LECTURTER NOTES ON ENGINEERING MATHEMATICS-I

(COMMON FOR ALL ENGINEERING BRANCH)

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MATRIX

A *matrix* is a rectangular array of numbers (or other mathematical objects) arranged in rows and columns for which operations such as addition and multiplication are defined.

$$\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 5 & 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 8 & 9 & 10 \\ 7 & 5 & 15 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Uses of matrix in real life:-

This is a football match. Here A passes the ball to B, B passes the ball to A, B passes the ball to C and C passes the ball to D. This can inform to the computer using matrix as follows

	A	B	C	D
A	0	1	0	0
B	1	0	1	0
C	0	0	0	1
D	0	0	0	0

$$\Longrightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Order of a matrix:-

The number of rows and columns that a matrix has is called its order. By convention, rows are listed first; and columns, second. Thus, we would say that the order (or dimension) of the matrix below is 3 x 4, meaning that it has 3 rows and 4 columns.

$$\begin{bmatrix} 1 & 2 & 2 & 5 \\ 8 & 9 & 1 & 10 \\ 7 & 5 & 9 & 15 \end{bmatrix}$$

The matrix with m rows and n column is m x n (m-by-n)

Types of matrix:-

Square matrix:-

If a matrix have same number of rows and columns.

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 2 & 0 & 4 \end{bmatrix}$$

Null matrix/zero matrix:-

If all the elements of a matrix are zero then the matrix is called a zero/ null matrix.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Row matrix & Column Matrix:-

If a matrix has only one row (or column) is called a row matrix (or column matrix)

$[1 \quad 1 \quad 2]$ $\qquad$ $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Diagonal Matrix:-

If all the non-diagonal entries of a square matrix are zero and at least one diagonal entry is non-zero then the matrix is called a Diagonal matrix.

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

<u>Properties of determinant</u>:-

i) If any two rows or columns of a matrix A are equal then the determinant of A is zero.

Ex:- $\begin{vmatrix} 2 & 2 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 5 \end{vmatrix}$ (calculate the determinant) =0

$\begin{vmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 5 \end{vmatrix}$ (Calculate the determinant)=0

$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$ (Calculate the determinant)=0

ii) If we exchange any two rows or columns then the absolute value of the determinant does not change. But the sign of the determinant is changed.

Ex:- $\begin{vmatrix} 5 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$ (Calculate the determinant)= -1

Exchanging 1st row and 2nd row

IAI= $\begin{vmatrix} 1 & 1 & 1 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$ (Calculate the determinant)= +1

iii) if k is multiplied to each element of any row or column of a matrix A then the determinant of the matrix so formed is k.det(A)

Ex- IBI= $\begin{vmatrix} 2 & 2 & 2 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$(Calculate the determinant)=+2

$\begin{vmatrix} 2 & 2 & 2 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$=2(1.1-1.2)-2(5.1-2.2)+2(5.1-2.1)=-2-2+6=2=2.1=2. $\begin{vmatrix} 1 & 1 & 1 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$

IBI=2.IAI

Ex- $\begin{vmatrix} 2 & 2 & 2 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1+1 & 1+1 & 1+1 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$

$$\begin{vmatrix} 5 & 2 & 3 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2+3 & 1+1 & 2+1 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 2 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 3 & 1 & 1 \\ 5 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} a+b & c+d & e+f \\ g & h & i \\ j & k & l \end{vmatrix} = \begin{vmatrix} a & c & e \\ g & h & i \\ j & k & l \end{vmatrix} + \begin{vmatrix} b & d & f \\ g & h & i \\ j & k & l \end{vmatrix}$$

PROBEM

I) $\begin{vmatrix} 3 & 6 & 9 \\ 1 & 1 & 1 \\ 4 & 8 & 12 \end{vmatrix}$ **(without expanding determinant and use the properties of determinant)**

$$\begin{vmatrix} 3.1 & 3.2 & 3.3 \\ 1 & 1 & 1 \\ 4.1 & 4.2 & 4.3 \end{vmatrix} (\textit{using 3rd property})$$

=3.4 $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ **=12 X 0 (using 1st property) (as the elements of 1st row and 3rd row are equal)**

$$\begin{vmatrix} 12 & 3 & 9 \\ 1 & 1 & 1 \\ 8 & 8 & 4 \end{vmatrix} = 3.4 \begin{vmatrix} 4 & 1 & 3 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 12$$

PROPERTY 4

If to each element of any row or column of a determinant the equimultiples of corresponding elements of other row(or column) are added , then the value of the determinant remains the same, operation is R1 by R1 + kR2 or C1 by C1+kC2

2ND PROBLEM $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$ **=4xyz (without expanding evaluate the determinant)**

$$= \begin{vmatrix} (y+z)+z+y & z+(z+x)+x & y+x+(x+y) \\ z & z+x & x \\ y & x & x+y \end{vmatrix} (R_1 \text{ BY } R_1+R_2+R_3)$$

$$= \begin{vmatrix} 2(y+z) & 2(z+x) & 2(x+y) \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

$2\begin{vmatrix} (y+z) & (z+x) & (x+y) \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$

$2\begin{vmatrix} (y+z)-((Z+X)+(X+Y)) & (z+x) & (x+y) \\ z-((Z+X)+X) & z+x & x \\ y-(X+(X+Y)) & x & x+y \end{vmatrix}$ **{C1→C1-(C2+C3)}**

$2\begin{vmatrix} (y+z)-Z-X-X-Y) & (z+x) & (x+y) \\ z-(Z-X)-X & z+x & x \\ y-X-X-Y) & x & x+y \end{vmatrix}$

$2\begin{vmatrix} -X-X & (z+x) & (x+y) \\ (-X)-X & z+x & x \\ -X-X) & x & x+y \end{vmatrix}$

$=2\begin{vmatrix} -2X & (z+x) & (x+y) \\ -2X & z+x & x \\ -2X & x & x+y \end{vmatrix}$

$=2\text{ X} - 2x\begin{vmatrix} 1 & (z+x) & (x+y) \\ 1 & z+x & x \\ 1 & x & x+y \end{vmatrix}$

$=\text{-}4\,x\begin{vmatrix} 1 & (z+x) & (x+y) \\ 0 & 0 & -y \\ 0 & -z & 0 \end{vmatrix}$ **(R2 by R2-R1 ,R3 by R3-R1)**

$=\text{-}4\,x \text{ x } 1\begin{vmatrix} 0 & -y \\ -z & 0 \end{vmatrix}$

$= -4\,x \text{ x } (0- (zy))$

$= 4\,xyz$

2) ***Prove without expanding*** $\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \frac{1}{a} * \frac{1}{b} * \frac{1}{c}\begin{vmatrix} bc*a & a*a & a^2*a \\ ca*b & b*b & b^2*b \\ ab*c & c*c & c^2*c \end{vmatrix}$ **(multiply 1st row by a, 2nd row by b, 3rd row by c)**

R1→aR1;R2→bR2;R3→cR3

$=\frac{1}{abc}\begin{vmatrix} abc & a^2 & a^3 \\ abc & b^2 & b^3 \\ abc & c^2 & c^3 \end{vmatrix} = \frac{1}{abc}\text{abc}\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = 1.\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

Transpose of a matrix:-

If A is a matrix then the transpose of A which is denoted by A^T is formed by exchanging elements of rows by elements of columns.

$A=\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$ $A^T=\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$

$$A=\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} \text{ then } A^T=\begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix}$$

$Det(A)=det(A^T)$

Minor and Co-factor , ADJ A , INVERSE OF A :-

Suppose A be a matrix and its determinant be det(A). Then the minor of an element is the determinant of the matrix so formed after deleting the corresponding row and column to which the element lie.

$C_{ij} = (-1)^{i+j} M_{ij}$

Ex- $A=\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$ $M_{11}= 3$ $C_{11}= (-1)^2 x\ 3 = 3$ $|A|= 3-(-8) = 11$

$M_{12}=4$ $C_{12}= (-1)^3\ x4= -4$ ADJ $A=\begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix}$

$M_{21}= -2$ $C_{21}= (-1)^3\ x\text{-}2= 2$

$M_{22}=1$ $C_{22}= (-1)^4\ x\ 1= 1$

$$A^{-1} = \frac{Adj\ A}{|A|} = \frac{\begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix}}{11} = \begin{vmatrix} 3/11 & 2/11 \\ -4/11 & 1/11 \end{vmatrix}$$

$A=\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ $|A|= 2(-20)-(-3)(-46)+5(30) = -40-138+150= -178+150= -28$

$M_{11}= \begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix}= 0-20= -20$ $C_{11}=(-1)^2\ x\ -20 =-20$

$M_{12}= -42-4=-46$ $C_{12}=(-1)^3\ x\ -46 = 46$

$M_{13}= 30-0=30$ $C_{13}=(-1)^4\ x\ 30 = 30$

$M_{21}= 21-25 = -4$ $C_{21}=(-1)^3\ x\ -4 = 4$

$M_{22}= -14- 5 = -19$ $C_{22}=(-1)^4\ x\ -19= -19$

$M_{23}= 10 -(-3) = 13$ $C_{23}=(-1)^5\ x\ 13= -13$

$M_{31}= -12-0 = - 12$ $C_{31}=(-1)^4\ x\ -12 = -12$

$M_{32}= 8-30 = - 22$ $C_{32}=(-1)^5\ x\ -22 = 22$

$M_{33}= 0-(-18) = 18$ $C_{33}=(-1)^6\ x\ 18 = 18$

$$Adj\ A = \begin{bmatrix} C11 & C21 & C31 \\ C12 & C22 & C32 \\ C13 & C23 & C33 \end{bmatrix} = \begin{bmatrix} -20 & 4 & -12 \\ 46 & -19 & 22 \\ 30 & -13 & 18 \end{bmatrix}$$

$$A^{-1} = \frac{Adj\ A}{|A|} = \frac{\begin{bmatrix} -20 & 4 & -12 \\ 46 & -19 & 22 \\ 30 & -13 & 18 \end{bmatrix}}{-28}$$

MATRIX ADDITION SUBSTRACTION MULTIPLICATION

A= $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ B = $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ $A+B$ = $\begin{bmatrix}1+1 & 2+1\\3+2 & 4+1\end{bmatrix}$= $\begin{bmatrix}2 & 3\\5 & 5\end{bmatrix}$

A= $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ B = $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ A- B = $\begin{bmatrix}1-1 & 2-1\\3-2 & 4-1\end{bmatrix}$ =$\begin{bmatrix}0 & 1\\1 & 3\end{bmatrix}$

A= $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ B = $\begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$ A. B = $\begin{bmatrix}1\,X\,1 \quad +2\,X\,2 & 1\,X\,1+2\,X\,1\\3\,X\,1+4\,X\,2 & 3\,X\,1+4\,X\,1\end{bmatrix}$ = $\begin{bmatrix}5 & 3\\11 & 7\end{bmatrix}$

ADDITION, SUBSTRACTION= ORDER SAME

MULTIPLICATION A= m x n B = n x p AB= M X N

A=$\begin{bmatrix}1 & 2\\1 & 1\\4 & 2\end{bmatrix}$ B=$\begin{bmatrix}1 & 1\\1 & 0\\5 & 4\end{bmatrix}$ C=$\begin{bmatrix}2 & 3 & 2\\1 & 1 & 0\end{bmatrix}$ A= 3 X 2 B = 3 X 2 C = 2 X 3

A+B = $\begin{bmatrix}1 & 2\\1 & 1\\4 & 2\end{bmatrix}+\begin{bmatrix}1 & 1\\1 & 0\\5 & 4\end{bmatrix}$ = $\begin{bmatrix}2 & 3\\2 & 1\\9 & 6\end{bmatrix}$

A-B = $\begin{bmatrix}1 & 2\\1 & 1\\4 & 2\end{bmatrix}-\begin{bmatrix}1 & 1\\1 & 0\\5 & 4\end{bmatrix}$ = $\begin{bmatrix}0 & 1\\0 & 1\\-1 & -2\end{bmatrix}$

A.C=$\begin{bmatrix}1 & 2\\1 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}2 & 3 & 2\\1 & 1 & 0\end{bmatrix}$=$\begin{bmatrix}1\,X\,2+2\,X\,1 & 1\,X\,3+2\,X\,1 & 1\,X\,2+2\,X\,0\\1X\,2+1X\,1 & 1\,X\,3+1\,X\,1 & 1\,X\,2+\ 1\,X\,0\\4X\,2+\ 2\,X\,1 & 4\,X\,3+2\,X\,1 & 4X\,2+2\,X\,0\end{bmatrix}$

= $\begin{bmatrix}4 & 5 & 2\\3 & 4 & 2\\10 & 14 & 8\end{bmatrix}$

C.A =$\begin{bmatrix}2 & 3 & 2\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 2\\1 & 1\\4 & 2\end{bmatrix}$=$\begin{bmatrix}2X1+3X1+2X\,4 & 2X\,2+3X1+2X2\\1X1+1X1+0X4 & 1X2+1X1+0X2\end{bmatrix}$=$\begin{bmatrix}13 & 11\\2 & 3\end{bmatrix}$

$\begin{bmatrix}1 & x\\2 & -2\end{bmatrix}$+$\begin{bmatrix}3 & 4\\3 & 6\end{bmatrix}$=$\begin{bmatrix}4 & 3\\5 & 4\end{bmatrix}$.

