii. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$,

then, show that, $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$.

TWO DIMENSIONAL GEOMETRY AND CIRCLE

1. 02 Mark Questions

i. Find the distance between two points (3, –4) and (3, 5).

ii. Find slope, x intercept and y intercept of the line. $x - 2y + 4 = 0$.

iii. Determine the area of the triangle with the vertices at (0,0), (4,0) and (4,10).

iv. Find the slope of the line which makes an angle of 45° with x-axis.

v. Find the slope of a line which passes through the points (3, 2) and (–1, 5).

vi. Determine x so that the line passing through (3, 4) and (x, 5) makes 135° angle with the positive direction of x –axis.

vii. If A(–2, 1), B(2, 3) and C(–2, –4) are three points, find the angle between BA and BC.

viii. Find slope of the line whose equation is y+2=0.

ix. Find slope of the line joining the point (-k,-k) and the origin.

x. Find the slope a line perpendicular to the line joining the points (6, 4) and (2, 12).

xi. Without using Pythagoras theorem, show that the points A(0, 4), B(1, 2) and C(3, 3) are the vertices of a right angled triangle.

xii. Find the equation of a line with slope 2 and y-intercept is 3.

xiii. Find the equation of a straight line cutting off an intercept of -1 units on negative direction of y –axis and being equally inclined to the axis.

xiv. Determine the equation of a line through the point(–4, –3) and parallel to x –axis.

xv. Find the point of intersection of the lines whose equations are $x + 3 = 0$ and $y - 4 = 0$.

xvi. Determine the x-intercept and y- intercept of the line $y = 2x + 3$.

xvii. Find the equation of the line passing through the origin and parallel to the line $y = 3x + 4$.

xviii. Find the equation of the line passing through the origin and perpendicular to the line $y = -2x + 4$.

xix. Find the equation of the line which is at a distance 3 from the origin and the perpendicular from the origin to the line makes an angle of 30° with the positive direction of $x - \text{axis}$.

xx. Reduce the equation $3x - 2y + 6 = 0$ to the intercept form and find the x- and y-intercepts.

xxi. Find the distance of the point (4, 5) from the straight line $3x - 5y + 7 = 0$.

xxii. Find centre and radius of the circle $2x^2 + 2y^2 - 5x + 6y + 2 = 0$.

xxiii. Find the equation of the circle whose two end points of a diameter are $(-1, 2)$ and $(4, -3)$.

2. 05 Mark Questions

i. Find the co-ordinates of a point whose distance from (3, 5) is 5 units and from (0, 1) is 10 units.

ii. A line AB is of length 5. A is the point (2, –3). If the abscissa of the point B is 5, prove that the ordinate of the B is 1 or 7.

iii. Show that the points (–2, 3), (1, 2) and (7, 0) are collinear.

iv. If the points (*a*, 0), (0, *b*) and (*x*, *y*) are collinear, prove that $\frac{x}{a} + \frac{y}{b} = 1$

v. Show that the points (7, 3), (3, 0), (0, –4) and (4, –1) are the vertices of a rhombus.

vi. Find the ratio in which the line segment joining (2, 3) and (-3, -4) divided by x- axis and hence find the co-ordinates of the point.

vii. Find the ratio in which the line $x - y - 2 = 0$ cuts the line segment joining (3,-1) and (8, 9).

viii. If the vertices of a right angled ΔABC are (0, 0) and (3,0), then find the third vertex.

ix. Determine the ratio in which the line joining the points (3,4) and (-3,-4) divided by the origin.

x. Show that the points (a, b+c), (b, c+a) and (c, a+b) are collinear.

xi. For what value of k, the points (k, 1), (5, 5) and (10, 7) are collinear.

xii. Find the equation of the perpendicular bisector of the line segment joining the points A(2, 3) and B(6, –5).

xiii. Find the equation of the line which passes through the point (3, 4) and the sum of its intercepts on the axes 14.

xiv. Find the equation of the perpendicular bisector of the line joining the points (1, 3) and (3, 1).

xv. Find equation of the circle whose centre is on x-axis and the circle passes through (4, 2) and (0, 0).

xvi. Find Co-ordinates of the point where the circle $x^2 + y^2 - 7x - 8y + 12 = 0$ meets the co-ordinates axes and hence find the intercepts on the axes.

xvii. Find equation of the circle which passes through the points (0, 0), (1,2) and (2,-1).

xviii. Find the equation of the circle which touches the lines $x = 0$, $y = 0$ and $x = a$.

xix. Find the equation of a circle passing through the point $(2, -1)$ and which is concentric with the circle $5x^2 + 5y^2 - 12x + 15y - 420 = 0$.

xx. Show that the points $(9, 1), (7, 9), (-2, 12)$ and $(6, 10)$ are concyclic.