Solution of system of linear equation by Cramer's method:-

Linear equation:- A linear equation is an equation consisting of only the variable with power one/ degree one.

System of linear equation:- Combination of two or more linear equations is known as system of linear equation.

Suppose 2x-y+3z=3 ; x+2y+9z=2; x-5y+3z=6 is the system of linear equation with variables x,y & z each variable has degree 1.

Solve by Cramers rule

2x-y+3z=3
x+2y+9z=2
x-5y+3z=6

Ans- AX=B

$\equiv\begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & 9 \\ 1 & -5 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$

|A|= 2(6+45)+1(3-9)+3(-5-2)=102-6-21=75

$A_x=\begin{pmatrix} 3 & -1 & 3 \\ 2 & 2 & 9 \\ 6 & -5 & 3 \end{pmatrix}$

$|A_x|$= 3(6+45)+1(6-54)+3(-10-12)=153-48-66=39

$A_y=\begin{pmatrix} 2 & 3 & 3 \\ 1 & 2 & 9 \\ 1 & 6 & 3 \end{pmatrix}$

$|A_y|$=2(6-54)-3(3-9)+3(6-2) = -96+18+12=-66

$A_z=\begin{pmatrix} 2 & -1 & 3 \\ 1 & 2 & 2 \\ 1 & -5 & 6 \end{pmatrix}$

$|A_z|$=2(12+10)+1(6-2)+3(-5-2)=44+4-21=48-21=27

$x=\frac{|A_x|}{|A|}$ =39/75

$y= \frac{|A_y|}{|A|}$=-66/75

$z=\frac{|A_z|}{|A|}$=27/75

problem2- 2x+y=1 solve by creamer's rule

3x-y=4

AX=B

$=\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

|A|= -2-3= -5

$|A_x|=\begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$ = -1 -4 =-5

$|A_y| = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = 8\text{-}3 = 5$

$x = \frac{|A_x|}{|A|} = $ -5/-5 = 1

$y = \frac{|A_y|}{|A|} = $ 5/-5 = -1

Finding Inverse of a Matrix Using Matrix Inversion Method

A is matrix. We find the inverse of A (A^{-1}) as follows

$A*(A^{-1}) = I$

Procedure for finding inverse of a matrix:-

$$\mathbf{A^{-1}} = \frac{adj(A)}{|A|}$$

Adj(A):-

1. Find the minor of each element
2. Find the cofactor of each element
3. Form the cofactor matrix Co(A)
4. Form the adj(A)=$[co(A)]^T$

$A = \begin{bmatrix} 1 & 1 & 2 \\ 5 & 2 & 7 \\ 0 & 1 & 3 \end{bmatrix}$ order of A=3x3

$|A|$ =-3

Minor of 1=	-1	co-factor of 1=	-1
Minor of 1=	15	co-factor of 1=	-15
Minor of 2=	5	co-factor of 2=	5
Minor of 5=	1	co-factor of 5=	-1
Minor of 2=	3	co-factor of 2=	3
Minor of 7=	1	co-factor of 7=	-1
Minor of 0=	3	co-factor of 0 =	3
Minor of 1=	-3	co-factor of 1=	3
Minor of 3=	-3	co-factor of 3=	-3

Co-factor matrix:-

Co(A)= $\begin{bmatrix} -1 & -15 & 5 \\ -1 & 3 & -1 \\ 3 & 3 & -3 \end{bmatrix}$ Adj(A)= $\begin{bmatrix} -1 & -1 & 3 \\ -15 & 3 & 3 \\ 5 & -1 & -3 \end{bmatrix}$

$$\mathbf{A^{-1}} = \frac{adj(A)}{|A|} = \frac{1}{-3} \begin{bmatrix} -1 & -1 & 3 \\ -15 & 3 & 3 \\ 5 & -1 & -3 \end{bmatrix}$$

pr –ii 2x+y=1 by matrix inversion method
3x-y=4

AX=B

X= $A^{-1}B$

$=\begin{bmatrix}2 & 1\\3 & -1\end{bmatrix}\begin{bmatrix}X\\Y\end{bmatrix}=\begin{bmatrix}1\\4\end{bmatrix}$

A= $\begin{bmatrix}2 & 1\\3 & -1\end{bmatrix}$ |A|= -2-3 =-5

M_{11}=-1 C_{11}= -1

M_{12}= 3 C_{12}= -3

M_{21}=1 C_{21}= -1

M_{22}=2 C_{22}= 2

Adj A = $\begin{bmatrix}-1 & -1\\-3 & 2\end{bmatrix}$

A^{-1} = Adj A/ |A| = $\frac{-1}{5}\begin{bmatrix}-1 & -1\\-3 & 2\end{bmatrix}$ = $\begin{bmatrix}1/5 & 1/5\\3/5 & -2/5\end{bmatrix}$

X= A^{-1} B

$\begin{bmatrix}x\\y\end{bmatrix}$ =X = $\begin{bmatrix}1/5 & 1/5\\3/5 & -2/5\end{bmatrix}\begin{bmatrix}1\\4\end{bmatrix}$ = $\begin{bmatrix}\frac{1}{5}+\frac{4}{5}\\\frac{3}{5}-\frac{8}{5}\end{bmatrix}$= $\begin{bmatrix}\frac{5}{5}\\-\frac{5}{5}\end{bmatrix}$ = $\begin{bmatrix}1\\-1\end{bmatrix}$

X= 1 y = -1

TRIGNNOMETRY

ASTC RULE

2nd quadrant (90°-180°)
Sin, cosec +ve
Tan, sec, cos, cot -ve

1st quadrant (0°-90°)
Sin, cos, tan, sec, cosec, cot
All +ve

3rd quadrant (180°-270°)
Tan, cot +ve
Sin, sec, cos, cosec -ve

4th quadrant (270°-360°)
Cos, sec +ve
Sin, cosec, cot, tan -ve

$\sin(90-\theta)=\cos\theta$

$\cos(90-\theta)=\sin\theta$

$\tan(90-\theta)=\cot\theta$

$\cot(90-\theta)=\tan\theta$

$\sec(90-\theta)=\mathrm{cosec}\,\theta$

$\mathrm{cosec}(90-\theta)=\sec\theta$

Θ (0<θ<90)	90-θ (1st qrd.)	90+θ (2nd qrd.)	180-θ (2nd qrd.)	180+θ (3rd qrd.)
Sin	cosθ	cosθ	sinθ	-sinθ
Cos	sinθ	-sinθ	-Cosθ	-Cosθ
Tan	cotθ	-cotθ	-Tanθ	Tanθ
Cot	tanθ	-tanθ	-Cotθ	Cotθ
sec	cosecθ	-cosecθ	-Secθ	-Secθ
cosec	secθ	secθ	cosecθ	-cosecθ

2275= 90 X 25+25 2ND QUADRENT

2275=360 X 6+ 115

Sin 3754° =sin (90 X 41 +64) = cos 64 **2ND QUADRENT**

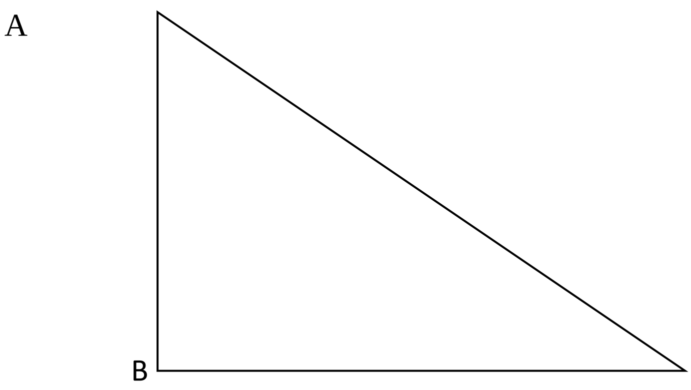
3754= 360 X 10 +154

3333=360 X 9 +93

3333=90 X 37+3

Sin 3754°= Sin 154°= Sin(90+64)=cos64=+ve

Trigonometric Ratios:-



$\text{Sin}\theta = \frac{p}{h}$; $\text{Cos}\theta = \frac{b}{h}$; $\text{Tan}\theta = \frac{p}{b}$; $\text{Cot}\theta = \frac{b}{p}$

$\text{Sec}\theta = \frac{h}{b}$; $\text{Cosec}\theta = \frac{h}{p}$

Pythagoras theorem

$p^2 + b^2 = h^2$; p- Perpendicular, b- Base, h- Hypotenuse

$sin^2\theta + cos^2\theta = (\frac{p}{h})^2 + (\frac{b}{h})^2 = \frac{p^2}{h^2} + \frac{b^2}{h^2} = \frac{p^2+b^2}{h^2} = \frac{h^2}{h^2} = 1$

$sec^2\theta - tan^2\theta = 1$

$cosec^2\theta - cot^2\theta = 1$

Sec θ = 1/cosθ

Cotθ=1/tanθ

Cosecθ=1/sinθ

Θ (0<θ<90)	90-θ (1^{st} qrd.)	90+θ (2^{nd} qrd.)	180-θ (2^{nd} qrd.)	180+θ (3^{rd} qrd.)
Sin	cosθ	cosθ	sinθ	-sinθ
Cos	sinθ	-sinθ	-Cosθ	-Cosθ
Tan	cotθ	-cotθ	-Tanθ	Tanθ
Cot	tanθ	-tanθ	-Cotθ	Cotθ
sec	cosecθ	-cosecθ	-Secθ	-Secθ
cosec	secθ	secθ	cosecθ	-cosecθ

F(-x)= -f(x) odd function

F(-x)= f(x) even function

Cos, sec even function

Sin, tan, cosec, cot odd function

Sin(-3333°)= - sin3333°= - sin(90^0 x37 + 3°) = - cos 3°

Sin (3423)= sin(90°x 38+3) = - sin 3°

Cot (3425^0)= cot(90° x38 + 5) = +cot 5^0

Cosec (6255^0) = cosec(90 x 69 + 45) =+ sec 45= $\sqrt{2}$

Sec(-7634^0) = sec 7634^0 = sec(90 x 84 +74)= + sec 74

1. **Find the value of cos1°.cos2°……..cos100°.**

Solution:-

cos1°.cos2°……..cos100°

=cos1°.cos2°……..cos90°…..cos100°

=0 [as cos90°=0]

find the value of tan1°.tan2°……tan89°

tan1°.tan2°……tan89°

=tan1°.tan2°……tan44°.tan45°.tan46°…….tan88°.tan89°

= tan1°.tan2°……tan44°.1. tan(90°-44°)…….tan(90°-2°).tan(90°-1°)

= tan1°.tan2°……tan44°.1.cot44°……..cot2°.cot1°

= (tan1°.cot1°).(tan2°.cot2°)……..(tan44°.cot44°).1

=1.1.1……….1

=1

2. **find the value of cos 24°+cos 5°+cos 175°+cos 204°+cos 300°**

solution: cos 24°+cos 5°+cos 175°+cos 204°+cos 300

= cos 24°+cos 5°+cos (180°-5°)+cos (180°+24°)+cos(90 x3°+ 30°)

= cos 24°+cos 5°-cos 5°-cos 24°+sin30°

= sin30° =1/2

Sol = cos 24°+cos 5°+cos (180°-5°)+cos (180°+24°)+cos(360°-60°)

= cos 24°+cos 5°-cos5°-cos 24°+cos 60°

=0+0+1/2

=1/2

find the value of tan1°.tan2°…….tan100°

Sol:-

tan1°.tan2°…….tan100°

=tan1°.tan2°……tan90°…..tan100°

=∞

$find\ the\ value\ of\ \begin{vmatrix} cosx & sinx \\ sinx & cosx \end{vmatrix}$

$\begin{vmatrix} cosx & sinx \\ sinx & cosx \end{vmatrix} = \cos^2 x - \sin^2 x = cos2x =$

$$maximum\ value\ 1\ , minimum\ value - 1$$

Some standard formula:-

1. Sin(A+B)=sinA.cosB+cosA.sinB
2. Sin(A-B)= sinA.cosB-cosA.sinB
3. Cos(A+B)= cosA.cosB-sinA.sinB
4. Cos(A-B)= cosA.cosB+sinA.sinB
5. Sin2A=sin(A+A)= sinA.cosA+cosA.sinA= 2sinA.cosA
6. Cos2A=cos(A+A)= cosA.cosA-sinA.sinA= $\cos^2 A - \sin^2 A$
7. Sin3A=3sinA-4$\sin^3$A

Sin3A= sin(A+2A)= sinA.cos2A+cosA.sin2A
sinA.($\cos^2 A - \sin^2 A$)+cosA.2sinA.cosA
=sinA(1-$\sin^2 A - \sin^2 A$)+2sinA. $\cos^2$A
=sinA-2$\sin^3$A +2sinA(1-$\sin^2$A)
=sinA-2$\sin^3$A+2sinA-2$\sin^3$A
=3sinA-4$\sin^3$A

8. Cos3A= cos(A+2A) = $4\cos^3 A - 3\cos A$

Problem- Sin18°

Θ=18°
=> 5θ=90°
=> 3θ+2θ=90°
=> 2θ=90°-3θ
=> Sin 2θ = sin (90°-3θ)
=> Sin 2θ= cos 3θ
=> 2sinθ.cosθ =4$\cos^3$θ - 3cosθ
=> 2sinθ.cosθ= cosθ(4$\cos^2$θ-3)
=> 2sinθ=(4$\cos^2$θ-3)
=> 2sinθ=4(1-$\sin^2$θ)-3=4-4$\sin^2$θ-3=1-4$\sin^2$θ
=> 2sinθ=1-4$\sin^2$θ
=> 2sinθ+4$\sin^2$θ-1=0
=>4$\sin^2$θ+2sinθ-1=0

$Ax^2 + bx + c = 0 \quad x = \frac{-b \mp \sqrt{b^2 - 4.a.(c)}}{2.a}$

A=4,b=2,c= -1

Sinθ=$\frac{-2 \mp \sqrt{2^2 - 4.4.(-1)}}{2.4} = \frac{-2 \mp \sqrt{20}}{2.4} = \frac{-2 \mp 2\sqrt{5}}{2.4} = \frac{-1 \mp \sqrt{5}}{4}$

Sin 18° =$\frac{-1 \mp \sqrt{5}}{4}$

Therefore Sin 18°=$\frac{-1+\sqrt{5}}{4}$ (sin 18 is always +ve)

Cos18°, sin 36°, cos36°

If sinθ + cosec θ = 2 . how that $\sin^n$θ + cosec^nθ = 2 for all positive integers n.

sinθ + cosec θ = 2

$\sin\theta + \frac{1}{\sin\theta} = 2$
$\sin^2\theta + 1 = 2\sin\theta$
$\sin^2\theta - 2\sin\theta + 1 = 0$
$\sin^2\theta - 2 * 1 * \sin\theta + 1 = 0$

$(\sin\theta - 1)^2 = 0$

$\sin\theta = 1$, cosec $\theta = 1$
$\sin^n\theta + \operatorname{cosec}^n\theta = 1^n + 1^n = 1 + 1 = 2$

Find the maximum value of 3 sinx +4 cos x ?

3 sinx +4 cos x
Lect 3= r cosθ , 4 = r sinθ
r cosθ sinx +r sinθ cos x
r(sin x. cos θ +cos x . sin θ)
r(sin(x+θ))
$3^2 + 4^2 = r^2\cos^2\theta + r^2\sin^2\theta$
$9+16 = r^2(\cos^2\theta + \sin^2\theta) = r^2.1$
$25 = r^2$
r= +5,-5
maximum value of 3 sinx +4 cos x is 5
minimum value of 3 sinx +4 cos x is -5

3 cosx +4 sinx
Lect 3= r sinθ , 4 = r cosθ
r sinθ cos x+ r cosθ sinx

<u>Some important rule</u>

1. Sin(A+B)=sinA.cosB+cosA.sinB
2. Sin(A-B)= sinA.cosB-cosA.sinB
3. Cos(A+B)= cosA.cosB-sinA.sinB
4. Cos(A-B)= cosA.cosB+sinA.sinB
5. Sin2A=sin(A+A)= sinA.cosA+cosA.sinA= 2sinA.cosA
6. Cos2A=cos(A+A)= cosA.cosA-sinA.sinA= $\cos^2 A - \sin^2 A$
7. Sin3A=3sinA-4$\sin^3$A
8. Sin 2A = $\frac{2\tan A}{1+\tan^2 A}$ Sin A = $\frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
9. Cos 2A = $\frac{1-\tan^2 A}{1+\tan^2 A}$ Cos A = $\frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}$
10. Cos 2A = $\cos^2 A - \sin^2 A = 1 - \sin^2 A - \sin^2 A$= 1-2$\sin^2$A
    1-2$\sin^2 A = \cos 2A$
    1- Cos2A =2$\sin^2$A   1+Cos2A =2$\cos^2$A
    $\sin^2 A = \frac{1-\cos 2A}{2}$
11. Sin A=$\sqrt{\frac{1-\cos 2A}{2}}$ Sin$\frac{A}{2}$=$\sqrt{\frac{1-\cos A}{2}}$
12. Cos A=$\sqrt{\frac{1+\cos 2A}{2}}$ Cos $\frac{A}{2}$=$\sqrt{\frac{1+\cos A}{2}}$

13. Tan A= $\sqrt{\frac{1-cos2A}{1+cos2A}}$ $\tan\frac{A}{2} = \sqrt{\frac{1-cosA}{1+cosA}}$

14. Sin C + Sin D= 2sin $\frac{C+D}{2}$. cos $\frac{C-D}{2}$

15. Sin C - Sin D=2cos $\frac{C+D}{2}$. sin $\frac{C-D}{2}$

16. Cos C + Cos D=2cos $\frac{C+D}{2}$. cos $\frac{C-D}{2}$

17. Cos C - Cos D =2sin $\frac{C+D}{2}$. sin $\frac{D-C}{2}$

PROBLEM, :- If A+B+C=π then prove that(PROVE THAT IN A TRIANGLE)
Sin 2A+sin 2B+sin 2C= 4 sinA.sinB.sinC

Proof:-
L.H.S
Sin 2A+Sin 2B+Sin 2C
=2 sin $\frac{2A+2B}{2}$. cos $\frac{2A-2B}{2}$ + 2 sinC. CosC
=2 Sin (A+B).cos(A-B)+2 SinC.cosC
[given A+B+C=π => A+B= π-C]
=2 sin(π-C). (cosA.cosB+sinA.sinB)+ 2 SinC.cosC
=2 sin C. (cosA.cosB+sinA.sinB)+ 2 SinC.cosC
=2 SinC.CosA.cosB+2SinC.sinA.sinB+ 2 SinC.cosC
=2 SinC.CosA.cosB+ 2 SinC.cosC+2.sinA.sinB. SinC
=2sinC[cosA.cosB+cosC]+2sinA.sinB.sinC
=2sinC[cosA.cosB+cos([π-(A+B))]+ 2sinA.sinB.sinC
=2sinC[cosA.cosB-Cos(A+B)]+ 2sinA.sinB.sinC
=2sinC[cosA.cosB-(cosA.cosB-sinA.sinB)]+ 2sinA.sinB.sinC
=2sinC[cosA.cosB-cosA.cosB+sinA.sinB]+ 2sinA.sinB.sinC
=2sinC.sinA.sinB+ 2sinA.sinB.sinC
=2sinA.sinB.sinC+2sinA.sinB.sinC
=4sinA.sinB.sinC

h.w-PROBLEM, :- If A+B+C=π then prove that(PROVE THAT IN A TRIANGLE)
cos2A+cos 2B+cos 2C+1 = - 4 cosA.cos B.cos C

Show that the equation $\sin\theta = a + \frac{1}{a}$ $does\ not\ have\ a\ solution\ for\ every\ real\ number\ \ a \neq 0$

$$sin\theta = \frac{a^2 + 1}{a}$$

If a=0

Then $sin\theta = 1$ we have solution for this .

Suppose a$\neq$ 0

For any positive value a $\frac{a^2+1}{a} > 1$

For any negative value a $\frac{a^2+1}{a} < -1$

But we know range of $sin\theta\ is\ [-1\ \ \ 1]$ so for any value

$\sin\theta = a + \frac{1}{a}\ does\ not\ have\ a\ solution\ for\ every\ real\ number\ \ a \neq\ 0$

Find the maximum value of 5sinx +12 cos x

Lect 5= r cosθ , 12 = r sinθ

r cosθ sinx +r sinθ cos x

r(sin x. cos θ +cos x . sin θ)

r(sin(x+θ))

$5^2 + 12^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta$

$25+144= r^2(\cos^2 \theta+ \sin^2 \theta) = r^2.1$

$169= r^2$

r= 13,-13

Find the maximum value of 5sinx +12 cos x is 13

2+3sinx+4cosx

3 sinx +4 cos x

Lect 3= r cosθ , 4 = r sinθ

2+r cosθ sinx +r sinθ cos x

2+r(sin x. cos θ +cos x . sin θ)

2+r(sin(x+θ))

$3^2 + 4^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta$

$9+16= r^2(\cos^2 \theta+ \sin^2 \theta) = r^2.1$

$25= r^2$

R=5,-5

Maximum value= 2+5= 7

Minimum value= 2-5= -3

Find the maximum value of 3sinx+4cosx-2

3sinx+4cosx-2

3 sinx +4 cos x -2

Lect 3= r cosθ , 4 = r sinθ

r cosθ sinx +r sinθ cos x-2

r(sin x. cos θ +cos x . sin θ)-2

r(sin(x+θ))-2

$3^2 + 4^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta$

$9+16= r^2(\cos^2 \theta+ \sin^2 \theta) = r^2.1$

$25= r^2$

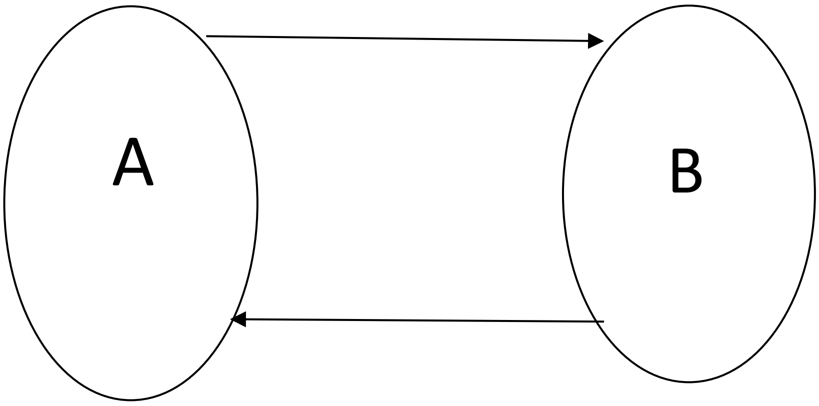
R=5,-5
Maximum value= 5-2= 3
Minimum value= -5-2= -7
Q-Find the maximum value of 2-3sinx-4cosx

2-3sinx-4cosx
2-3 sinx -4 cos x
Lect 3= r cosθ , 4 = r sinθ
2-r cosθ sinx -r sinθ cos x
2-r(sin x. cos θ +cos x . sin θ)
2-r(sin(x+θ))
$3^2 + 4^2 = r^2 \cos^2 \theta + r^2 \sin^2 \theta$
$9+16= r^2(\cos^2 \theta+ \sin^2 \theta) = r^2.1$
$25= r^2$
R=5,-5
Minimum value= 2-5= -3
Maximum value= 2-(-5)= 7

INVERSE TRIGNOMETRIC FUNCTION:

F:A→B then f^{-1}:B→A

Sin:R→[-1,1] sin^{-1}:[-1,1]→R



Sin$(sin^{-1}x) = x$ $sin^{-1}(sinx) = x$ $sin^{-1}(1) =$ $sin^{-1}(\sin 90) = 90^{O}$

cos$(cos^{-1}x) = x$ $cos^{-1}(cosx) = x$

tan$(tan^{-1}x) = x$ $tan^{-1}(tanx) = x$ $tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = tan^{-1}(tan30)$=30^{O}

sec $(sec^{-1}x) = x$ $sec^{-1}(secx) = x$ $sec^{-1}(2) =$ $sec^{-1}(\sec 60) =$ 60^{o}

cosec$(cosec^{-1}x) = x$ $cosec^{-1}(cosec\ x) = x$

cot $(cot^{-1}x) = x$ $cot^{-1}(cotx) = x$

$cosec^{-1}x = sin^{-1}(\frac{1}{x})$;|x|≥1 , $sin^{-1}(x\ ) = cosec^{-1}(\frac{1}{x})$

$sec^{-1}x = cos^{-1}(\frac{1}{x})$;|x|≥1 $sec^{-1}(\frac{1}{x}) = cos^{-1}(x)$

$cot^{-1}x = tan^{-1}(\frac{1}{x})$, x>0 $cot^{-1}(\frac{1}{x}) = tan^{-1}(x)$, x>0

$cot^{-1}x = \pi + tan^{-1}(\frac{1}{x})$, x<0 $cot^{-1}(\frac{1}{x}) = \pi + tan^{-1}(x)$, x<0

$sin^{-1}x + cos^{-1}x = \frac{\pi}{2}$, x€[-1,1]

$tan^{-1}x + cot^{-1}x = \frac{\pi}{2}$, x€R

$sec^{-1}x + cosec^{-1}x = \frac{\pi}{2}$, x€(-∞,-1]U[1,∞)

$$tan^{-1}x + tan^{-1}y = \begin{cases} tan^{-1}\frac{x+y}{1-xy} \text{ if } xy < 1 \\ \pi + tan^{-1}\frac{x+y}{1-xy} \text{ if } xy > 1,\ x>0,\ y>0 \end{cases}$$

$-\pi + tan^{-1}\frac{x+y}{1-xy}$ if xy >1, x<0, y<0

$tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1+xy}$ if x≥0,y≥0

$problem\ 1:\ tan^{-1}(\frac{1}{2})\ + tan^{-1}(\frac{1}{3})$

[x=1/2 , y=1/3 => xy=1/6<1]

$= tan^{-1}\frac{(\frac{1}{2})+(\frac{1}{3})}{1-(\frac{1}{2})(\frac{1}{3})} = tan^{-1}\frac{(\frac{5}{6})}{(\frac{5}{6})} = tan^{-1}1 = tan^{-1}(\tan\frac{\pi}{4}) = \pi/4$

Problem 2: Find the value of cos tan^{-1} cot cos^{-1} $(\frac{\sqrt{3}}{2})$?

cos tan^{-1} cot cos^{-1} $(\frac{\sqrt{3}}{2})$

=cos tan^{-1} cot cos^{-1} $(\cos\frac{\pi}{6})$

= cos tan^{-1} cot 30°

= cos tan^{-1} $\sqrt{3}$

=cos tan^{-1} (tan 60°)

=cos60°

=1/2

Q- $\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}\frac{3}{\sqrt{10}}$ =?