3. 10 Mark Questions

i. If the point (x, y) be equidistant from the points $(a + b, b - a)$ and $(a - b. a + b)$, prove that $bx = ay$.

ii. Find the area of a quadrilateral whose vertices are (1, 1), (7, –3), (12, 2) and (7, 21).

iii. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. Find the third vertex.

iv. Find equation of the circle whose centre is on the line 8x + 5y = 0 and the circle passes through (2, 1) and (3, 5).

v. ABCD is a square whose side is ‗a‘, taking AB and AD as axes, prove that the equation to the circle circumscribing the square is $x^2 + y^2 = a(x + y)$.

CO-ORDINATE GEOMETRY IN THREE DIMENSIONS

1. 02 Mark Questions

i. Find the distance of the point P(1,2,3) from z axis.

ii. Find the direction cosines of the line joining the points (8 ,-1 ,5) and (2 , -4 ,3).

iii. Determine the direction cosines of the line equally inclined to both the axes.

iv. Find the number of lines making equal angles with coordinate axes.

v. If a line a line is perpendicular to z-axis and makes an angle measuring 60^0 with x -axis then find the angle it makes with y-axis.

vi. Find the projection of line segment joining (1,3,-1) and (3,2,4) on z axis.

vii. Find the image of the point (2,-4,7) with respect to xz plane.

viii. For what value of z, the distance between the points (-1,1,2) and (-1,-1,z) is 4.

ix. Find the centre of the sphere $x^2+y^2+(z+2)^2 =0$.

x. If the centre and radius of a sphere are (1,0,0) and 2 respectively, then find the equation of the sphere.

xi. If the segment of line joining the points (1,0,0) and (0,0,1) is a diameter of a sphere, then find equation of the sphere.

2. 05 Mark Questions

i. Prove that angle between two main diagonals of cube is $\cos^{-1}\frac{1}{3}$

ii. Find the ratio in which the line through (1 ,-1 ,3) and (2 , -4 ,1) is divided by XY- plane.

iii. Find the ratio in which the line through (1 ,-1 ,3) and (2 , -4 ,1) is divided by YZ- plane.

iv. If P(x , y , 2) lies on the line through (1 ,-1 ,0) and (2 , 1 ,1). Find the values of x and y.

v. Find the ratio in which the line joining the points (2 , -3 , 1) and (3 , -4 , -5) is divided by the locus $2x - y + 3z - 4 = 0$.

vi. Find the foot of perpendicular drawn from the point (1 ,1 , 2) on the line joining (1 ,4 ,2) and (2 , 3 ,1).

vii. Find the value of k, if the distance between the points (-1 ,-1 ,k) and (1 ,-1 ,1) is 2.

viii. Find the value of ‗a' such that two planes $2x + y + az - 2 = 0$ and $3x - y + 5z - 2 = 0$ are perpendicular to each other.

ix. Find angle between the planes $3x - y + 5z - 2 = 0$ and $3x - y + 5z - 2 = 0$

x. Find the equation of a plane passing through the points (1,2,3), (1,-2,-3) and perpendicular to the plane $3x - 3y + 5z - 2 = 0$.

xi. Find the equation of plane passing through intersection of planes $3x + y + z - 2 = 0$ and $x - 2y + 3z - 1 = 0$ and parallel to the plane $x - y + z - 6 = 0$.

xii. Find the equation of plane passing through intersection of planes $3x + 2y + z + 2 = 0$ and $x - 2y + 2z - 3 = 0$ and perpendicular to the plane $4x - y + 3z - 7 = 0$.

xiii. Find the equation of plane passing through the points (1,-1,-2) and perpendicular to the planes $4x - 2y + 3z - 1 = 0$ and $x + 2y + 3z - 2 = 0$

xiv. Find the equation of plane passing through the points (1,-2,3), (1,-1,-3) and (1,-3,0).

xv. Show that the points (1,2,3), (-1,1,0) (2,1,3) and (1,1,2) are coplanar.

xvi. Find the equation plane passing through the point (2, 3, -1) and parallel to the plane $x - y + z - 6 = 0$.

xvii. Find the equation of plane passing through the foot of the perpendicular drawn from points (1,2,3) on the co-ordinate planes .

xviii. Find the distance between the parallel planes $2x - 3y + 6z + 1 = 0$ and $4x - 6y + 12z - 5 = 0$

xix. Find the equation of a sphere having centre at $(2, -1, 4)$ and the sphere touches the plane $2x - y - 2z + 6 = 0$

xx. Find the condition that the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z - 6 = 0$ will touch the plane $x + y + z - a = 0$

xxi. Find the equation of a sphere passing through the points $(0,0,0)$, $(0, 1, -1)$, $(-1,\ 2,\ 0)$ and $(1,\ 2,\ 3)$

xxii. Find centre and radius of the sphere $x^2 + y^2 + z^2 - x - y - z - 6 = 0$ and $3x^2 + 3y^2 + 3z^2 - 4x + 3y - z - 6 = 0$

xxiii. Find the equation of a sphere having the two end points of a diameter as $(0, 1, -1)$, $(-1,\ 2,\ 2)$.