$\sin^{-1}x + \cos^{-1}y = \sin^{-1}x + (\frac{\pi}{2} - \sin^{-1}y)$

$=\frac{\pi}{2} + \sin^{-1}x - \sin^{-1}y = \frac{\pi}{2} + \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$

$\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2} + \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$

$\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}\frac{3}{\sqrt{10}} = \frac{\pi}{2} + \sin^{-1}(\frac{1}{\sqrt{5}}\sqrt{1-\frac{3}{\sqrt{10}}^2} - y\sqrt{1-\frac{1}{\sqrt{5}}^2}$

$= \frac{\pi}{2} + \sin^{-1}(\frac{1}{\sqrt{5}}\sqrt{1-\frac{9}{10}} - \frac{3}{\sqrt{10}}\sqrt{1-\frac{1}{5}}) = \frac{\pi}{2} + \sin^{-1}(\frac{1}{\sqrt{5}} * \frac{1}{\sqrt{10}} - \frac{3}{\sqrt{10}} * \frac{2}{\sqrt{5}}) = \frac{\pi}{2} + \sin^{-1}\left(-\frac{5}{5\sqrt{2}}\right)$

$=\frac{\pi}{2} + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2} + \sin^{-1}\left(\sin\left(-\frac{\pi}{4}\right)\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

Q-4 $\cos^{-1} x + \sin^{-1} x = \pi$ what is the value of x

$4\cos^{-1} x + \sin^{-1} x = \pi$

$2.2\ \cos^{-1} x + \sin^{-1} x = \pi$

$2.\cos^{-1}(2x^2 - 1) + \sin^{-1} x = \pi$

$\cos^{-1}(2(2x^2 - 1)^2 - 1) + \sin^{-1} x = \pi$

$\cos^{-1}(2(2x^2 - 1)^2 - 1) + \frac{\pi}{2} - \cos^{-1} x = \pi$

$\cos^{-1}(2(2x^2 - 1)^2 - 1) - \cos^{-1} x = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

Q-Find $\sec^2(\tan^{-1} 2) + cosec^2(\cot^{-1} 3)$

$\sec^2(\tan^{-1} 2) + cosec^2(\cot^{-1} 3)$

$= 1 + \tan^2(\tan^{-1} 2) + 1 + \cot^2(\cot^{-1} 3)$

$= 1 + (\tan(\tan^{-1} 2))^2 + 1 + (\cot(\cot^{-1} 3))^2$

$=1+2^2 + 3^2 = 1 + 4 + 1 + 9 = 15$

$tan^2 x = (tan x)^2$

If $tan^{-1}2$ *and* $tan^{-1}3$ are two angle of a triangle then third angle=?

$tan^{-1}2 + tan^{-1}3 + tan^{-1}x = 180^0$

$tan^{-1}(\frac{2+3+x-6x}{}) = 180^0$

$2 + 3 + x - 6x = 0$

5-5x=0,x=1

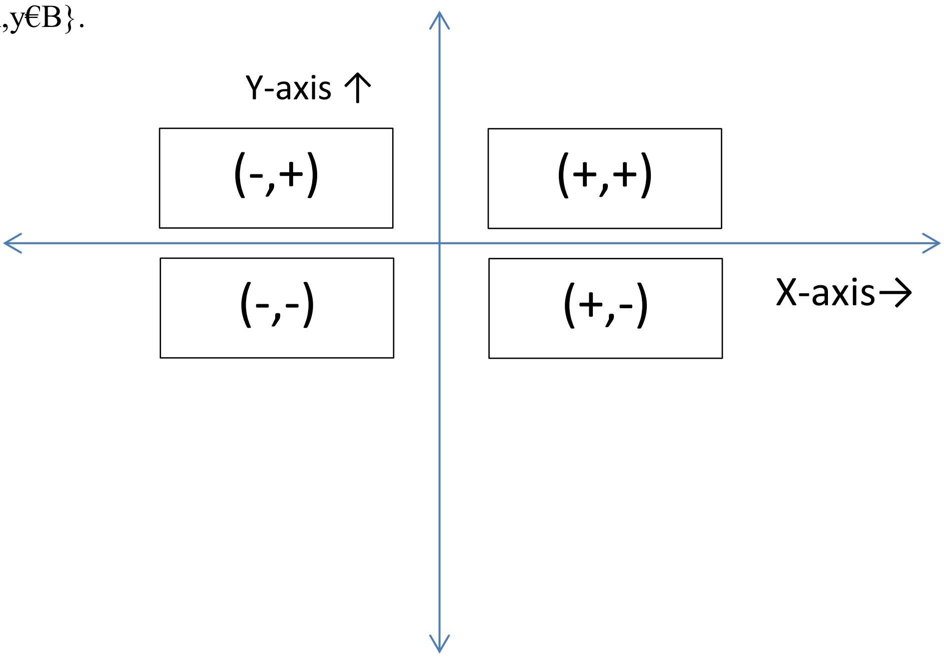
$tan^{-1}1 = 45^0$

Co-ordinate geometry in two dimensions

Cartesian product:-

A & B are two sets then their AXB is the order pair (x,y) such that x€A & y€B.

AXB={(x,y): x€A,y€B}.



Distance formula:-

Let P(x_1, y_1) & Q(x_2, y_2) are two points on the co-ordinate geometry. Then their distance can be calculated as follows

d= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ex:- distance between (3,5) & (2,1)

$x_1 = 3, y_1 = 5\ , x_2 = 2, y_2 = 1$

d= $\sqrt{(2-3)^2 + (1-5)^2}$=$\sqrt{(-1)^2 + (-4)^2}$=$\sqrt{1+16}$= $\sqrt{17}$

ex:- find the distance between two points (-1,1) and (3,3);

d= $\sqrt{(3-(-1))^2 + (3-1)^2}$=$\sqrt{(4)^2 + (2)^2}$=$\sqrt{16+4}$= $\sqrt{20}$

:- find the distance between two points(2,0) and (-3,7)

d= $\sqrt{(-3-2)^2 + (7-0)^2}$=$\sqrt{(-5)^2 + (7)^2}$=$\sqrt{25+49}$= $\sqrt{74}$

Division Formula (Internal):-

Suppose P(x,y) intersects the line segment joining the points (x_1, y_1) & (x_2, y_2) internally in the ration m:n then the co-ordinate of (x,y) is

x= $\frac{mx_2 + nx_1}{m+n}$ and y= $\frac{my_2 + ny_1}{m+n}$

Ex- find the point which intersects the line segment joining the points (5,4) and (2,1) internally in the ration 2:1?

Sol- here $x_1 = 5$ x_2=2 y_1=4 $y_2 = 1$ m=2 , n=1

Therefore by internal division formula

The point P(x,y) which intersects the line segment joining the points (3,4) and (2,1) is

x= $\frac{mx_2+nx_1}{m+n}$ and y= $\frac{my_2+ny_1}{m+n}$
x= $\frac{2*2+1*5}{2+1}$; y= $\frac{2*1+1*4}{2+1}$
x=9/3 ; y= 6/3=2
Therefore the point is (3,2)

Division Formula (External):-

Suppose P(x,y) intersects the line segment joining the points (x_1, y_1) & (x_2, y_2) externally in the ration m:n then the co-ordinate of (x,y) is

x= $\frac{mx_2-nx_1}{m-n}$ and y= $\frac{my_2-ny_1}{m-n}$

Ex- find the point which intersects the line segment joining the points (3,4) and (2,1) externally in the ration 2:1

here $x_1 = 3$ x_2=2 y_1=4 $y_2 = 1$ m=2 , n=1
x= $\frac{2*2-1*3}{2-1}$ and y= $\frac{2*1-1*4}{2-1}$
x= 1/1 = 1, y = (2-4)/1 = -2

**** Suppose P(x,y) bisects the line segment joining the points (x_1, y_1) & (x_2, y_2) then the co-ordinate of (x,y) is*

x= $\frac{x_2+x_1}{2}$ *and y=* $\frac{y_2+y_1}{2}$

Area of Triangle:-

Suppose ABC is a triangle with A (x_1, y_1); B (x_2, y_2) & C (x_3, y_3) are the vertices. Then the area of the triangle ABC is

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

Ex:- find the area of the triangle whose vertices are (0,1), (1,0) and (1,1)

Solution- Let ABC is the triangle with vertices as A(0,1), B(1,0) and C(1,1) then the its area is

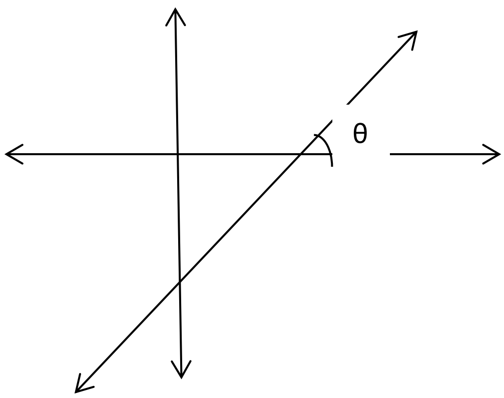
$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \frac{1}{2}[1(1-0)-1(0-1)+1(0-1)] = \frac{1}{2}(1+1-1) = \frac{1}{2} \text{ sq. unit}$$

If the area of triangle ABC is zero then the points A (x_1, y_1); B (x_2, y_2) & C (x_3, y_3) are collinear.

1.h.w= Find the area of the triangle whose vertices are (0,0) , (2,0) and (5,0)
2.find the mid point of line joining (2,3) and (4,5)

Slope of a line:-

Suppose a line L makes an angle θ with respects to X-axis. Then the slope of L is the tanθ. Slope of a line is denoted by m. here θ is the angle of inclination.



ex:- let angle of inclination of the line L is 60°. Then what is

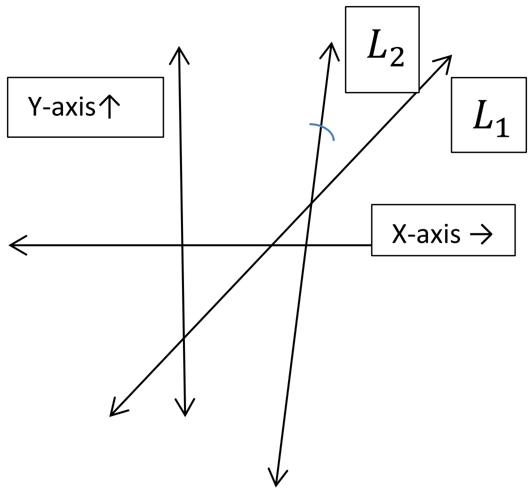
it slope.
Here θ=60° therefore slope, m=tan 60°=√3

H.W- Find the slope of line whose angle of inclination is
i) 45° (m=1) ii) 30° (m= 1/√3) iii) 120°(m=-√3)
ii) Find the angle of inclination of the line whose slope is
i)1 (θ=45°) ii) √3 (θ =60°) iii)1/√3 (θ =30°) iv)0(θ =0°)

<u>Angle between two lines:-</u>

Suppose L_1 & L_2 are two lines with slope m_1 & m_2 respectively. Then the angle between two line L_1 & L_2 is

$\theta = tan^{-1}(\frac{m_1-m_2}{1+m_1m_2})$



Ex:-find the angle between two lines whose slopes are 1 & 0.

$\theta = tan^{-1}(\frac{m_1-m_2}{1+m_1m_2})$

$\Theta = tan^{-1}(\frac{1-0}{1+1*0}) = tan^{-1}1 = tan^{-1}(\tan 45°) = 45° = \pi/4$

$\frac{m_1-m_2}{1+m_1m_2}=0$

$=m_1 - m_2 = 0 \quad m_1 = m_2$ parallel

$\frac{m_1-m_2}{1+m_1m_2}=\frac{1}{0}$

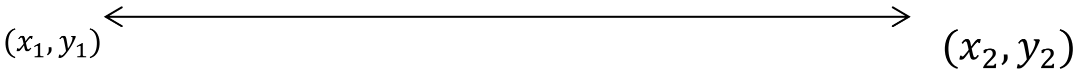
$0=1 + m_1m_2$

$m_1m_2 = -1$ perpendicular

Suppose L_1 & L_2 are two lines with slope m_1 & m_2 respectively then
i) $L_1||L_2 \leftrightarrow m_1 = m_2$ (condition of Parallelism)
ii) $L_1 \perp L_2 \leftrightarrow m_1m_2 = -1 \Rightarrow m_1 = -\frac{1}{m_2}$ (condition of Perpendicularity)

Ex:- if L_1 & L_2 are two lines with slopes 1 & -1 then what you can tell about L_1 & L_2.
$m_1m_2 = 1 * -1 = -1 \Rightarrow L_1 \perp L_2$

Slope of a line which passes through two given points (x_1, y_1) & (x_2, y_2)

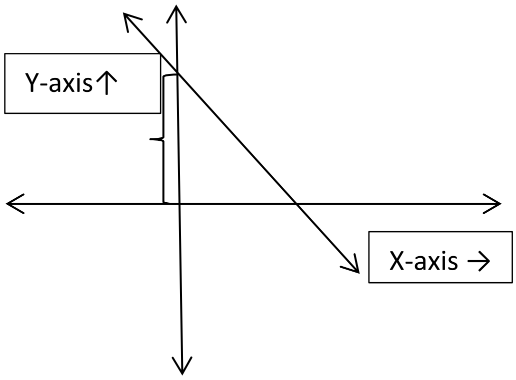


m=$\frac{y_2-y_1}{x_2-x_1}$

Ex:- suppose a line passes through (1,2) and (2,3) then find it's slope.

m=$\frac{y_2-y_1}{x_2-x_1} = \frac{3-2}{2-1}$ = 1/1 =1

it's angle of inclination is 45°



Equation of a straight line:-

1. **Slope intercept form:-**

Equation of a line with slope m and y-intercept as c is **y=mx+c**

m=2 c=4

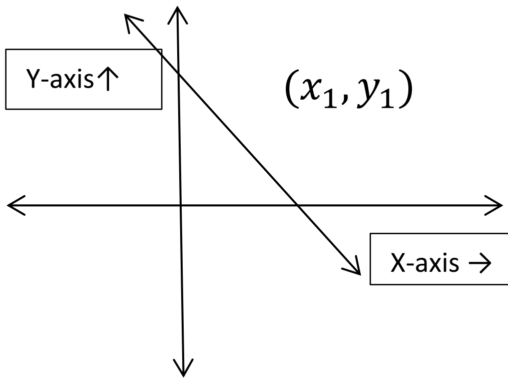
y=2x+4

ex:- find the equation of a line with slope 1 and Y-intercept as 3.

y=x+3

2. **Slope Point form:-**

Equation of a line with slope m and which passes through (x_1, y_1) **is y-$y_1 = m(x - x_1)$**



Ex:- Find the equation of a line with slope 1 and passes through (2,3)

$x_1 = 2, y_1$=3 m=1

y-3=1(x-2)

y-3 =x -2

y-x-1=0

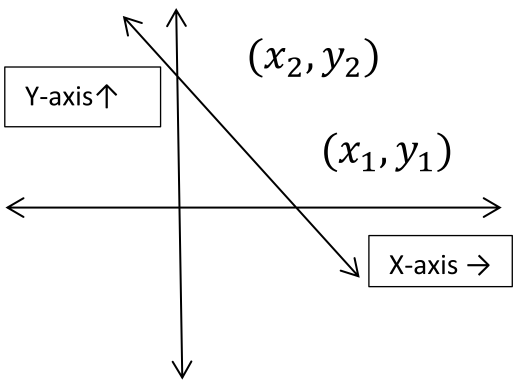
y=x+1

3.

4. **Two Point form**:-

Equation of a line which passes through (x_1, y_1) & (x_2, y_2) is

y-y_1= $\frac{y_2-y_1}{x_2-x_1}$ (x-x_1)



Ex:- Find the equation of a line which passes through (-1,2) and (2,3)

$x_1 = -1 y_1$= 2 $x_2, = 2, y_2$=3

$y-2=\frac{3-2}{2-(-1)}\,(x-(-1))$

$y-2=\frac{3-2}{2+1}\,(x+1)$

$y-2=\frac{1}{3}\,(x+1)$

3(y-2)=x+1

3y-6=x+1

3y-x-7=0

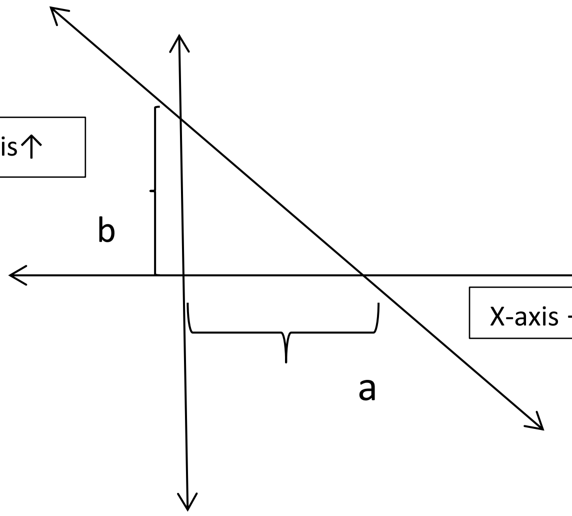
3y=x+7

$Y=\frac{1}{3}\,x+\frac{7}{3}$

4. <u>Intercept form:-</u>

Equation of a line with X-intercept and Y-intercept as a & b respectively is

$\boldsymbol{\frac{x}{a}+\frac{y}{b}=1}$



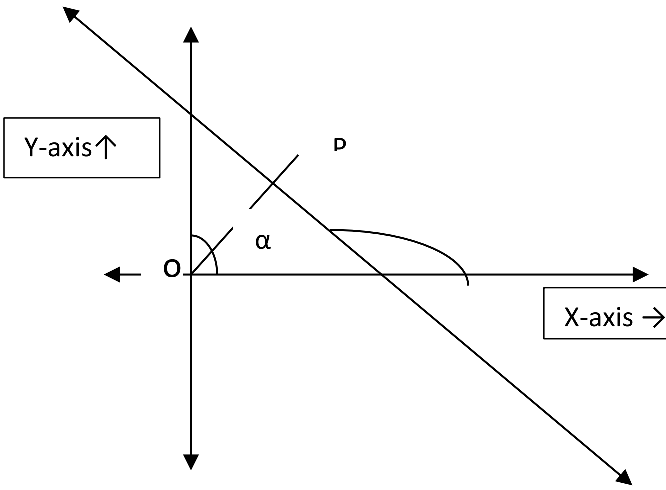
Ex:- find the equation of a line with 2 as X-intercept, 3 as Y-intercept.

Ans- a=2,b=3

$\frac{x}{2}+\frac{y}{3}=1$

3x+2y-6=0

5. <u>Normal (Perpendicular) Form:-</u>



The equation of a line L which is at a distance d from the origin.

x cos α+y sin α=d, d=|OP|

Ex:- find the equation of line which is at a distance 4 from origin and makes an angle 135° with respect to X-axis.

Here α=45°;d=4

Therefore equation of line x cos 45°+ y sin 45°=4

=> $x\,\frac{1}{\sqrt{2}}+y\,\frac{1}{\sqrt{2}}=4 \Rightarrow \frac{1}{\sqrt{2}}(x+y)=4 \Rightarrow x+y=4\sqrt{2}$

Q-reduce $x + \sqrt{3}\, y + 8 = 0$ *to normal form of equation of straight line.*
General form of eq of a line ax+by+c=0.

Normal form x$\cos\alpha + y\, \sin\alpha - p = 0$
We know by comparing these two equation we get

$$\frac{a}{\sqrt{a^2+b^2}}x + \frac{b}{\sqrt{a^2+b^2}} = \frac{-c}{\sqrt{a^2+b^2}}$$

$$x + \sqrt{3}\, y + 8 = 0 \; here\; a = 1\,, b = \sqrt{3}\,, c = 8$$

So equation in normal form will be

$$\frac{1}{\sqrt{1^2+\sqrt{3}^2}}x + \frac{\sqrt{3}}{\sqrt{1^2+\sqrt{3}^2}}y = \frac{-8}{\sqrt{1^2+\sqrt{3}^2}}$$

$$\frac{1}{2}x + \frac{\sqrt{3}}{2}y = -\frac{8}{2}$$

$$\cos\frac{\pi}{3}x + \cos\frac{\pi}{6}y = -4$$

Ex-find the equation of the line with normal of length 4 unit and inclined with an angle 60°

$\propto$=60°, p= 4

x cos$\propto$ +y sin $\propto$ =p

xcos60°+ ysoin 60° =4

x($\frac{1}{2}$)+y($\frac{\sqrt{3}}{2}$)=4

x+$\sqrt{3}\, y = 8$

General form of a Equation of a line

ax+by+c=0
by+c= -ax
by= -ax-c
y= $\frac{-a}{b}\, x - \frac{c}{b}$,**(slope)m=** $\frac{-a}{b}$ **C(y intercept)=** $-\frac{c}{b}$

ex- Find slope and y-intercept of the equation 2x+3y+8=0

2x+3y+8=0
3y= -2x-8
y=$\frac{-2}{3}\, x + \left(-\frac{8}{3}\right)$

m=$\frac{-2}{3}$ $c = \left(-\frac{8}{3}\right)$

Ex-Find angle of inclination of the equation 2x+3y+8=0

2x+3y+8=0

3y= -2x-8

y=$\frac{-2}{3}\ x + \left(-\frac{8}{3}\right)$

m=$\frac{-2}{3}$ tanθ=$\frac{-2}{3}$

θ=$\tan^{-1}(\frac{-2}{3})$

Ex-Find angle of inclination of the equation 2x-2y+8=0

2x-2y+8=0

-2y= -2x-8

y=$\frac{2}{2}\ x + \left(\frac{8}{2}\right)$

y=x+4

m=1, tanθ=1

θ=$\tan^{-1}1$= 45^0

1. **<u>find the equation</u>** of a line which passes through (1,-1) and having inclination 150°

What u think:-

What to do	What is given	Thinking's
Find Equation of a line	Passes through (1,-1)	1. Slope-intercept form 2. Slope-Point form 3. Two point form 4. Intercept form 5. Normal form
	Angle of inclination 150°	Slope = tan θ Slope = tan 150°= -$\frac{1}{\sqrt{3}}$

Find the equation of line passing through (1,-2) and having Y-intercept as 2.

What to do	What is given	Thinking's
Find Equation of a line	Passes through (1,-2)	1. Slope-intercept form 2. Slope-Point form 3. Two point form 4. Intercept form 5. Normal form
	Y-intercept:-2	As it has 2 as Y-intercept. Hence the line passes through point (0,2).

PROBLEM ON THESE CONCEPT DISCUSSED EARLIER

$\boldsymbol{\theta = tan^{-1}(\frac{m_1-m_2}{1+m_1m_2})}$

$\boldsymbol{m_1 = m_2}$ **parallel**

$\boldsymbol{m_1m_2 = -1}$ **perpendicular**

EX- Find angle between two lines x+y+6=0 and 2x-y+3=0

Slope of the line x+y+6=0 is

y=-x-6,

m_1= -1

slope of the line 2x-y+3=0

y=2x+3

m_2= 2

$\theta = tan^{-1}(\frac{m_1-m_2}{1+m_1m_2})$

$\theta = tan^{-1}(\frac{-1-2}{1+(-1).2})$

$\theta = tan^{-1}(\frac{-3}{-1})$

$\theta = tan^{-1}(3)$

Ex- find the line perpendicular to x+y+2=0 whose y-intercept is 3

let the equation of the line perpendicular to x+y+2=0 is

y=mx+c

given c=3

y=mx+3

since two lines are perpendicular then product of their slope is -1

slope of x+y+2=0 is -1

(y= -x-2)

m.(-1)=-1

m=-1/-1 = 1

so equation of the line is y=x+3

x-y+3=0 (ans)

Ex- find the line parallel to x+y+2=0 whose y-intercept is 3

let the equation of the line parallel to x+y+2=0 is

y=mx+c

given c=3

y=mx+3

if two lines are parallel the slope are equal to each other

Then slope of the line x+y+2=0(y= -x-2) is -1

so equation of the line is y=-x+3

x+y-3=0 (ans)

Ex- check whether these two lines are perpendicular or parallel to each other.
x+2y+3=0 ,2x+4y+1=0

slope of the line x+2y+3=0 is $m_1 = \frac{-1}{2}$

2y=-x-3

Y= $-\frac{-1}{2}x - \frac{3}{2}$

slope of the line 2x+4y+1=0 is $m_2 = \frac{-1}{2}$

4y=-2x-1

y=$-\frac{-2}{4}x - \frac{1}{4}$

$m_1 = m_2$

so these two lines are parallel

ex- check whether these two lines are perpendicular or parallel to each other.
x+2y+3=0 ,4x-2y+1=0

slope of the line x+2y+3=0 is $m_1 = \frac{-1}{2}$

2y=-x-3

Y= $-\frac{-1}{2}x - \frac{3}{2}$

slope of the line 4x-2y+1=0 is m_2= 2

2y=4x+1

y= $\frac{4}{2}x + \frac{1}{2}$

$m_1 . m_2 = \frac{-1}{2}. 2 = -1$

so these two lines are perpendicular

$\theta = tan^{-1}(\frac{m_1 - m_2}{1+m_1 m_2})$

$\theta = tan^{-1}(\frac{\frac{-1}{2}-2}{1+\frac{-1}{2}.2})$

$\theta = tan^{-1}(\frac{\frac{-5}{2}}{0})= tan^{-1}(\infty)= 90^o$

<u>Condition for coincidence of lines</u>

.$a_1x+b_1y+c_1$=o

.$a_2x+b_2y+c_2$=o

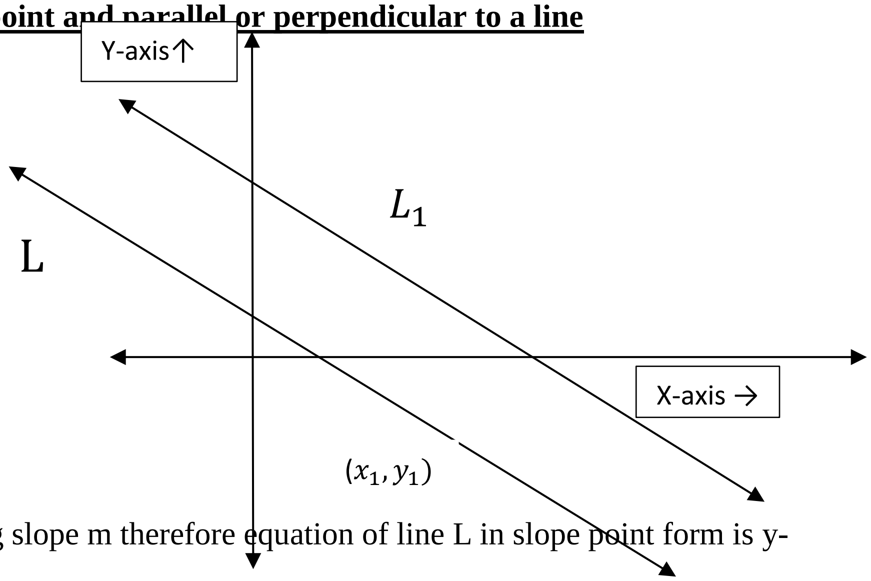
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Ex 2x+3y+6=0
4x+6y+12=0

$\frac{2}{4} = \frac{3}{6} = \frac{6}{12} = \frac{1}{2}$

Equation of a line passing through a point and parallel or perpendicular to a line

Find the equation a line L which passes through (x_1, y_1) and parallel to L_1



The equation of L_1 is y= mx+c

Slope of $L_1 = m$

As L || L_1

Therefore slope of L= m

As L passes through (x_1, y_1) *and* having slope m therefore equation of line L in slope point form is y-y_1= m(x-x_1)

Ex- find equation of a line which passes through a point(4,3) and parallel to the line 2x-3y+5=0

Ans- passing through a poin (x_1,y_1)=(4,3)

Line is parallel to 2x-3y+5=0 so slope will be equal to the slope of the given line.

Slope of 2x-3y+5=0

3y=2x+5

Y=$\frac{2}{3}x + \frac{5}{3}$

m =$\frac{2}{3}$

(2x-3y+k=0

2.4-3.3+k=0

8-9+k=0

k=1, 2x-3y+1=0)

so equation of the line passing through (4,3) and slope $\frac{2}{3}$

y-y_1= m(x-x_1)

is y-3= $\frac{2}{3}$ (x-4)

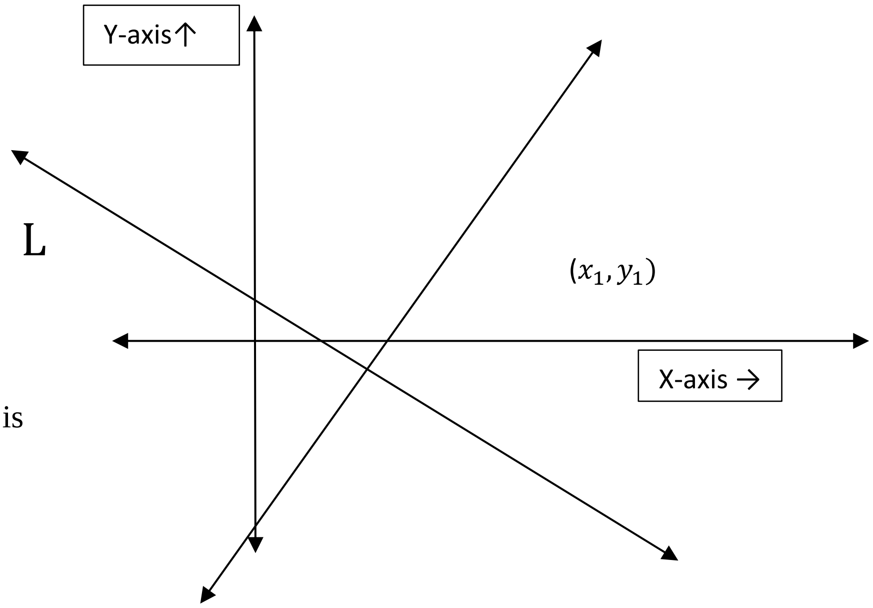
3(y-3)=2(x-4)

3y-9=2x-8

2x-3y+1=0

Equation of line passing through a point and perpendicular to a line:-

Suppose L is a line
Which passes through
(x_1, y_1) and perpendicular
To the line L:y=mx+c
We know that if two
Lines are perpendicular
Then the multiplication of
Their slopes is -1.
Therefore slope of L X slope of L L
is -1.
Slope of L=-1/m
Equation of line passing through
(x_1, y_1) and having -1/m as slope is
y-$y_1 = -\frac{1}{m}(x - x_1)$



Ex- find equation of a line which passes through a point(4,3) and perpendicular to the line 2x-3y+5=0

Ans- passing through a poin (x_1,y_1)=(4,3)
Line is perpendicular to 2x-3y+5=0 so slope will be equal to -1/slope of the given line.
Slope of 2x-3y+5=0

3y=2x+5

Y=$\frac{2}{3}x + \frac{5}{3}$

m =$\frac{2}{3}$ slope of the line perpendicular to it= $\frac{-3}{2}$

so equation of the line passing through (4,3) and slope $\frac{-3}{2}$

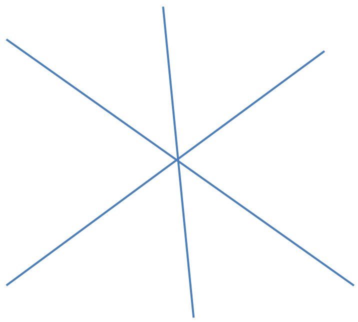
is y-3= $\frac{-3}{2}$(x-4)

2(y-3)=-3(x-4)

2y-6=-3x+12

3x+2y-18=0(ans)eq of the line

Equation of a line passing through intersection of two lines



If a line passing through intersection of two lines L_1 and L_2 (GIVEN)
Then the equation of the line is $L_1 + kL_2=0$

Obtaion equation of the line passing through the intersection of $3x-2y+7=0$, $x+3y+3=0$ and point $(1, -1)$

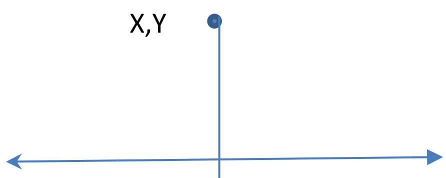
Ans- equation of the line $3x-2y+7+k(x+3y+3)=0$

$$3.1-2(-1)+7+k(1+3(-1)+3)=0$$
$$3+2+7+k(1-3+3)=0$$
$$12+k=0$$
$$K=-12$$

So equation of the line is $3x-2y+7+k(x+3y+3)=0$

$$3x-2y+7-12(x+3y+3)=0$$
$$3x-2y+7-12x-36y-36=0$$
$$-9x-38y+29=0$$
$$9x+38y-29=0 \text{(ans)}$$

Perpendicular Distance of the point from a line



Let d be the distance from the point (x_1,y_1) to the line $ax+by+c=0$

$$d=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

Ex- Find the length of the perpendicular drawn from the point (-5,3) to the line $3x+4y-6=0$

$x_1 = -5$,$y_1= 3$
$a=3$, $b=4$,$c=-6$

d=$\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

d=$\left|\frac{3(-5)+4(3)-6}{\sqrt{3^2+4^2}}\right|$

d=$\left|\frac{-15+12-6}{\sqrt{25}}\right|=\left|\frac{-9}{5}\right|=\frac{9}{5}$ (ans)

If the area of triangle ABC is zero then the points A (x_1, y_1); B (x_2, y_2) & C (x_3, y_3) are collinear.

<u>Condition for coincidence of lines</u>

.$a_1x+b_1y+c_1=o$

.$a_2x+b_2y+c_2=o$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

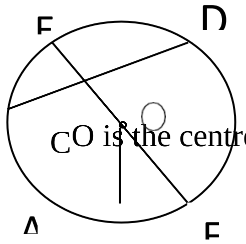
Ex **2x+3y+6=0**

4x+6y+12=0

$\frac{2}{4}=\frac{3}{6}=\frac{6}{12}=\frac{1}{2}$

CIRCLE

Circle:- A circle is the locus of points which are equidistancefrom a fixed point.
Here the fixed point is the centre of the circle and
The fixed distance is called as the radius of the circle.



O is the centre of the circle and OA is

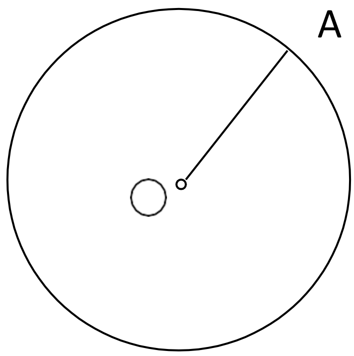
Chord of a circle:- The line segment joining any two pointson the circumference of the circle is called as the chord.

Here in the diagram CD is the chord.

Diameter:-The chord which passes through the centre of the circle is called as thediameter. Here EF is the diameter.

Equation of the circle:-

The equation the circle, whose centre is at (h,k) and theradius is r, is
$(x-h)^2+(y-k)^2=r^2$



If the centre is the origin i.e (0,0) then the equation of theCircle is $x^2+y^2=r^2$

Ex:- find the equation of the circle whose centre is at (2,3)and radius is 4.

Solution:-
Here the centre of the circle is at (2,3)Radius r = 4

Therefore the equation of circle is $(x-2)^2+(y-3)^2=4^2$

$$=> x^2+4-2x+y^2+9-6y=16$$
$$=> x^2+y^2-2x-6y-3=0$$

Ex:- find the equation of the circle whose centre is at (-1,4) and radius is 3

Solution:- Here the centre of the circle is at (-1,4)
Radius r = 3

Therefore the equation of circle is $(x-(-1))^2+(y-4)^2=3^2$

$$=> (x+1)^2+(y-4)^2=3^2$$
$$=> x^2+1+2x+y^2+16-8y=9$$
$$=> x^2+y^2+2x-8y-8=0$$

find the equation of the circle whose centre is at origin and radius is 1H..W

Equation of a circle passing through three points

Example- Find Equation of a circle which passes through (0,0)(1,0)(0,1)?

Ans-
We know equation of a circle
$X^2+y^2+2gx+2fy+c=0$
As circle passes through (0,0)
$0^2+0^2+2g.0+2f.0+c=0$
C=0--------(1)
As circle passes through (1,0)
$1^2+0^2+2g.1+2f.0+c=0$
1+2g+C=0--------(2)
As circle passes through (0,1)

$0^2+1^2+2g.0+2f.1+c=0$

$1+2f+C=0$--------(3)

Now we will solve these three equation

$C=0$--------(1)

$1+2g+C=0$--------(2)

$1+2f+C=0$--------(3)

Putting c= 0 in eq- 2 we get

$1+2g=0$

$2g=-1$

$g=\frac{-1}{2}$

Putting c= 0 in eq- 3 we get

$1+2f=0$

$2f=-1$

$f=\frac{-1}{2}$

we get $g=\frac{-1}{2}, f = \frac{-1}{2}$, c=0

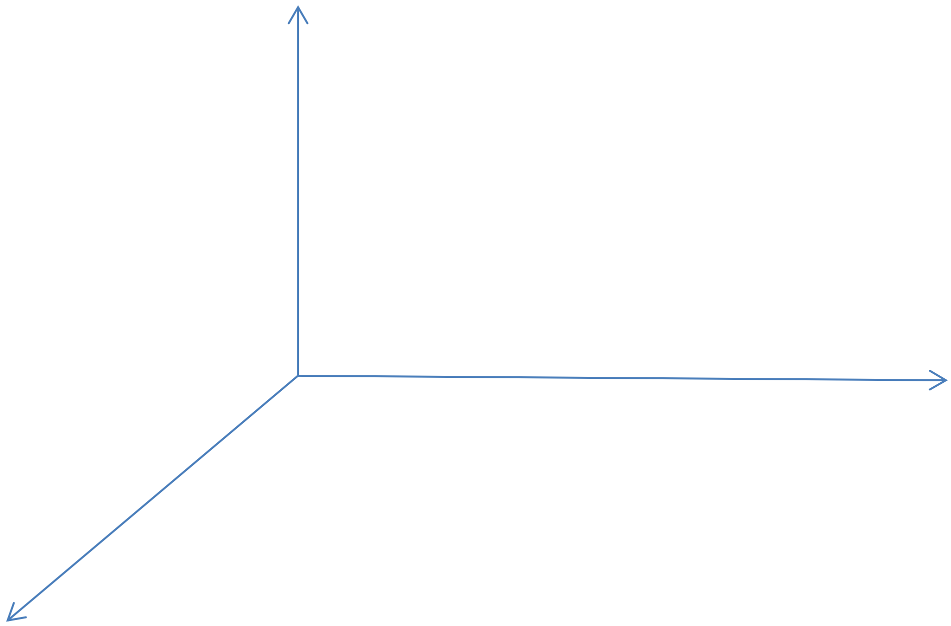
so equation of the circle is

$X^2+y^2+2gx+2fy+c=0$

$X^2+y^2+2.\frac{-1}{2}x+2\frac{-1}{2}y+0=0$

$X^2+y^2-x-y=0$(ans)

Three dimensional geometry



Distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2)

d=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

ex- Find distance between two points (1,2,4) and (-5,6,7)?

Ans- we know distance between two points is

d=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

here (x_1,y_1,z_1)= (1,2,4) (x_2,y_2,z_2)= (-5,6,7)

d=$\sqrt{(-5-1)^2+(6-2)^2+(7-4)^2}$

d=$\sqrt{36+16+9}$=$\sqrt{61}$

Ex- Find distance between two points (1,-2,4) and (-1,3,1)?

Ans- we know distance between two points is

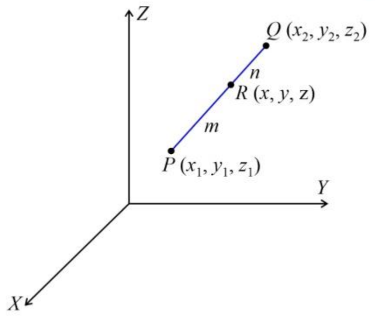
d=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

here (x_1,y_1,z_1)= (1,2,4) (x_2,y_2,z_2)= ()

d=$\sqrt{(-1-1)^2+(3-(-2))^2+(1-4)^2}$

d=$\sqrt{4+25+9}$=$\sqrt{38}$

section formula



if a point (x,y,z) divide the line sement joining two points (x_1,y_1,z_1) and (x_2,y_2,z_2,) in m:n ratio internally

then the point (x,y,z)= $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

if a point (x,y,z) divide the line sement joining two points (x_1,y_1,z_1) and (x_2,y_2,z_2,) in m:n ratio externally

then the point (x,y,z)= $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right)$

Ex- Find the coordinates of the points that divide the line segment joining the points (2,3−2) and (6,32) internally in the ratio 3:1?

Ans-

$(\mathbf{2,3,-2})$ **3:1** $(\mathbf{6,3,2})$

We know if a point (x,y,z) divide the line sement joining two points (x_1,y_1,z_1) and (x_2,y_2,z_2,) in m:n ratio internally

then the point (x,y,z)= $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

here(x_1,y_1,z_1)=(2,3,-2) (x_2,y_2,z_2,)=($\mathbf{6,3,2}$) **m=3 n=1**

(x,y,z)= $\left(\frac{3.6+1.2}{3+1}, \frac{3.3+1.3}{3+1}, \frac{3.2+1.-2}{3+1}\right) = \left(\frac{20}{4}, \frac{12}{4}, \frac{4}{4}\right) = (5,3,1)$

Find the ***ratio in which the point* (9,−11,1) *divides the line segment joining the points* (1,5,−3) *and* (3,1,−2).**

(9,-11,1)K:1

$(\mathbf{1}, \mathbf{5}, -\mathbf{3})$ $(\mathbf{3}, \mathbf{1}, -\mathbf{2})$

We know if a point (x,y,z) divide the line sement joining two points (x_1,y_1,z_1) and (x_2,y_2,z_2,) in m:n ratio internally

then the point (x,y,z)= $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

here(x_1,y_1,z_1)=(1,5,-3) (x_2,y_2,z_2,)=($\mathbf{3}, \mathbf{1}, -\mathbf{2}$) **m=k n=1**

(x,y,z)= $\left(\frac{k.3+1.1}{k+1}, \frac{k.1+1.5}{k+1}, \frac{k.-2+1.-3}{k+1}\right) =$

$$\left(\frac{3k+1}{k+1}, \frac{k+5}{k+1}, \frac{-2k-3}{k+1}\right) = (9, -11, 1)$$

$$\frac{3k+1}{k+1} = 9$$

$$3k + 1 = 9(k + 1)$$

$$3k + 1 = 9k + 9$$

$$6k + 8 = 0$$

$$k = -\frac{8}{6} = -\frac{4}{3}$$

Therefore the point divides the line in a ratio 4:3 externally.

Centroid of a triangle:

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

Mid point formula

Midpoint(x,y,z) of the line segment joining two points (x_1,y_1,z_1) and (x_2,y_2,z_2,) is

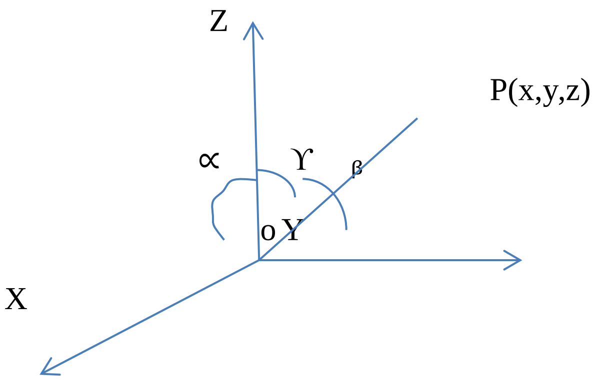
(x,y,z)= $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}, \frac{z_2+z_1}{2}\right)$

***Question- What is the midpoint of the line joining the points* (−3,4,7) *and* (9,0,3)?**

Ans- Midpoint(x,y,z) of the line segment joining two points (x_1,y_1,z_1)=(-3,4,7) and (x_2,y_2,z_2,)=(9,0,3) is

$(x,y,z)= \left(\frac{9+(-3)}{2}, \frac{0+4}{2}, \frac{3+7}{2}\right) = \left(\frac{6}{2}, \frac{4}{2}, \frac{10}{2}\right) = (3,2,5)$

DIRECTION COSINE AND DIRECTION RATIO



l= cos ∝

m= cos β

n= cos ϒ

(l,m,n) is direction cosine of OP.

Let a,b,c be direction ratio of a line and and l,m,n be direction cosine then

$\frac{l}{a}=\frac{m}{b} = \frac{n}{c} = k$

l=ak,m=bk,n=ck

$l^2 + m^2 + n^2$**=1(proof not in syllabus)**

$a^2k^2 + b^2k^2 + c^2k^2=1$

$k^2(a^2 + b^2 + c^2)=1, k=\frac{1}{\sqrt{a^2+b^2+c^2}}$

$l = ak=\frac{a}{\sqrt{a^2+b^2+c^2}}$

$m= bk=\frac{b}{\sqrt{a^2+b^2+c^2}}$

$n= ck=\frac{c}{\sqrt{a^2+b^2+c^2}}$

direction ration of a line joining two points $(x_1,y_1,z_1)(x_2,y_2,z_2)$ is

$(x_2- x_1, y_2- y_1, z_2- z_1)$

EX- IF A LINE has direction ratio 2,-1.-2 . determine its direction cosine

Given a=2,b=-1,c=-2

$l =\frac{a}{\sqrt{a^2+b^2+c^2}}=\frac{2}{\sqrt{2^2+(-1|)^2+(-2)^2}}=\frac{2}{\sqrt{9}}=\frac{2}{3}$

$m =\frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{-1}{\sqrt{2^2+(-1|)^2+(-2)^2}}=\frac{-1}{\sqrt{9}}=\frac{-1}{3}$

$n =\frac{c}{\sqrt{a^2+b^2+c^2}}= \frac{-2}{\sqrt{2^2+(-1|)^2+(-2)^2}} = \frac{-2}{\sqrt{9}}=\frac{-2}{3}$

ex- Find the direction cosines of the line passing through the two points (-2,4,-5) and(1,2,3)?

ans= direction ratio of the line joining the two points (-2,4,-5) and(1,2,3) is(1-(-2), 2-4 ,3- (-5))

= (3,-2,8)

This means (a,b,c)=(3,-2,8)

$\boldsymbol{l} = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{3}{\sqrt{3^2+(-2)^2+(8)^2}} = \frac{3}{\sqrt{77}}$

m $= \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{-2}{\sqrt{3^2+(-2)^2+(8)^2}} = \frac{-2}{\sqrt{77}}$

n $= \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{8}{\sqrt{3^2+(-2)^2+(8)^2}} = \frac{8}{\sqrt{77}}$

Ex- show that the points A(2,3,-4), B(1,-2,3) and C(3,8,-11) are collinear.

Ans- Direction ratios of line joining A and B are

1-2,-2-3, 3-(-4) that is -1,-5,7=$(a_{1,}b_1, c_1)$

Direction ratios of line joining B and C are

3-1,8+2,-11-3 that is 2,10,-14=$(a_{2,}b_2, c_2)$

two lines are collinear if the ration of their direction ration is same

that is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{-1}{2} = \frac{-5}{10} = \frac{7}{-14} = \frac{-1}{2}$

Angle between two lines

If two lines having direction ratio $(a_{1,}b_{1,}c_1)$ ***and*** $(a_2, b_2. c_2)$ **given or direction cosine** $(l_{1,}m_{1,}n_1)$ ***and*** $(l_2, m_2. n_2)$ **given**

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}$$

Condition of parlallelism:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

Condition of perpendicularity

$$a_1.a_2 + b_1b_2 + c_1c_2 = 0$$
$$l_1.l_2 + m_1m_2 + n_1n_2 = 0$$

Ex-find the acute angle between two lines whose direction ratios are 2,3,6 and 1,2,2 respectively.

Ans- here direction ratio is given so angle between two lines is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}$$

Here $a_1, b_1, c_1 = 2,3,6$ $a_2, b_2, c_2 = 1,2,2$

$$cos\theta = \frac{2.1 + 3.2 + 6.2}{\sqrt{2^2 + 3^2 + 6^2}\sqrt{1^2 + 2^2 + 2^2}} = \frac{20}{\sqrt{49}\,\sqrt{9}} = \frac{20}{7.3} = \frac{20}{21}$$

$\theta = cos^{-1}\left(\frac{20}{21}\right)$ (ans)

Ex- Find angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4,}\frac{1}{4},\frac{\sqrt{3}}{2}\right)$ $\left(\frac{\sqrt{3}}{4,}\frac{1}{4},\frac{-\sqrt{3}}{2}\right)$

ans if we know the direction cosines then angle between two lines is

$$\boldsymbol{cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2}$$

Here $l_1, m_1, n_1 = \left(\frac{\sqrt{3}}{4,}\frac{1}{4},\frac{\sqrt{3}}{2}\right)$ $l_2, m_2, n_2 = \left(\frac{\sqrt{3}}{4,}\frac{1}{4},\frac{-\sqrt{3}}{2}\right)$

$\cos\boldsymbol{\theta} = \frac{\sqrt{3}}{4,}.\frac{\sqrt{3}}{4,} + \frac{1}{4}.\frac{1}{4} + \frac{\sqrt{3}}{2}.-\frac{\sqrt{3}}{2} = \frac{3}{16} + \frac{1}{16} - \frac{3}{4} = \frac{3+1-12}{16} = -\frac{8}{16} = -\frac{1}{2}$

$\cos\boldsymbol{\theta} = -\frac{1}{2} = \boldsymbol{cos}\ 120^o$

$\boldsymbol{\theta} = 120^o$**(ans)**

<u>PLANE</u>

Equation of a plane

ax+by+cz+d=0

equation of xy plane z=0

equation of yz plane x=0

equation of zx plane y=0

<u>equation of a plane through three non collinear points</u>

suppose three non collinear points are (x_1,y_1,z_1), (x_2,y_2,z_2) (x_3,y_3,z_3)

<u>then equation of the plane</u>

$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix}$=0

<u>Equation of plane in intercept form</u>

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

ex- find the equation of the plane with intercepts 2,-1,5 on x,y,and z axis respectively

ans= we know equation of a plane having intercept a,b,c is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\frac{x}{2} + \frac{y}{-1} + \frac{z}{5} = 1$$

$$5x-10y+2z-10=0$$

Transformation of general equation of a plane to normal form

ax+by+cz+d=0

normal form lx+my+nz-p=0

$$\frac{l}{a}=\frac{m}{b}=\frac{n}{c}=-\frac{p}{d}$$

$\frac{a}{\sqrt{a^2+b^2+c^2}}$x+$\frac{b}{\sqrt{a^2+b^2+c^2}}$y+$\frac{c}{\sqrt{a^2+b^2+c^2}}$z=p

P=$\frac{d}{\sqrt{a^2+b^2+c^2}}$

ex- find direction cosine of the normal to the plane x-y+1=0

direction ratio (a,b,c) is (1,-1,0)

direction cosines are l=$\frac{a}{\sqrt{a^2+b^2+c^2}}=\frac{1}{\sqrt{1^2+(-1)^2+0^2}}=\frac{1}{\sqrt{2}}$

m=$\frac{b}{\sqrt{a^2+b^2+c^2}}=\frac{-1}{\sqrt{1^2+(-1)^2+0^2}}=\frac{-1}{\sqrt{2}}$

n=$\frac{c}{\sqrt{a^2+b^2+c^2}}=\frac{0}{\sqrt{1^2+(-1)^2+0^2}}=0$

Angle between two planes

$$cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Ex- find angle between the two planes 2x+2y-3z=5 and 3x-3y+5z=3

Ans= here $(a_1,b_1,c_1)=(2,2,-3)(a_2,b_2,c_2)=(3,-3,5)$

We know $cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}=\frac{2.3+2.-3+(-3).5}{\sqrt{2^2+2^2+(-3)^2}\sqrt{3^2+(-3)^2+5^2}}$

$$= \frac{6-6-15}{\sqrt{17}\sqrt{43}} = -\frac{15}{\sqrt{731}}$$

$$\theta = \cos^{-1} - \frac{15}{\sqrt{731}}$$

Distance of a point from a plane

Distance from a point$(x_1, y_1, z_1) to\ a\ line\ ax + by + cz + d = 0$

d=$\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

Ex- find the distance from the point (2,-3,-1) to the plane 2x-3y+6z+7=0.

Ans-we know Distance from a point$(x_1, y_1, z_1) to\ a\ line\ ax + by + cz + d = 0$

d=$\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

here $(x_1, y_1, z_1) =$ (2,-3,-1) a=2,b=-3,c=6

d=$\left|\frac{2.2+(-3)(-3)+6.(-1)+7}{\sqrt{2^2+(-3)^2+6^2}}\right| = \left|\frac{4+9-6+7}{\sqrt{4+9+36}}\right| = \frac{14}{7} = 2$

Equation of a plane parallel to another plane and passing through a point

If equation of a plane ax+by+cz+d=0 then equation of plane parallel to this and pass through the point (x_1, y_1, z_1) is

$ax + by + cz + k = 0$-----eq-1

$ax_1 + by_1 + cz_1 + k = 0$

K=? and put it in eq 1

Ex- Find the equation of the plane which passes through the point (1,-1,4) and is parallel to the plane 2x-3y+7z-11=0

Ans : the equation of plane parallel to 2x-3y+7z-11=0 is

2x-3y+7z+k=0-------(1)

But it is given that the plane passes through (1,-1,4)

So 2.1-3(-1)+7.4+k=0

2+3+28+k=0

K=-33

Now we will put k=-33 in eq-----(1)

2x-3y+7z-33=0

Therefore equation of the plane parallel to 2x-3y+7z-11=0 and passes through(1,-1,4) is2x-3y+7z-33=0

Equation of a plane perpendicular to another plane and passing through a point

If equation of a plane ax+by+cz+d=0 then equation of plane perpendicular to this and pass through the point (x_1, y_1, z_1) is

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

Ex- Find the equation of the plane which passes through (4,-2,1) and is perpendicular to the line whose direction ratio are 7,2,-3

Ans-

Equation of the plane passes through (4,-2,1) is

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

$$a(x-4)+b(y-(-2))+c(z-1)=0$$

Here a=7,b=2,c=-3

$$7(x-4)+2\big(y-(-2)\big)+(-3)(z-1)=0$$

$$7x-28+2y+4-3z+3=0$$

$$7x+2y-3z-21=0$$

Therefore $7x+2y-3z-21=0$ is the required equation of the plane.

Plane through the intersection of two given plane

If equation of the given plane are

$$P_1: a_1x+b_1y+c_1z+d_1=0$$

$$P_2: a_2x+b_2y+c_2z+d_2=0$$

Then the new plane will be

$P_1+kP_2=0$, k is a constant.

Ex- Find the equation of the plane which is perpendicular to the plane 5x+3y+6z+8=0 and contains the line of intersection of the planes x+2y+3z-4=0 &2x+y-z+5=0.

Ans- equation of the plane through the line of the intersection of the planes x+2y+3z-4=0 &2x+y-z+5=0. is

(x+2y+3z-4)+k(2x+y-z+5)=0

(1+2k)x+(2+k)y+(3-k)z+(5k-4)=0---------eq-1

Since the plane is perpendicular to 5x+3y+6z+8=0

So (1+2k)5+(2+k)3+(3-k)6=0 (by using perpendicular condition)

5+10k+6+3k+18-6k=0

7k+29=0

K=$\frac{-29}{7}$

By putting k=$\frac{-29}{7}$ in eq----1 we get

(1+2. $\frac{-29}{7}$)x+(2+($\frac{-29}{7}$))y+(3-($\frac{-29}{7}$))z+(5($\frac{-29}{7}$)-4)=0

$$\left(1-\frac{58}{7}\right)x+\left(2-\frac{29}{7}\right)y+\left(3+\frac{29}{7}\right)z+5\left(-\frac{29}{7}\right)-4=0$$

$$51x+15y-50z+173=0$$

Therefore $51x+15y-50z+173=0$ $is\ requried\ equation\ of\ the\ plane.$

SPHERE

THE LOCOUS OF ALL THE POINT IN A SPACE WHICH ARE EQUIDISTANCE FROM A FIXED POINT IS KNOWN AS A SPHERE.

The fixed point is known as centre of the sphere and the constant distance is known as radius of the sphere.

Equation of a sphere whose centre at origin

$$(x-0)^2+(y-0)^2+(z-0)^2=r^2$$

$$x^2+y^2+z^2=r^2$$

General equation of a sphere

Equation of a sphere having centre at (a,b,c) *is* $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

Where center of the sphere is (-$u,-v,-w)$ *and radius* $r=\sqrt{u^2+v^2+w^2-d}$

Equation of a sphere having co-ordinates of end point of diameter (x_1,y_1,z_1) *and* (x_2,y_2,z_2)

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$$

Ex- Find the centre and radius of a sphere
$\mathbf{4x^2+4y^2+4z^2-16x-24z+3=0}$

Ans= We know general equation of a sphere is

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$-----eq-1

Given equation of a sphere is

$4x^2+4y^2+4z^2-16x-24z+3=0$

$x^2+y^2+z^2-4x-6z+\frac{3}{4}=0$-----eq 2

$x^2+y^2+z^2+2ux+2vy+2wz+d=0$-----eq-1

By copmparing eq-1 and eq-2

$2u=-4 \quad 2v=0, \quad 2w=-6 \quad d=\frac{3}{4}$

$u=-2, \quad v=0, \quad w=-3, \quad d=\frac{3}{4}$

Centre of the sphere is $(-u,-v,-w)=(2,0,3)$

Radius of the sphere is $r=\sqrt{u^2+v^2+w^2-d}$

$r=\sqrt{u^2+v^2+w^2-d}$

$$r=\sqrt{(-2)^2+0^2+(-3)^2-\frac{3}{4}}=\sqrt{4+9-\frac{3}{4}}=\sqrt{\frac{49}{4}}=\frac{7}{2}$$

Hence the centre is (2,0,3) and radius is 7/2 .

ex- Find the centre and radius of a sphere
$\boldsymbol{x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0}$

We know general equation of a sphere is
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$-----eq-1
Given equation of a sphere is
$x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0$-------eq-2
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$-----eq-1
By copmparing eq-1 and eq-2
$2u = -2 \quad 2v = -4, \quad 2w = -6 \quad d = -11$
$u = -1, \quad v = -2, \quad w = -3, \quad d = -11$
Centre of the sphere is $(-u, -v, -w) = (1,2,3)$
Radius of the sphere is $r = \sqrt{u^2 + v^2 + w^2 - d}$
$r = \sqrt{u^2 + v^2 + w^2 - d}$
$r = \sqrt{(-1)^2 + (-2)^2 + (-3)^2 - (-11)} \quad = \sqrt{1 + 4 + 9 + 11} \quad = \sqrt{25} \quad = 5$
Hence the centre is $(1,2,3)$and radius is 5.

Ex- Find equation of the sphere on the join of (2,3,5) and (4,9,-3) as diameter.

Equation of a sphere having co-ordinates of end point of diameter (x_1, y_1, z_1) *and* (x_2, y_2, z_2)

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + (z - z_1)(z - z_2) = 0$$

$(x_1, y_1, z_1) = (\mathbf{2}, \mathbf{3}, \mathbf{5})(x_2, y_2, z_2) = (\mathbf{4}, \mathbf{9}, -\mathbf{3})$
Equation of the sphere is

$$(x - 2)(x - 4) + (y - 3)(y - 9) + (z - 5)(z - (-3)) = 0$$

$$x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$$

hence $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$ is the required equation of the sphere.

<u>EQUATION OF A SPHERE PASSING THROUGH FOUR POINTS</u>

EX- Find equation of a sphere which passes through the points (0,0,0), (1,0,0), (0,1,0) and (0,0,1)
Ans-
Let equation of the sphere be
$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$-------1
Since (0,0,0) lies on eq-1
So $0^2 + 0^2 + 0^2 + 2u0 + 2v0 + 2w0 + d = 0$
$d = 0$-----2
Since (0,1,0) lies on eq-1
$0^2 + 1^2 + 0^2 + 2u.0 + 2v.1 + 2w.0 + d = 0$
1+$2v + d = 0$-------eq3
Since (0,0,1) lies on eq-1
$0^2 + 0^2 + 1^2 + 2u.0 + 2v.0 + 2w.1 + d = 0$

$1+2w+d=0$-------eq4

Since (1,0,0) lies on eq-1

$1^2+0^2+0^2+2u.1+2v.0+2w.0+d=0$

$1+2u+d=0$-------eq5

$d=0$-----2

$1+2v+d=0$-------eq3

$1+2w+d=0$-------eq4

$1+2u+d=0$-------eq5

By solving eq-2 to eq-4 we will get

$$d=0, u=-\frac{1}{2}, v=-\frac{1}{2}, w=-\frac{1}{2}$$

Now we get the equation of sphere by putting value of u,v,w and d in equation 1

$$x^2+y^2+z^2+2ux+2vy+2wz+d=0$$

$$x^2+y^2+z^2+2\left(-\frac{1}{2}\right)x+2\left(-\frac{1}{2}\right)y+2\left(-\frac{1}{2}\right)z+0=0$$

$$x^2+y^2+z^2-x-y-z=0$$

EX-- Find equation of a sphere which passes through the points (1,0,0), (0,1,0) and (0,0,1) AND WHOSE CENTRE LIES ON THE PLANE 3X-Y+Z=2

3(-u)-(-v)+(-w)=0

-3u+v-w=0

$1+2w+d=0$

$1+2u+d=0$

$1+2v+d=0$

Ex- Find the Equation of the sphere whose centre is on the point (1,2,3) and which touches the plane 3x+2y+z+4=0

Ans=

Since the sphere touches the plane 3x+2y+z+4=0

Its radius = length of perpendicular from its centre (1,2,3) to the plane 3x+2y+z+4=0

d=$\left|\frac{3.1+2.2+1.3+4}{\sqrt{3^2+2^2+1^2}}\right|=\frac{14}{\sqrt{14}}=\sqrt{14}$

the required equation of the sphere is $(x-1)^2+(y-2)^2+(z-3)^2=\sqrt{14}^2$

$$(x-1)^2+(y-2)^2+(z-3)^2=14$$

$$x^2+y^2+z^2-2x-4y-6z=0$$